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Flexural Tensile Strength Measurement and Determination for Kerbstones with Unsymmetrical Section

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Moment of inertia,
Unsymmetrical section.

Abstract: In this study, the determination of flexural tensile strength of concrete kerbstone is investigated. Currently, the flexural tensile strength of concrete kerbstone is determined using simple bending formula. However, due to the unsymmetrical cross-section of kerbstone, the bending take place on the section of kerbstone is not simple bending but bi-axial bending. The study shows how the flexural tensile strength for unsymmetrical sections should be determined. Additionally, flexural strength results obtained using the simple bending formula suggested by certain specifications and bi-axial bending formula used in the study were compared. As a result of the study, it was found that bi-axial bending formula gave higher values than that of the simple bending formula.

Simetrik Kesitli Olmayan Bordür Taşları için Eğilmede Çekme Dayanımı Ölçümü ve Tayini

Anahtar Kelimeler
Bordür taşı,
Çift eksenli eğilme dayanımı,
Atalet momenti,
Simetrik olmayan kesit.

Özet: Bu çalışmada, beton bordür taşının eğilmede çekme dayanımının belirlenmesi konusu araştırılmıştır. Günümüzde, beton bordür taşının eğilmede çekme dayanımı, basit eğilme formülü kullanılarak belirlenmektedir. Ancak, bordür taşının simetrik olmayan enine kesitinden dolayı, bordür taşı kesitinde meydana gelen eğilme, basit bir eğilme değil, iki eksenli bir eğilme olmaktadır. Çalışma da, simetrik olmayan kesitler için eğilmede çekme dayanımının nasıl belirlenmesi gerektiği teorik olarak gösterilmiştir. Ek olarak, standartların önerdiği basit eğilme formülü ve çalışmada kullanılan iki eksenli eğilme formülü kullanılarak elde edilen eğilme dayanımı sonuçları karşılaştırılmıştır. Çalışma sonucuna göre, iki eksenli eğilme formülü ile basit eğilme formülüne göre daha yüksek eğilmede çekme dayanımı değerleri elde edildiği tespit edilmiştir.

1. Introduction

There are three different approaches in the measurement of the tensile strength of concrete materials. These are direct tensile strength testing, splitting tensile strength testing, and bending tensile strength testing measurement. The information on these tensile strength testing measurements are available in the relevant literature [1, 2]. In this work, bending tensile strength testing and determination would be under consideration.

In general, bending tensile strength of concrete can be found by simply supported beam bending test using a concrete prism sample with square or rectangular section. In the testing, three-point or four-point loading testing apparatus are used [1, 2].

Kerbstone or border stone is generally used in the border of road to separate pedestrian walking way and motorized vehicle way, also it is used to make traffic island, or to separate double roads properly, or in construction of parking lots.

It is expected that kerbstone, used in the border of access way or road in urban, to have specified dimensions with appropriate tensile strength. Turkish Standard that specifies the dimensions and strength of kerbstone made with concrete is TS 436 EN 1340 [3]. Apart from describing dimensions, standard specification specifies the lower limit of flexural

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strength of kerbstone. This study is focused on measurement and determination of flexural strength of concrete kerbstone.

Kerbstone bending test is carried out according to relevant standard TS 436 EN 1340 [3], using three-point load testing by simply supported beam simulation.

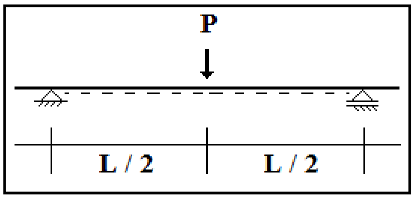

Figure 1. Kerbstone bending test by simply supported beam simulation [6]

Modeling of three-point load by simply supported beam simulation can be seen in Figure 1, and application of testing can be seen in Figure 2 Bending strength is determined using the equation given by relevant standard, in the equation, kerbstone working dimensions and section parameters, breaking load obtained from laboratory testing, the distance between supports and position of point load are used.

