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PEER PROBLEM SOLVING AS AN INSTRUCTIONAL STRATEGY TO ENHANCE MATHEMATICAL DISCOURSE IN 6TH GRADE MATHEMATICS[1]

AKRAN GRUPLARINDA PROBLEM ÇÖZMENİN 6. SINIF MATEMATİK DERSİNDE MATEMATİKSEL TARTIŞMAYI GELİŞTİRME AMAÇLI BİR ÖĞRETİM YÖNTEMİ OLARAK KULLANILMASI

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ABSTRACT: Recent studies on mathematics education focus on improving higher order thinking skills instead of merely attaining and using knowledge. Deep understanding of mathematics requires to engage in the processes of mathematical thinking. Defining and solving problems, discovering patterns, making conjectures, inferences and justifying one's own thinking are among those mathematical processes. Using mathematical talk, discussion and discourse to promote new goals of mathematics education have been widely researched in the last decades. For these reasons, creating a classroom atmosphere which provides a fruitful communication is an important area for research in mathematics education. The purpose of the study was to implement and investigate an instructional strategy to enhance mathematical discourse among students. Students' approaches to mathematical discourse as a learning tool is also examined. Qualitative methodology was used to investigate the effectiveness of group problem solving as an instructional strategy in fostering mathematical discourse. A two hour lesson plan was developed and implemented to 20 students attending 6th grade. Students filled open-ended question forms before and after implementation. Researchers also made observations and took field notes during the implementation. Content analysis was used to analyze data. Findings of the study indicated that group problem solving is an effective way to foster mathematical discourse in the classroom. Students pointed out that they enjoy and learned from mathematical talk they engaged in this group activity. Observation and field notes also indicated if the instruction presents a problem to solve through groupwork students need to engage in mathematical talk.

Keywords: mathematical communication, mathemtical discussion, mathemtical discourse, peer learning, problem solving, mathematics education.

ÖZET: Matematik eğitimi üzerine yapılan son çalışmalarda bilginin edinilmesi ve kullanılmasından çok öğrencilerde düşünme becerilerinin geliştirilmesine odaklanılmaktadır. Anlamlı matematik öğrenimi matematiksel düşünme süreçleri içerisine girmeyi gerektirmektedir. Problemi tanımlama ve çözme, örüntüleri keşfetme, tahminlerde ve çıkarımlarda bulunma ile kendi fikir ve düşünme süreçlerini savunma matematiksel düşünme süreçleri kapsamına girmektedir. Matematik eğitiminin söz konusu amaçlarına ulaşmak için matematiksel konuşma ve tartışma kavramları üzerinde yapılmış bir çok araştırma bulunmaktadır. Buradan hareketle sınıf ortamında verimli bir matematiksel tartışma deneyiminin oluşturulması matematik eğitimi alanında önemli bir çalışma konusu olarak ön plana çıkmaktadır. Çalışmanın amacı sınıf ortamında matematiksel tartışmayı geliştirmek için bir öğretim yöntemi uygulamak ve etkilerini incelemektir. Öğrencilerin bir matematik öğrenme yöntemi olarak matematiksel tartışma hakkında neler düşündükleri de çalışma kapsamında ele alınmıştır. Akran gruplarında problem çözmenin matematiksel tartışma ortamını geliştirme üzerindeki etkisini incelemek üzere nitel araştırma deseni kullanılmıştır. Bunun için iki saatlik bir ders planı geliştirilmiş ve 6.sınıfa devam eden 20 öğrenciye uygulanmıştır. Öğrenciler uygulama öncesi ve sonrasında açık uçlu sorulardan oluşan anketleri doldurmuşlardır. Araştırmacılar uygulamayı gözlemleyerek saha notları tutmuşlardır. Verilerin

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çözümlenmesi için içerik analizi kullanılmıştır. Bulgular akran gruplarında problem çözmenin matematiksel tartışma ortamı oluşturmada etkili olduğunu işaret etmektedir. Öğrenciler grup çalışması sırasında gerçekleştirdikleri matematiksel tartışmadan öğrendiklerini ve keyif aldıklarını dile getirmişlerdir. Yapılan gözlemler ile de öğrenciler için tasarlanan grup çalışmalarının onları matematiksel konuşmaya teşvik ettiği görülmüştür.

