Blow Up and Exponential Growth to a Petrovsky Equation with Degenerate Damping

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Article Info

Abstract

This paper deals with the initial boundary value problem of Petrovsky type equation with degenerate damping. Under some appropriate conditions, we study the finite time blow up and exponential growth of solutions with negative initial energy.

1. Introduction

We investigate the following initial boundary value problem:

$$
\begin{cases}
u_{tt} + \Delta^2 u - \int_0^t \mu(t-s)\Delta^2 u(s)\,ds + |u|^{\upsilon}\, \partial j(u_t) = |u|^{\gamma-1} u & \text{in } \Omega \times (0,+\infty), \\
u(x,0) = u_0(x), \ u_t(x,0) = u_1(x) & \text{on } x \in \Omega, \\
u(x,t) = \frac{\partial}{\partial n} u(x,t) = 0 & \text{on } x \in \partial\Omega,
\end{cases}
\tag{1.1}
$$

where $\partial j(s)$ denotes the sub-differential of $j(s)$ [1], n is the outer normal and Ω is a bounded domain in R^n with a smooth boundary $\partial\Omega$. The Petrovsky type equations are orginated from the study of beams and plates and so often arise in many branches of physics such as optics, nuclear physics, geophysics and ocean acoustics. Rivera et al. [2] considered

$$
u_{tt} + \Delta^2 u - \int_0^t \mu(t-s)\Delta^2 u(s)\,ds - \gamma\Delta u_{tt} = 0,
$$

and proved the asymptotic behaviour of solution with the initial and dynamical boundary conditions.
The following problem was studied by Alabau-Boussouira et al. [3]

$$
u_{tt} + \Delta^2 u - \int_0^t \mu(t-s)\Delta^2 u(s)\,ds = f(u).
\tag{1.2}
$$

The authors studied exponential and polynomial decay results of solutions when the memory μ decay exponentially and polynomially, respectively. Afterwards, Tahamtani ve Shahrouzi [4] investigated the existence of weak solutions for problem (1.2). In addition, the authors proved blow up of solutions with positive and negative initial energy in finite time.
In [5], Li and Gao discussed the following equation

$$
u_{tt} + \Delta^2 u - \int_0^t \mu(t-s)\Delta^2 u(s)\,ds + |u_t|^{p-2} u_t = |u|^{\gamma-2} u.
\tag{1.3}
$$

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The authors studied blow up result of solutions under suitable conditions of the initial datum and the relaxation function. Furthermore, problem (1.3) has been studied by Liu et al. [6] and the finite time blow-up of solutions with arbitrary high initial energy has been proved. Recently, Liu et al. [7] investigated problem (1.3) with case ($p = 2$) and proved blow up of solution with $E(0) \leq M$, M is positive constant. Furthermore, the authors studied blow up of solutions with $E(0) > M$ by applying concavity method.

On the other hand, Messaoudi [8] investigated the following problem

$$u_{tt} + \Delta^2 u + |u_t|^{p-2} u_t = |u|^{\gamma-2} u. \tag{1.4}$$

The author studied an existence result and global solution in case $p \geq \gamma$. Then, blow-up of solutions with negative initial energy and $p < \gamma$ was proved. Then, Chen and Zhou [9] discussed blow up with positive initial energy for (1.4) and showed that the solution blows up in finite time for vanishing initial energy case ($p = 2$). Moreover, the problem (1.4) with Δu_t term has been considered by Pişkin and Polat [10] and the authors proved decay estimates of the solution by using Nakao's inequality. Some other studies on Petrovsky equations are [11], [12], [13], [14].

The hyperbolic models with degenerate damping also are of much interest in material science and physics. It particularly appears in physics when the friction is modulated by the strains. There are a lot of studies that has degenerate damping terms, namely $\delta(u)h(u_t)$ here $\delta(u)$ is a positive function and h is nonlinear, (see [15–21]).

Motivated by previous results, we prove several results concerning the blow up and exponential growth of solution for the problem (1.1). It should be noted here that we can say that the study is both quite difficult and interesting and the analysis are more subtle because of the degenerate damping.

The remaining part of this paper is organized as follows: In the next section, we study the nonexistence of solutions. The exponential growth result is presented in Section 3.

2. Preliminaries

Now, we present some preliminary material which will be helpful in the proof of our results. Throughout this paper, we denote the standart $L^2(\Omega)$ norm by $\|.\| = \|.\|_{L^2(\Omega)}$ and $L^q(\Omega)$ norm $\|.\|_q = \|.\|_{L^q(\Omega)}$.