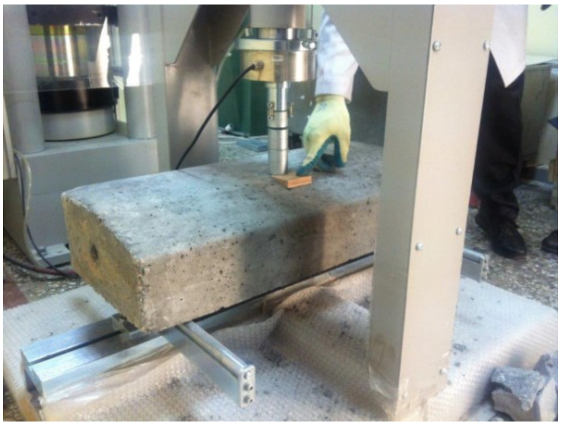

Figure 2. Kerbstone bending application picture [6]

Bending strength determination formula specified by current Turkish Standard [3] is presented in the following Equation 1.

$$\mathrm{T} = \frac{PL}{4I} y \qquad (1)$$

"P", is breaking load (in Newton). "L", is span length of beam or distance between supports (mm). "I", is moment of inertia for working or breaking section of kerbstone, (mm4). "y", is distance between centroid of working section and outer fibre of beam in tensile zone (mm). "T" is flexural tensile strength of kerbstone in MPa.

It is known that above given formula by Equation 1 is valid for symmetrical section under simple bending. However, in general, the cross-section of kerbstone is not a symmetrical, thus, given formula is under question.

Therefore, this case should be analyzed, and the situation has to be clarified. This clarification is taken as the main target of the study. To achieve this goal a theoretical and experimental program were planned and carried out.

2. Material and Method

2.1. Method for determination of flexural bending strength

As it was stated above that the given formula by Equation 1, is valid for simple bending with symmetrical section, it is not valid for simple bending of unsymmetrical section. In Equation 1, there appears a "y" term, the definition of "y" value by the standard is that "distance between centroid of working section and outer fibre of beam". This definition of y value is also thought to be inappropriate. The realistic definition of "y" value should be that "distance of outer fibre in tension zone to neutral axis".

Strength determination can be carried out for a general section regardless if the section is either symmetrical or unsymmetrical, using the formula described by mechanics of materials rule (Equation 2) presented below [4,5].

$$\sigma_z = \frac{M_x(I_y Y - I_{xy} X) + M_y(I_{xy} Y - I_x X)}{I_x I_y - (I_{xy})^2} \qquad (2)$$

In the formula, M_x is bending moment about x axis of section. M_y is bending moment about y axis of section. I_x is inertial moment of section about x axis. I_y is inertial moment of section about y axis. I_{xy} is product inertial moment of section. X and Y are apsis and ordinate of a point where stress is calculated, with respect to the axes located at centroid of section. σ_z is stress at any point located at X and Y coordinates.

In addition, the equation of neutral axis is given in Equation 3 that is presented below [4, 5]. The definition of neutral axis is that the stress distribution of a cross-section becomes zero on a line that passes through centroid, such line is called neutral axis, and it can be found by equating the stress values, given in Equation 2, to zero, then equation of neutral axis is found as presented in Equation 3.

$$Y = \frac{X(I_{xy} M_x - I_x M_y)}{I_y M_x - I_{xy} M_y} \qquad (3)$$

In the case of symmetrical kerbstone section, the formula (Equation 1) suggested by relevant standard to be used in the determination of the strength of kerbstone section, can be used. However, when the section is unsymmetrical then bending becomes biaxial. When a symmetrical section is subjected to such a moment with two components about axes then also bending becomes biaxial, not simple. Thus, the general formula (Equation 2) should be used in the strength calculation. In case of kerbstone flexural tensile strength determination, the value of maximum moment about x axis can be determined using the three-point testing using the formula described by Equation 4, and suggested by the relevant Standard.

$$\mathrm{M} = \frac{PL}{4} \tag{4}$$

The general dimensions and shape of the kerbstone used in this study are presented in Figure 3. It can be clearly seen from the figure that kerbstone section is not symmetrical. The bending moment that breaks the section would be about x axis, there would not be moment about y axis. Therefore, moment about x axis would be determined after bending testing that is to be determined using Equation 4, and moment about y axis M_y is zero.