Anahtar Kelimeler: matematiksel iletişim, matematiksel tartışma, matematiksel konuşma, akran öğrenimi, problem çözme

INTRODUCTION

Recent studies which have been investigating on mathematics education reveal that it is not adequate for students to learn and use only procedural and declarative knowledge (Kostos & Shin, 2010; Lynch & Bolyard, 2012). Recent research focuses on improving higher order thinking skills instead of merely attaining and using knowledge. For instance, several researchers, including Kilpatrick, Swafford and Findell (2001), indicated that mathematics education refers to conceptual understanding, strategic competence, adaptive reasoning, productive dispositions and procedural fluency. In other words, learning mathematics needs to address problem solving, showing and expressing ideas, recognizing patterns and transferring learned skills to original situations (Trafton & Claus, 1994). Romberg and Kaput (1999), on the other hand, emphasized mathematical expression, reasoning and generalization as the objectives of mathematical education. National Research Council (2001 cited from Walshaw & Anthony, 2008) defines mathematical proficiency as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. From these points on, it can be inferred that in mathematics education, there is a shift from doing mere calculations and applying procedural knowledge, towards developing students' higher order thinking skills.

Although the goals of mathematics education are widely accepted by scholars and educators, the issue of creating classroom environments to reach those goals still needs an effort to elaborate on (Hiebert et al. , 2005; Pape, Bell & Yetkin, 2003). Deep understanding of mathematics requires engaging in the processes of mathematical thinking. Defining and solving problems, discovering patterns, making conjectures, inferences and justifying one's own thinking are among those mathematical processes (Stein, Grover & Henningsen, 1996). To engage students in this kind of activities there is a need for active learning environments, where students are seen as communities spending effort on mathematical sense making (Schoenfeld, 1992).

Using mathematical talk, discussion and discourse to promote new goals of mathematics education have been widely researched in the last decades. The theoretical background of these strategies is socio-cultural models in learning. According to socio-cultural perspective on learning knowledge, skills and dispositions are developed through social interactions with more skilled others (Bandura, 1969). Vygotsky (1978), also argues that knowledge is socially constructed and mediated by language. Mathematics inquiry is also described as an apprenticeship model where mathematical thinking skills are developed within reflective classroom communication (Cobb, Boufi, McClain & Whitenack, 1997). Various forms of communication are considered central to explore and deepen students' understanding of mathematical ideas, and make connections between other concepts of mathematics and other fields of knowledge (Hiebert, 1992). Facilitating student talk on mathematical problems, concepts and procedures enhance students' understanding so that they can make deeper and clearer connections (Chapin, O'Connor & Anderson, 2003). Furthermore, Sfard (2001) pointed out the importance of mathematical communication by describing thinking as a case of communication. In other words, thinking is a dialogical effort, where one asks questions, investigate possible solutions and reflect upon them.

Fostering mathematical talk in classroom is widely accepted to enhance students' higher order thinking processes and mathematical discussion, explanation and defense of ideas are seen as the essential features of quality mathematics instruction (Walshaw & Anthony, 2008). In a similar manner, peer discussions are also employed for improving conceptual understanding (Brown & Pallincsar, 1989; National Council of Teachers of Mathematics, 1989; Resnick, 1990).