(**A1**) $\upsilon, p \geq 0$, $\gamma > 1$; $\upsilon \leq \frac{n}{n-2}$, $\gamma + 1 \leq \frac{2n}{n-2}$ if $n \geq 3$. There exist positive constants c, c_0, c_1 such that for all $s, k \in R$ $j(s): R \to R$ be a C^1 convex real function satisfies
- $j(s) \geq c|s|^{p+1}$,
- $j'(s)$ is single valued and $|j'(s)| \leq c_0|s|^p$,
- $(j'(s) - j'(k))(s - k) \geq c_1|s - k|^{p+1}$.

(**A2**) $u_0(x) \in H_0^2(\Omega)$, $u_1(x) \in L^2(\Omega)$.

(**A3**) Assume $\mu(\tau): R^+ \to R^+$ satisfies

$$\mu(\tau) \geq 0, \ \mu'(\tau) \leq 0,$$

for all $s \in R^+$ and

$$\int_0^t \mu(\tau) d\tau < 1.$$

(**A4**) $\int_0^t \mu(\tau) d\tau < \frac{\gamma - 1}{\gamma + 1}$.

We use the following notations

$$l = 1 - \int_0^t \mu(\tau) d\tau,$$

$$(\mu \diamond \theta)(t) = \int_0^t \mu(t - \tau) \int_\Omega |\theta(t) - \theta(\tau)| \, dx d\tau.$$

The said solution of (1.1) satisfies the energy identity

$$E(t) + \frac{1}{2} \mu(t) \|\Delta u\|^2 - \frac{1}{2} (\mu' \diamond \Delta u)(t) + \int_0^t \int_\Omega |u(\tau)|^\upsilon j(u_t)(\tau) \, dx d\tau = E(0), \tag{2.1}$$

where

$$E(t) = \frac{1}{2} \left[\|u_t\|^2 + \left(1 - \int_0^t \mu(s) \, ds\right) \|\Delta u\|^2 + (\mu \diamond \Delta u)(t) \right] - \frac{1}{\gamma + 1} \|u\|_{\gamma+1}^{\gamma+1} \tag{2.2}$$

and

$$E(0) = \frac{1}{2} \left[\|u_1\|^2 + \|\Delta u_0\|^2 \right] - \frac{1}{\gamma + 1} \|u_0\|_{\gamma+1}^{\gamma+1}. \tag{2.3}$$

Moreover, by computation, we get

$$E(t) \leq E(0). \tag{2.4}$$

3. Blow up

In this section, we shall prove the blow up results of the solutions for problem (1.1).

Theorem 3.1. *Let (A1)-(A4) hold. Assume further that $\gamma > \upsilon + p$, $E(0) < 0$ and u be a any solution to (1.1) on the interval $[0,T]$, then T is necessarily finite, i.e. u can't be continued for all $t > 0$.*

Proof. We assume that the solution exists for all time and we arrive to a contradiction. Set

$$H(t) = -E(t). \tag{3.1}$$

By using (2.1), we get

$$
\begin{aligned}
H'(t) &= -E'(t) \\
&= \frac{1}{2}\mu(t)\|\Delta u\|^2 - \frac{1}{2}\left(\mu' \diamond \Delta u\right)(t) + \int_\Omega |u(t)|^\upsilon j(u_t) u_t dx \\
&\geq \int_\Omega |u(t)|^\upsilon j(u_t) u_t dx \\
&\geq c_0 \int_\Omega |u(t)|^\upsilon |u_t|^{p+1} dx.
\end{aligned}
\tag{3.2}
$$

Hence, we find

$$0 < H(0) \leq H(t) \leq \frac{1}{\gamma+1}\|u\|_{\gamma+1}^{\gamma+1}, \quad t \geq 0. \tag{3.3}$$

