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Journal of Military and Information Science
Institute of Science

ICOVACS 2015, International Conference on Value Chain Sustainability

A Heuristic Approach for a Shelf Space Allocation Problem

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Abstract- A shelf space allocation problem (SSAP) is a special form of multi-constraint knapsack problem. The main difference between a knapsack problem and a SSAP is that a knapsack problem has only capacity constraints. Commercial space management systems use many different heuristic approaches for allocating shelf space due to the NP-hard complexity of the SSAP. These heuristics are usually based on simple intuitive rules that could be easily used in practice to implement shelf space allocation decisions. This paper develops a new heuristic to obtain a good allocation of shelf space for different products in order to increase profitability under different constraints such as limited shelf space and elasticity factors.

Keywords- Evolutionary algorithms; heuristic methods; shelf space allocation problem.

1. Introduction

The knapsack problem is a combinatorial optimization problem. Given a set of items, each with a mass and a value, the goal is to determine how many of items to include in collection so that the total weight can be compared with a given capacity and the total value is as high as possible. Different knapsack problems exist in combinatorics, applied mathematics, and optimization.

The knapsack problem has been studied since 1897. Tobias Dantzig was the first researcher that referred to this problem. Dantzig suggested the name that is obtained in myths before a mathematical problem is fully defined. For all knapsack problems, capable reduction algorithms have been proposed that permit one to fix numerous decision variables for objective functions (Pisinger, 1995). There are many variations of knapsack problems, such as multi-objective knapsack problem, multidimensional knapsack problem, quadratic knapsack problems, and subset sum problem.

The multidimensional knapsack problem is similar to the bin packing problem. In this problem, a subset of items can be chosen to be placed in a bin so that all the items have to be packed in the minimum number of bins. The concept is that the items have multiple dimensions. This variation is apparent in many problems in scheduling/loading in Operations Research (OR) and polynomial-time approximation schemes.

Due to the NP-hard computational complexity of bin packing problems, optimal solutions to the large size instances cannot be obtained. To solve these problems, new heuristics have been developed such as first fit algorithm, tabu search algorithm, and genetic algorithm. There are certain similarities between bin packing problems and parallel machine scheduling problems where the objective is the minimization of the make span.

The shelf space allocation problem (SSAP) was then also viewed as a two-dimensional packing problem which was studied by different researchers. Gilmore and Gomory proposed the first model for two-dimensional packing problems, by modifying their column generation approach for one-dimensional packing problems (Gilmore and Gomory, 1961). Beasley also worked on bin packing problems and formulated an Integer Linear Programming (ILP) model for two-dimensional packing problems (Beasley, 1985). Hadjiconstantinou and Christofides developed a similar model for this kind of problem (Hadjiconstantinou and Christofides, 1995). Fekete and Schepers studied a new bin packing problem based on graph theory (Fekete and Schepers, 1998), and Lodi et.al. worked on a special kind of problem where the products are packed by levels (Lodi et. al., 2002). Michael and Moffitt showed that item sizes and the capacity of bins span a vector of values, requiring that a feasible or an optimal assignment of the items must satisfy the capacity constraints in all dimensions (Michael and Moffitt, 2013).

In the logistics field, shelf space is one of the main resources to attract more consumers. Effective shelf space management can reduce the inventory levels, and increase wholesaler relationship and customer satisfaction (Fancher, 1991). Shelf space is one of the major resources in retail environment (Hwang et al., 2009). Therefore, the current shelf space management decision is an essential issue in retail operation management.

SSAP is a kind of multi-constraint knapsack problem. SSAP with some policy constraints is not necessary in a knapsack problem. The main assumption of a knapsack problem is that there are one or more dominant resources in the problem such as budget or capacity. Commercial space management systems use many different heuristic approaches for allocating shelf space due to the NP-hard complexity of the SSAP. These heuristics are usually based on simple intuitive rules that could be easily used in practice to obtain shelf space allocation decisions (Zufryden, 1986). The concern for practicability and simplicity for these approaches is on the performance of the space allocation decisions. According to technological growth, the development of optimization approaches to solve SSAP has reached a feasible solution to space management systems stage (Yang, 2001).