There are a number of forms of discussion that can be implemented in mathematics classrooms. However, all forms of mathematical discussion involve talking about one's thinking publicly in one way or another (Jansen, 2006). For example, Yackel, Cobb & Wood (1991), used small group problem solving to create an environment, where students can talk and discuss about their ideas. In their study they found that this strategy provides students with opportunities to learn, verbalize their thinking, explain or justify their solutions, and ask for clarifications. Classroom discourse also facilitates students' access to mathematical ideas and task interpretations (Lampert, 1990). Research provides evidence that mathematical discourse, when used consistently over periods of time, provides students with opportunities for effective learning by presenting an appropriate level of challenge and increasing students' sense of control. Mathematical discourse also helps students gain positive dispositions towards mathematics (Walshaw & Anthony, 2008).

Unfortunately not all classroom activities including some form of discussion support processes that leads to deep understanding. Stein, Grover & Henningsen (1996) pointed out that a considerable amount of designed mathematical tasks were not implemented as appropriately to produce desired outcomes. Challenges become non-problems, inappropriateness of task for students, too much or too little time, lack of accountability and classroom management problems are among the factors causing a decline in the quality of the task. On the other hand, task builds on student prior knowledge, appropriate amount of time, high-level performance modeled, sustained pressure for explanation and meaning and teacher draws conceptual connections were found to be factors that support mathematical tasks produce outcomes like deep and conceptual understanding. To summarize, it is necessary to spend an effort on tasks that really involves sophisticated thinking and participation processes. According to McNair (2000), a mathematical discussion must have a mathematical subject and a mathematical purpose. He argues that if the mathematical talk is not as high as expected it is because of the content of the discussion not because of the students low level of skills or knowledge. Discussions that provide confusion and conflict help student achieve conceptual understanding in mathematics. Because, 'holding shared meanings is essential to the development of successful human interactions' (Wood, 1999, p. 174).

Teachers' careful planning of classroom discussions is very important for effective discussions. Establishing social norms for discussion and participation, as well as, designing tasks suitable for students' characteristics are very crucial to achieve desired outcomes. To foster students' mathematical thinking process, teachers have to listen attentively to the mathematics in students' reasoning and ideas. This will also encourage students further elaboration and understanding of the concepts (Walshaw & Anthony, 2008). It is important that teachers encourage their students to give reasons for their ideas. This will shift the attention from only finding answers toward understanding, explaining and justifying procedures (McNair, 2000). For fruitful classroom discussions students' motivations to participate is another important factor. To involve all students in the discussion process teachers have to take into account social concerns and beliefs of their students (Jansen, 2006). In other words, it is necessary that teachers use various strategies to increase their students' motivation to participate in class discussions. In the discussion process teachers' role is to model ways to negotiate, practice skills such as reaching agreement and help students to stay on mathematically productive paths (Moschkovich, 1996).

In this vein, the purpose of the study was to implement and explore an instructional strategy to enhance mathematical discourse among students. Students' approaches to mathematical discourse as a learning tool are also examined.

METHOD

2.1. Research Design

The study used qualitative methodology to investigate the effectiveness of group problem solving as an instructional strategy in fostering mathematical discourse. According to Creswell (2012), qualitative methodology is suitable for addressing research problems in which researchers do not know exact variables affecting the phenomenon. Therefore, qualitative methodology includes deeper exploration techniques to infer variables framing the central phenomenon. The purpose of the study was to reach a deep understanding about the effectiveness of peer problem solving strategies on mathematical discourse in classroom settings.

To address the purpose of the study a two hour lesson plan was developed by the researchers and implemented to twenty 6th grade students. The subject of the lesson plan was the relationship between perimeter and the area of quadrilaterals. The perimeter and area of quadrilaterals is a subject that most students are fluent on. However, many students do not have a deeper understanding on the meaning of perimeter, area and how the calculation formulas were derived. Therefore the subject was found to be suitable to initiate fruitful discussions.

The basic instructional strategy was peer problem solving followed by a whole class discussion. Introductory whole class discussion involved teacher led questions to warm up the students to discuss on mathematical contexts. In other words the purpose of this initial whole class discussion was to stimulate and encourage students to talk about mathematics. After the whole class discussion students started to work in pairs on a worksheet involving questions and activities that were designed to encourage pairs to talk, discuss and come up with a common solution. To increase the effectiveness of the discussion among the peers, pairs were formed heterogeneously before the classroom session regarding the mathematical development.