Define

$$K(t) = H^{1-\rho}(t) + \varepsilon \int_\Omega u u_t dx,$$

where $\rho = \min\left\{\frac{\gamma-p-\upsilon}{p(\gamma+1)}, \frac{\gamma-1}{2(\gamma+1)}\right\}$ and ε is a positive constant.
Taking the derivative of $K(t)$ and using Eq.(1.1), we get

$$
\begin{aligned}
K'(t) &= (1-\rho)H^{-\rho}(t)H'(t) + \varepsilon\|u_t\|^2 - \varepsilon\|\Delta u\|^2 + \varepsilon\int_0^t \mu(t-s)\int_\Omega \Delta u(s)\Delta u(t)dxds \\
&\quad - \varepsilon\int_\Omega |u(t)|^\upsilon u(t)\partial j(u_t)(t)dx + \varepsilon\|u\|_{\gamma+1}^{\gamma+1} \\
&= (1-\rho)H^{-\rho}(t)H'(t) + \varepsilon\|u_t\|^2 - \varepsilon\|\Delta u\|^2 + \varepsilon\int_0^t \mu(s)ds\|\Delta u\|^2 + \varepsilon\int_0^t \mu(t-s)\int_\Omega \Delta u(t)(\Delta u(s)-\Delta u(t))dxds \\
&\quad - \varepsilon\int_\Omega |u(t)|^\upsilon u(t)\partial j(u_t)(t)dx + \varepsilon\|u\|_{\gamma+1}^{\gamma+1}.
\end{aligned}
\tag{3.4}
$$

By applying Young's inequality to estimate the fifth term of (3.4) as follows

$$\left|\int_0^t \mu(t-s)\int_\Omega \Delta u(t)(\Delta u(s)-\Delta u(t))dxds\right| \leq \int_0^t \mu(s)ds\|\Delta u\|^2 + \frac{1}{4}(\mu \diamond \Delta u)(t). \tag{3.5}$$

From $(A3)$, since $0 < l \leq 1$. Then it follows from the definition of $H(t)$ that

$$-\|\Delta u\|^2 = \frac{2}{l}H(t) + \frac{1}{l}\|u_t\|^2 + \frac{1}{l}(\mu \diamond \Delta u)(t) - \frac{2}{l(\gamma+1)}\|u\|_{\gamma+1}^{\gamma+1}. \tag{3.6}$$

Combining (3.4)-(3.6), we obtain

$$
\begin{aligned}
K'(t) &\geq (1-\rho)H^{-\rho}(t)H'(t) + \varepsilon\left(1+\frac{1}{l}\right)\|u_t\|^2 + \frac{2}{l}H(t) + \left(\frac{1}{l}-\frac{1}{4}\right)(\mu \diamond \Delta u)(t) \\
&\quad - \varepsilon\int_\Omega |u(t)|^\upsilon u(t)\partial j(u_t)(t)dx + \varepsilon\left(1-\frac{2}{l(\gamma+1)}\right)\|u\|_{\gamma+1}^{\gamma+1}.
\end{aligned}
\tag{3.7}
$$

By assumption $\int_0^t \mu(\tau)d\tau < \frac{\gamma-1}{\gamma+1}$, we have $1 - \frac{2}{l(\gamma+1)} > 0$.
In order to estimate fifth term in (3.7), since $q > \upsilon + p$, from assumption $(A1)$ and thanks to Holder's inequality and Young's inequality, we obtain

$$
\begin{aligned}
\left|\int_\Omega |u(t)|^\upsilon u(t)\partial j(u_t)(t)dx\right| &\leq \int_\Omega |u(t)|^{\upsilon+1-\frac{\upsilon+p+1}{p+1}}|u(t)|^{\frac{\upsilon+p+1}{p+1}}|u_t(t)|^p dx \\
&\leq C_0\left(\int_\Omega |u(t)|^\upsilon |u_t(t)|^{p+1}dx\right)^{\frac{p}{p+1}}\left(\int_\Omega |u(t)|^{\upsilon+p+1}dx\right)^{\frac{1}{p+1}} \\
&\leq C_0|\Omega|^{\frac{\gamma-\upsilon-p}{\gamma+1}}\left(\int_\Omega |u(t)|^\upsilon |u_t(t)|^{p+1}dx\right)^{\frac{p}{p+1}}\|u(t)\|_{\gamma+1}^{\frac{\upsilon+p+1}{p+1}} \\
&\leq \beta\left(H'(t)\right)^{\frac{p}{p+1}}\|u(t)\|_{\gamma+1}^{\frac{\upsilon+p+1}{p+1}} \\
&\leq \beta\left(\delta^{-\frac{1}{p}}H'(t) + \delta\|u(t)\|_{\gamma+1}^{\upsilon+p+1}\right),
\end{aligned}
\tag{3.8}
$$

where constant $\delta > 0$ is specified later and $\beta = C_0 C_1^{-\frac{p}{p+1}} |\Omega|^{\frac{\gamma - \upsilon - p}{\gamma + 1}}$.