In retail store, SSAP is used as a decision problem to attain the best possibly objective using operational constraints. The commercial space management systems generally use relatively simple intuitive rules to develop operating procedures designed easily to make decisions of shelf space allocation in practice (Yang and Chen, 1999). Space allocation affects store profitability through both the demand function considering main and cross space elasticities together, and through the cost function (procurement, carrying and out-of-stock costs) (Corstjens and Doyle, 1981) . Previous studies, usually focused on a limited number of brands and only a few shelves (Dreze et al., 1994).

Hwang et al. proposed a shelf space design mathematical model and item allocation problem to maximize the retailer’s profit. Space elasticity on demand and location effect are included in their model. They developed a Genetic Algorithm to solve the problem (Hwang et al., 2009).

In this study, a model is proposed as an aggregated optimization model for shelf space allocation. A modified integer programming model was developed to increase the applicability in practice. The objective is to determine the best allocation of the product items to the most suitable shelf space in order to maximize objective function adding space elasticity factor. A Simulated Annealing (SA) algorithm is proposed to allocate items to shelf space, subject to given constraints.

2. Literature Survey

Carvalho studied the formulation of arc flow including side constraints for one dimensional bin packing problems. A branch and price procedure that unifies overdue variable generation and branch and bound are used for the proposed model. OR Library test data sets were used for this study. A strong lower bound was derived and the linear relaxation leads to tractable branch and bound trees for these instances (Carvalho, 1999).

Lodi et al. (1999) explored the problem class arising from all combinations of requirements that the items were obtained through the sequence of edge-to-edge cuts parallel to the bin edges. A heuristic algorithm and a combined tabu search approach were adapted to change the neighborhood for a specific problem (Lodi et al., 1999).

Fekete and Schepers (2001) studied dual feasible solutions and proposed a simple generic approach to obtain fast lower bounds of bin packing problems. This study also provides a general framework for establishing new bounds (Fekete and Schepers, 2001).

Retailers benefit from the optimum allocation of products into shelves in two ways: they reduce the costs of shelf replacement and inventory, and increase sales. The sales quantity of products depends on many factors such as location of the product within shelf, product facings and adjacent products (Dreze et al., 1994). Anderson and Amato (1973) showed that the companies increased the demand for a product by increasing the display area on the shelf.

Table 1 summarizes the research, algorithms, and references in this area.

Yang presented a greedy algorithm to generate good solutions (Yang, 2001). Lim et al. improved Yang’s

Bingüler, A.H.E., Bulkan, S., Ağaoğlu, M., (2016). A Heuristic Approach for Shelf Space Allocation Problem, Journal of Military and Information Science, Vol4(1),38-44.

heuristic approach and compared the original and the improved heuristics with three metaheuristic algorithms. Their algorithm that incorporates local search found the best results (Lim et al., 2004).

Table 1. Research/Algorithms and references

Research/Algorithm	**Reference**
Demand model for a product depends on direct elasticity	Corstjens and Doyle (1981)
Dynamic programming solution to a simplified version	Zufryden (1986)
Greedy algorithm	Yang (2001)
Squeaky Wheel Optimization algorithm	Lim et al. (2004)
Integrated mathematical model on multi-level shelves	Hwang et al. (2005)
Data mining approach and association rule mining	Chen and Lin (2007)
A model for two local chains using proprietary data in supermarket	Fadıloğlu et al. (2007)
The wholesalers effect pricing on retailers allocation decisions	Martinez-de-Albeniz and Roels (2011)
Model with elasticities at different aggregation levels	Eisend (2014)

3. Problem Definition

P_{ik} is the profit of the product i on shelf k , X_{ilk} is the decision variable to identify if product i is in the l^{th} *position* on shelf k, E_{ij} is the cross price elasticity if the product i is to the right of the product j on the shelf, then the objective function can be formulated as:

$$Max\ P = \sum_{k=1}^{K}\sum_{i=1}^{I} P_{ik} * X_{i1k} + \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=2}^{L}\sum_{j=1}^{I} (P_{ik} * X_{i1k}) * (E_{ij} * X_{j(l-1)k}) \qquad (1)$$