By all preparations to implementation researchers worked with the classroom teacher collaboratively. In other words, in all phases of the implementation part of the research the classroom teacher had a major role and gave his opinion and consent on lesson plan, instructional strategy, materials and formation of pairs.

The lesson plan was implemented by the classroom teacher. The main flow of the lesson was as follows:

- Initial discussion questions about students' existing knowledge of area and perimeter.
- A short individual work on calculating the area of an irregular shape such as their hand.
- Main activity of pair problem solving: Students start to work on the worksheet using unit squares. The items of the worksheet are presented below:
    - What is the area of maximum land you can construct with 24 unit squares? Why? Can the shapes of the land be different although they have the same area?
    - With the same amount of unit squares can you construct land with different areas? Why or why not?
    - Construct a land with the unit squares and calculate the area and the perimeter. Compare your measurements with another pair. Discuss about what is the relationship between area and perimeter? When the perimeter of a quadrilateral changes does the area also change? How it is possible that quadrilaterals with different perimeters have the same area?
- Discussion about these follow up questions:
    - Can we calculate the perimeter of a rectangle by knowing its area? Why or why not?
    - Can we calculate the area of a rectangle by knowing its perimeter? Why or why not?

Researchers observed the whole class session and took field notes. The teacher was actively involved with the students during peer work. He moved from one group to the next, observing and intervening when necessary.

2.2. Participants

Participants of the study were 20 sixth grade students attending a private school in Istanbul, the largest metropolitan city of Turkey.

2.3. Data Collection

The data was mainly collected through open-ended questionnaires that students filled before and after implementation. Researchers also made observations and took field notes during the implementation. The open-ended questionnaires involved questions about what students think about the nature of learning mathematics, learning mathematics through discussion and how they perceived the implemented lesson plan. By the observation the researchers took on a role of a participant observer where "they take part in activities in the settings they observe" (Creswell, 2012). In other words, researchers carefully observed the classroom session made interventions when necessary. Field notes during the observation provided a detailed outline of and interesting anecdotes about the session.

2.4. Data Analysis

Content analysis was used to analyze data gathered from open-ended questionnaires and field notes. The analysis of data was done by hand. For the analysis, steps of (1) exploring the general sense of data, (2) coding the data and (3) specifying the themes were followed (Creswell, 2012).

FINDINGS AND RESULTS

In this section results of the data gathered from open-ended questionnaires before and after implementation are presented with the support of field notes of the researchers.

With the first two items of the questionnaire before the implementation researchers' purpose was to explore how students describe the way they learn mathematics and how they understand that they learned a mathematical concept. Most of the students (n=12) indicated that they learn mathematics by paying attention to teachers' explanation of the topic and by practicing after that. One of the students said "If the teacher does not explain the topic well, it is not possible to learn". Another student emphasizing the importance of hard work and practice to learn mathematics indicated that "Mathematics is learned by working on tests and studying hard". These are expected results considering the main understanding of teaching and learning mathematics in Turkey. Although mathematics curriculum embraced a constructivist approach, tests are still an important of mathematics education. Therefore, it is not a surprise that students think mathematics is learned better with practice, solving problems and working on many tests. However, there are also students (n=5) who think mathematics learning should involve hands-on activities to be more effective. One student underlined this issue by saying "mathematics cannot be learned by sitting". Two of the students indicated that they learn mathematics better when they ask teacher questions and participate in classroom discussions. These results point out that most of the students does not think or have an experience of mathematical discussion as an effective way of learning mathematics. On the other hand, in the second question a considerable number of students (n=7) indicated that they evaluate their understanding of topics by explaining the subject to another person. For example one student said "I think I understand the topic well, that I can explain the concepts when my mother asks what I have learned in school." This opinion of students points out that although they usually do not think mathematical communication/discussion as an effective way of learning they find communication of their learning to other people as an important to evaluate their understanding of the topics. Rest of the students (n=12) evaluates their learning by solving problems correctly and getting high grades in exams. This is also not surprising regarding the answers they gave to first question.