Hence, (3.7) becomes

$$K'(t) \geq \left[(1-\rho) H^{-\rho}(t) - \varepsilon \beta \delta^{-\frac{1}{p}} \right] H'(t) + \varepsilon \left(1 + \frac{1}{l} \right) \|u_t\|^2 + \varepsilon \frac{2}{l} H(t) + \varepsilon \left(\frac{1}{l} - \frac{1}{4} \right) (\mu \diamond \Delta u)(t)$$

$$+ \varepsilon \left(1 - \frac{2}{l(\gamma+1)} \right) \|u\|_{\gamma+1}^{\gamma+1} - \varepsilon \beta \delta \|u(t)\|_{\gamma+1}^{\upsilon+p+1}. \tag{3.9}$$

The choice of δ $\left(\text{i.e. } \delta = \frac{1}{\beta} \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right) \|u\|_{\gamma+1}^{\gamma-\upsilon-p} \right)$, then

$$\varepsilon \beta \delta \|u(t)\|_{\gamma+1}^{\gamma+p+1} = \varepsilon \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right) \|u\|_{\gamma+1}^{\gamma+1}.$$

Furthermore, since $\|u\|_{\gamma+1} \geq [(\gamma+1) H(0)]^{\frac{1}{\gamma+1}}$ by (3.3) and $\upsilon + p - \gamma + p(\gamma+1)\rho \leq 0$, then

$$(1-\rho) H^{-\rho}(t) - \varepsilon \beta \delta^{-\frac{1}{p}} = H^{-\rho}(t) \left[1 - \rho - \varepsilon \beta \delta^{-\frac{1}{p}} H^{\rho}(t) \right]$$

$$\geq H^{-\rho}(t) \left[1 - \rho - \varepsilon \beta^{1+\frac{1}{p}} \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right)^{-\frac{1}{p}} (\gamma+1)^{-\rho} \|u\|_{\gamma+1}^{\frac{p+\upsilon-\gamma+p(\gamma+1)\rho}{p}} \right]$$

$$\geq H^{-\rho}(t) \left[1 - \rho - \varepsilon \beta^{1+\frac{1}{p}} \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right)^{-\frac{1}{p}} (\gamma+1)^{-\rho - \frac{q-p-\upsilon}{p(q+1)}} H(0)_{\gamma+1}^{\rho - \frac{\gamma-\upsilon-p}{p(\gamma+1)}} \right]$$

$$\geq H^{-\rho}(t) \left[1 - \rho - \varepsilon \beta^{1+\frac{1}{p}} \chi \right], \tag{3.10}$$

where $\chi = \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right)^{-\frac{1}{p}} (\gamma+1)^{\rho - \frac{q-p-\upsilon}{p(q+1)}} H(0)_{\gamma+1}^{\rho - \frac{\gamma-\upsilon-p}{p(\gamma+1)}}$. Now, we choose ε to be sufficiently small such that

$$1 - \rho - \varepsilon \beta^{1+\frac{1}{p}} \chi > 0.$$

Then (3.10) and (3.9) yield

$$K'(t) \geq \varepsilon C \left[H(t) + \|u_t(t)\|^2 + \|u\|_{\gamma+1}^{\gamma+1} + (\mu \diamond \Delta u)(t) \right], \tag{3.11}$$

where $C > 0$ is a constant that does not depended on ε. Especially, (3.11) means that $K(t)$ is increasing on $[0,T)$, with

$$K(t) = H^{1-\rho}(t) + \varepsilon \int_{\Omega} u u_t \, dx \geq H^{1-\rho}(0) + \varepsilon \int_{\Omega} u_0 u_1 \, dx.$$

We also select ε to be sufficiently small such that $K(0) > 0$, thus $K(t) \geq K(0) > 0$ for $t \geq 0$.