Subject to:

$$\sum_{i=1}^{I}\sum_{l=1}^{L} a_i * X_{i1k} \le T_k \qquad k = 1,2,\dots,K\ (shelf\ space\ constraint) \qquad (2)$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} X_{ilk} \ge L_i \qquad i = 1,2,\dots,I\ \ (lower\ bound\ for\ each\ product) \qquad (3)$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} X_{ilk} \le U_i \qquad i = 1,2,\dots,I\ \ (upper\ bound\ for\ each\ product) \qquad (4)$$

$$X_{ilk} = \begin{cases} 1 \text{ if the product } i \text{ is located to the } l^{th} \text{ position in the } k^{th} \text{ shelf} \\ 0 \text{ otherwise} \end{cases}$$

Where:

k = 1,2, …, K — *the number of shelves*
i, j= 1,2, …, I — *the number of products*
l = 1,2, …, L — *the position of products*
T_k *: the length of shelf k*
a_i *: the length of product i*
L_i *: the lower bound to allocate product i*
U_i *: the upper bound to allocate product i*

4. Simulated Annealing

SA is one of the first available meta-heuristics. Therefore it is not astonishing that it is also the first one to be applied to Quadratic Assignment Problem (QAP) (Ji et al., 2006). SA is a local search that relies on the process of statistical mechanics. Kirkpatrick et al. (1983) were the first researchers who used the Metropolis algorithm as a heuristic to solve the traveling salesman problem. They proposed an iterative local search method called SA to solve combinatorial optimization problems (Kirkpatrick et al., 1983).

Kirkpatrick et al. (1983) proposed a solution procedure to deal with these problems. Annealing is used to obtain a "well ordered" solid state of minimal energy as an experimental technique. This technique includes heating material with a high temperature then lowering the temperature slowly. The SA method includes two parameters; annealing and temperature coefficients (Kirkpatrick et al., 1983) (Dreo et al., 2006).

Figure 1 shows the flow chart of the SA algorithm. When this algorithm is adapted to the placement problem of components, simulated annealing operates a disorder-order transformation (Dreo et al., 2006). The pseudocode of SA is given in Figure 2.

SA starts with an initial solution s (obtained either randomly or from a simple construction heuristic) and generates a new solution s' in each step. Acceptance or rejection of this solution s' is done according to the acceptance criteria. In order to implement an SA algorithm, effective parameters and functions must be specified. For an SA algorithm, an annealing schedule is very important. In this schedule, T0 is an initial temperature, new temperature is obtained from the previous temperature (UpdateTemp), a number of iterations must be performed at each temperature (inner loop criterion), and a termination condition (outer loop criterion) is used (Stützle, 1998).

Since the convergence of the algorithm to an optimum solution under certain conditions can be proved, SA can attract mathematicians on solving the NP-hard problems. Mathematically, the theory of Markov chains can be used to model SA. The SA algorithm converges asymptotically to the optimal solution (Stützle, 1998).

Burkard and Rendl's (1984) motivated simulation procedure for combinatorial optimization problems is one of the first applications of SA to the QAP. It was demonstrated that SA outperformed most of the existing heuristics for the QAP at that time. The corresponding algorithm yields a promising improvement of the trade-off between computation time and solution quality (Burkard and Çela, 1995). Thonemann and Bölte (1994) proposed an improved SA algorithm for the QAP. A metaheuristic closely related to SA is also applied to QAP by Nissen and Paul (1995) (Erol, 2010).