In the third item of the questionnaire researchers sought answers for what students think about participating in classroom discussions. A considerable number of students (n=7) find discussion methods beneficial for learning as they provide opportunities to discover their weaknesses. Students also think by discussing about mathematical concepts they give and receive help from their peers. One student presented his opinion as “when I participate in classroom discussions I become aware of my weaknesses and I catch a chance to fix them”. Similarly, another student indicated that “to learn best I have to comment on the concepts we learn.” One student also pointed out the importance of participation to classroom discussion as “I understand the problems better, when we talk about them”. Two of the students find discussions necessary to assert their opinion regardless of its truth. One of those students said “I talk about my ideas even if they are not true”. The other student indicated that by participating “I contribute to the solution of the problem”. Summarizing these answers one can conclude that students find participation in classroom discussions necessary and beneficial for learning mathematics. Although students did not indicate participation of discussions as a major mathematics learning strategy these results indicate that they are aware of its contributions to the learning process.

In the second questionnaire which students filled after the implementation researchers’ purpose was to explore how students perceived the overall instructional process. When they were asked about the distinctive aspect of the instruction the majority of the students underlined the activity part of the instruction. In other words they perceived the peer problem solving section as an activity on which they found a chance to participate in discussions. These findings are in line with the purpose of the research which was to explore the effectiveness of an instructional strategy to enhance mathematical discourse in the classroom setting. Students moreover indicated that they found an opportunity for deeper learning of the concepts. For example one of them said “With the activities I saw the details of the topic”. These comments also provide evidence about the effectiveness of the instructional strategy. Researchers also asked students whether they experienced any difficulties by working in pairs. Almost all of the students reported that they did not have such difficulties. However, researchers observed in some pairs only one student answered the questions on the worksheet or had a more dominant role. This conflict among students’ reports and researchers’ observations can be because of the fact that students do not have enough experience to evaluate the quality of their group working process.

CONCLUSIONS AND RECCOMENDATIONS

From the findings of the study it was found that peer problem solving is a promising instructional strategy to support effective classroom communication. Students feel the need to talk when they were presented a problem or conflict to solve with their peers. These findings are also supported by the literature. Yackel, Cobb & Wood (1991) used small group problem solving as major instructional strategy to enhance mathematical communication in classroom and they concluded that this strategy poses a learning opportunity for students. By small group problem solving students are encouraged to talk to each other because they are presented with a conflict and have to reach a common solution (Wood, 1999).

Another aspect that the results of the study indicate that student’ beliefs about learning mathematics is an important issue when implementing a relatively new instructional strategy. In this study students mostly believe that mathematics is learned by listening to teacher and practicing. In line with this they assess their performance mostly with the tests they work on. However when asked about their participation preferences, they usually indicate that they like to participate in class discussions and report the benefits of participation. To sum up, although they are aware of the benefits of participating to classroom discussions they do not perceive discussion as a major learning tool yet. In Jansen’s (2006) study the relationship between beliefs about mathematics and participation preferences is also emphasized.

A prominent result of the study is that students like the hands-on activity part, which is integrated with peer problem solving, most. This fact points out that a hands-on, active learning section integrated with mathematical communication strategies is appreciated more by the students.

Therefore, a careful instructional planning is a need to create a classroom atmosphere that involves an effective mathematical discourse. Chapin, O'Connor & Anderson (2003) also underlines the importance of planning for good mathematical communication.

For further studies, firstly, studying on varying and larger sample groups is recommended. This study covers a relatively small sample of students in a private school. Further studies can be done on larger class sizes and different types of schools. Studying the nature of mathematical discourse in other grades will also contribute to the literature. Moreover, research on different instructional strategies to create contexts for mathematical discussion is important to actualize the many positive aspects of mathematical discussion in classroom settings.

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