Let $\eta = \frac{1}{1-\rho}$. Since $0 < \rho < \frac{1}{2}$, it is evident that $2 > \eta > 1$. By using the following inequality

$$|x+y|^{\eta} \leq 2^{\eta-1} \left(|x|^{\eta} + |y|^{\eta} \right) \quad \text{for } \eta \geq 1,$$

applying Young's inequality, we get

$$K^{\eta}(t) \leq 2^{\eta-1} \left(H(t) + \varepsilon \|u(t)\|^{\eta} \|u_t(t)\|^{\eta} \right)$$

$$\leq C \left(H(t) + \|u_t(t)\|^2 + \|u(t)\|_{\gamma+1}^{\frac{1}{2}-\rho} \right). \tag{3.12}$$

By the choice of ρ, we have $\frac{1}{2} - \rho > \frac{1}{\gamma+1}$. Now applying the inequality

$$a^{\sigma} \leq \left(1 + \frac{1}{b} \right)(b+a), \quad a \geq 0, \ 0 \leq \sigma \leq 1, \ b > 0,$$

and taking $a = \|u(t)\|_{\gamma+1}^{\gamma+1}$, $\eta = \frac{1}{(\frac{1}{2}-\rho)(\gamma+1)} < 1$, and $b = H(0)$, we obtain

$$\|u(t)\|_{\gamma+1}^{\frac{1}{\frac{1}{2}-\rho}} \leq \left(1 + \frac{1}{H(0)} \right) \left(H(0) + \|u(t)\|_{\gamma+1}^{\gamma+1} \right)$$

$$\leq C \left(H(t) + \|u(t)\|_{\gamma+1}^{\gamma+1} \right). \tag{3.13}$$

Therefore, by combining of (3.12) and (3.13), we obatin

$$K^{\eta}(t) \leq C \left(H(t) + \|u_t(t)\|^2 + \|u(t)\|_{\gamma+1}^{\gamma+1} \right)$$

$$\leq C \left(H(t) + \|u_t(t)\|^2 + \|u(t)\|_{\gamma+1}^{\gamma+1} + (\mu \diamond \Delta u)(t) \right). \tag{3.14}$$

Thus, (3.11) and (3.14) arrive at

$$K'(t) \geq C K^{\eta}(t), \quad t \in [0,T]. \tag{3.15}$$

In the end, from (3.15) and $\eta = \frac{1}{1-\rho} > 1$, we see that $K(t) = H^{1-\rho}(t) + \varepsilon \int_{\Omega} u u_t \, dx$ blow up in finite time. This completes the proof. $\qquad \square$

4. Growth

In this section, we goal to show that the energy grow up as an exponential function as time as goes to infinity.

Theorem 4.1. *Let (A1)-(A4) hold. Assume further that $\gamma > \upsilon + p$ and $E(0) < 0$ and u be a any solution to (1.1) grows exponentially.*

Proof. We define

$$Z(t) = H(t) + \varepsilon \int_{\Omega} u u_t \, dx, \tag{4.1}$$

where $H(t) = -E(t)$ and $0 < \varepsilon \le 1$. By derivating (4.1) and using Eq.(1.1), we have

$$
\begin{aligned}
Z'(t) &= H'(t) + \varepsilon \|u_t\|^2 - \varepsilon \|\Delta u\|^2 + \varepsilon \int_0^t \mu(t-s) \int_{\Omega} \Delta u(s) \Delta u(t) \, dx \, ds - \varepsilon \int_{\Omega} |u(t)|^{\upsilon} u(t) \partial j(u_t)(t) \, dx + \varepsilon \|u\|_{\gamma+1}^{\gamma+1} \\
&= H'(t) + \varepsilon \|u_t\|^2 - \varepsilon \|\Delta u\|^2 + \varepsilon \int_0^t \mu(s) \, ds \, \|\Delta u\|^2 + \varepsilon \int_0^t \mu(t-s) \int_{\Omega} \Delta u(t) \left(\Delta u(s) - \Delta u(t) \right) dx \, ds \\
&\quad - \varepsilon \int_{\Omega} |u(t)|^{\upsilon} u(t) \partial j(u_t)(t) \, dx + \varepsilon \|u\|_{\gamma+1}^{\gamma+1}.
\end{aligned}
\tag{4.2}
$$

By using (3.5), the assumption (A3) and the definition $H(t)$, we have $0 < l \le 1$ and

$$Z'(t) \ge H'(t) + \varepsilon \left(1 + \frac{1}{l} \right) \|u_t\|^2 + \frac{2}{l} H(t) + \left(\frac{1}{l} - \frac{1}{4} \right) (\mu \diamond \Delta u)(t) - \varepsilon \int_{\Omega} |u(t)|^{\upsilon} u(t) \partial j(u_t)(t) \, dx + \varepsilon \left(1 - \frac{2}{l(\gamma+1)} \right) \|u\|_{\gamma+1}^{\gamma+1}. \tag{4.3}$$