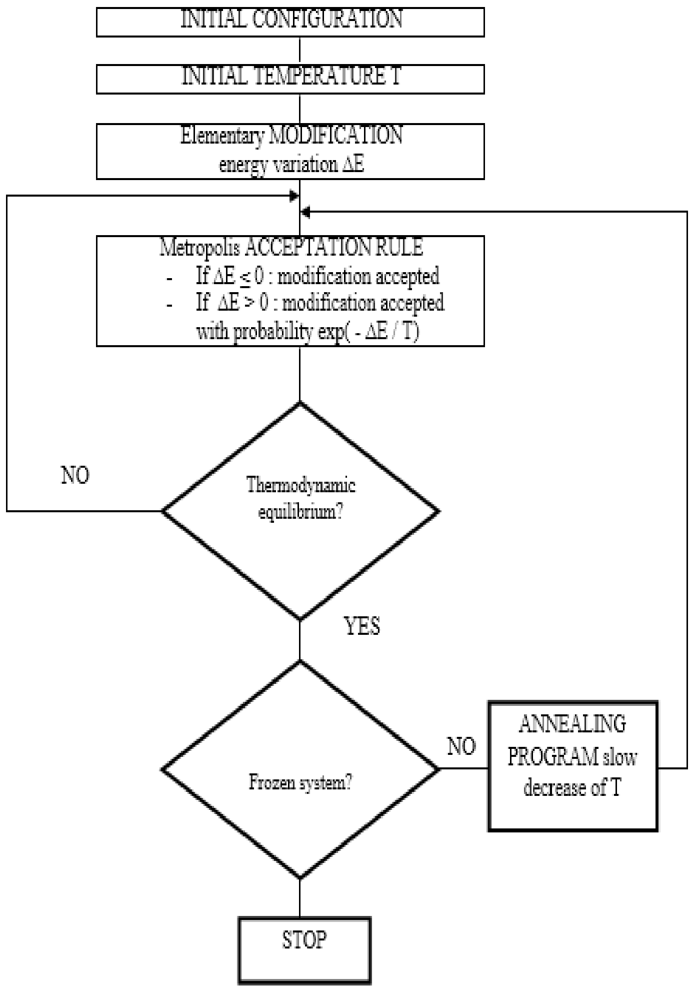

Fig 1. Simulated Annealing algorithm flow chart (Dreo et al., 2006)

procedure *Simulated Annealing*

generate initial solution k, $k_{best} = k$, initial value for T_0, i=0,

while outer-loop criterion not satisfied **do**

while inner-loop criterion not satisfied **do**

k' = Generate_Random_Solution (k)

k = Acc_Solution (T_i ; k ; k')

if (f(k) < f(k_{best})

$k_{best} = k$

end

T_{i+1}= Update_Temp (T_i), i = i+1

end

return k_{best}

end *Simulated Annealing*

Fig 2. Simulated Annealing algorithm outline. (Stützle, 1998)

Burkard and Rendl (1984) developed a general local search heuristic based on a simulated cooling process applicable to any combinatorial optimization problems once a neighborhood structure was introduced in the set of feasible solutions (Burkard and Rendl, 1984). In particular, Burkard et al. apply SA to the QAP (Burkard et al., 1998). Other approaches for applying SA to QAP are the studies of

Bos (1993), Yip and Pao (1994), Burkard and Çela (1995), Peng et al. (1996), Tian et al. (1996 and1999), Mavridou and Pardalos (1997), Chiang and Chiang (1998), Misevicius (2000 and 2003), Tsuchiya et al. (2001), Siu and Chang (2002). These studies differ from each other on the implementation of cooling process or the thermal equilibrium (Erol, 2010).

The advantages of the SA method are the flexibility on the evaluations of the problem and the ease of implementation. On the other hand, the main disadvantage of SA is the difficulty of adjustments of temperature decrease. SA obtained excellent results for large size problems (Dreo et al., 2006).

5. Simulated Annealing Application

The definitions of the initial temperature and the freezing temperature or the convergence condition are very essential in increasing the speed and accuracy of the SA algorithm. For this reason, many tests are performed to find effective temperatures. The repetition reduction rate is chosen by trial and error. For producing new generation, the switching operator is used in the inner loop thus, this operator can find the best possible result. Process optimization is performed in a Java program.

6. Results and Conclusion

The result of the Simulating Annealing algorithm is summarized in Table 2. In order to evaluate the efficiency of SA, average result of 100 independent runs are given.

Table 2. Comparison of algorithm results

Problem name	Min z	Mean z	Max z	Average run time
Simulated Annealing Algorithm	106.85	162.38	193.85	200.18
Genetic Algorithm (Bilsel et al., 2013)	145.95	162.38	176.25	253.13
Greedy Algorithm (Yang, 2001)	135.90	135.90	135.90	--
Greedy Algorithm with improvements (Ayhan et al., 2007)	146.10	146.10	146.10	--

As can be seen in Table 2, the proposed SA algorithm performs better than the heuristics of both Yang (2001) and Ayhan et al. (2007) its average performance is the same as the genetic algorithm of Bilsel et al. However, for the best (max) solution, the proposed SA algorithm outperforms Bilsel et al.'s genetic algorithm. These results show that SA is more suitable to solve SSAP.

As a future study, more problem instances will be solved to demonstrate the success of proposed SA heuristic.

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