By the assumption $\int_0^t \mu(\tau) \, d\tau < \frac{\gamma-1}{\gamma+1}$ and using (3.8), we get

$$
\begin{aligned}
Z'(t) &\ge \left[1 - \varepsilon \beta \delta^{-\frac{1}{p}} \right] H'(t) + \varepsilon \left(1 + \frac{1}{l} \right) \|u_t\|^2 + \varepsilon \frac{2}{l} H(t) + \varepsilon \left(\frac{1}{l} - \frac{1}{4} \right) (\mu \diamond \Delta u)(t) \\
&\quad + \varepsilon \left(1 - \frac{2}{l(\gamma+1)} \right) \|u\|_{\gamma+1}^{\gamma+1} - \varepsilon \beta \delta \|u(t)\|_{\gamma+1}^{\upsilon+p+1}.
\end{aligned}
\tag{4.4}
$$

The choice of δ $\left(\text{i.e. } \delta = \frac{1}{\beta} \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right) \|u\|_{\gamma+1}^{\gamma-\upsilon-p} \right)$, then

$$\varepsilon \beta \delta \|u(t)\|_{\gamma+1}^{\upsilon+p+1} = \varepsilon \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right) \|u\|_{\gamma+1}^{\gamma+1}.$$

Furthermore, since $\|u\|_{\gamma+1} \ge \left[(\gamma+1) H(0) \right]^{\frac{1}{\gamma+1}}$ by (3.3) and assumption $\upsilon + p - \gamma \le 0$, then

$$
\begin{aligned}
1 - \varepsilon \beta \delta^{-\frac{1}{p}} &\ge 1 - \varepsilon \beta^{1+\frac{1}{p}} \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right)^{-\frac{1}{p}} (\gamma+1)^{-\frac{\gamma-p-\upsilon}{p(\gamma+1)}} H(0)_{\gamma+1}^{-\frac{\gamma-\upsilon-p}{p(\gamma+1)}} \\
&\ge 1 - \varepsilon \beta^{1+\frac{1}{p}} P,
\end{aligned}
$$

where $P = \left(\frac{1}{2} - \frac{1}{l(\gamma+1)} \right)^{-\frac{1}{p}} (\gamma+1)^{-\frac{\gamma-p-\upsilon}{p(\gamma+1)}} H(0)_{\gamma+1}^{-\frac{\gamma-\upsilon-p}{p(\gamma+1)}}$. Now, we choose ε to be sufficiently small such that

$$1 - \varepsilon \beta^{1+\frac{1}{p}} P > 0.$$

Thus,

$$Z'(t) \ge \varepsilon C \left[H(t) + \|u_t(t)\|^2 + \|u\|_{\gamma+1}^{\gamma+1} + (\mu \diamond \Delta u)(t) \right] \tag{4.5}$$

where $C > 0$ is a constant that does not depended on ε.

Now, applying Young's inequality, and Sobolev Poincare inequality we have

$$
\begin{aligned}
Z(t) &\le H(t) + \varepsilon \|u\| \|u_t\| \\
&\le C \left(H(t) + \|u_t\|^2 + \|u\|^2 \right).
\end{aligned}
$$

Now, in order the estimate $\|u\|^2$ term we apply the inequality $a^l \le (a+1) \le (1 + \frac{1}{b})(a+b)$ for $a = \|u\|_{\gamma+1}^{\gamma+1}$, $l = 2/\gamma+1 < 1$, $b = H(0)$, we have

$$
\begin{aligned}
\|u\|^2 &\le C \|u\|_{\gamma+1}^2 \\
&= C \left(\|u\|_{\gamma+1}^{\gamma+1} \right)^{\frac{2}{\gamma+1}} \\
&\le \left(1 + \frac{1}{H(0)} \right) \left(\|u\|_{\gamma+1}^{\gamma+1} + H(0) \right) \\
&\le C \left(\|u\|_{\gamma+1}^{\gamma+1} + H(t) \right).
\end{aligned}
\tag{4.6}
$$

Thus,

$$Z(t) \leq C \left[H(t) + \|u_t(t)\|^2 + \|u\|_{\gamma+1}^{\gamma+1} + (\mu \diamond \Delta u)(t) \right]. \tag{4.7}$$

Therefore, (4.5) and (4.7) arrive at

$$Z'(t) \geq \xi Z(t), \quad t \geq 0$$

This completes the proof. $\qquad\square$

5. Conclusion

In this work, we obtained the finite time bolw up and growth of solutions for a Petrovsky equation with degenerate damping in a bounded domain. This improves and extends many results in the literature.

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Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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