While strength calculation carried out for kerb stone section, Equation 2 have to be used, since the section is unsymmetrical and the formula is general. Therefore, the parameters that used in Equation 2, i.e. I_x, I_y, Ixy, X ve Y are to be determined using the section dimensions provided in Figure 3. Determinations of those parameters can be carried out using mechanics of materials rule. As a result of mechanics of materials rule, the following equations are obtained to determine these inertial moment values (Equation 5, 6, 7).

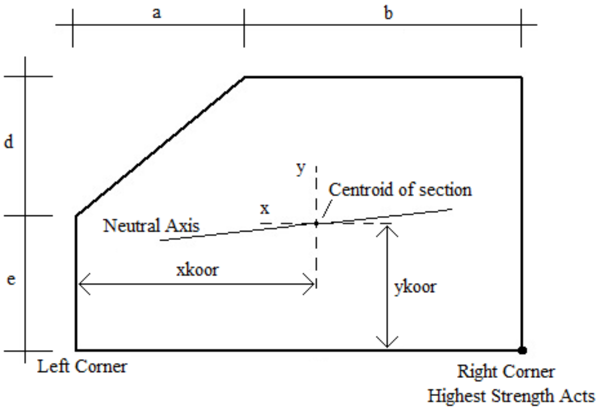

Figure 3. Typical kerb stone section, dimension and axis used in the study [6]

Where, a, b, d, e is side length of cross-section as defined in Fig.3. Abscissa and ordinate values of centroid of section can also be obtained using mechanics of materials rule. In this work, abscissa and ordinate values of centroid of entire section are obtained with respect to axis located at left corner of section and provided by Equation 8 and 9.

$$I_x = \frac{6b^2(d+e)^4 + 6ab(d+e)(d^3+4d^2e+4de^2+2e^3)}{72b(d+e)+36a(d+2e)} + \frac{a^2(d^4+6d^3e+12d^2e^2+12de^3+6e^4)}{72b(d+e)+36a(d+2e)} \tag{5}$$

$$I_y = \frac{6b^4(d+e)^2 + 12ab^3(d+e)(d+2e) + 12a^2b^2(d+e)(d+3e)}{72b(d+e)+36a(d+2e)} + \frac{6a^3b(d+e)(d+4e) + a^4(d^2+6de+6e^2)}{72b(d+e)+36a(d+2e)} \tag{6}$$

$$I_{xy} = \frac{ad(6b^2(d+e)(d+3e)+6ab(d+e)(d+4e)}{72(2b(d+e)+a(d+2e))} + \frac{a^2(d^2+6de+6e^2)}{72(2b(d+e)+a(d+2e))} \tag{7}$$

$$x_{koor} = \frac{\frac{-(a^2d)}{6} + \frac{1}{2}(a+b)^2(d+e)}{a(\frac{d}{2}+e)+b(d+e)} \tag{8}$$

$$y_{koor} = \frac{3b(d+e)^2 + a(d^2+3de+3e^2)}{6b(d+e)+3a(d+2e)} \tag{9}$$

Therefore, stress value at any point on the entire section can be calculated using these moments of inertia and other parameters. It is known that compressive strength of concrete is much higher than flexural tensile strength of concrete. Thus, the breaking occurs due to tensile stress on the section. Highest breaking stress should be calculated, since it causes the breaking of section. The highest breaking stress is also called flexural tensile strength of material. In the section presented in Figure 3, neutral axis is also plotted. It is known that the highest stress acts on the fibre that is the farthest from the neutral axis. In our case, the fibre farthest from neutral axis is located at right corner of kerbstone section. That point is also marked on the section presented in Figure 3. Coordinates of the above mention points with respect to the axes of whose origins located at centroid of entire section, where the highest tensile stress acts, can be stated by Equation 10 and 11. These coordinates are used in determination of flexural strength of kerbstone.

$$X = a + b - x_{koor} \tag{10}$$

$$Y = -y_{koor} \tag{11}$$

Above, it was stated that value of bending moments existed about y axis My was zero. If M_y value is substituted in stress formula given in Equation 2. Then below statement can be obtained in Equation 12.

$$\sigma_z = \frac{M_x(I_yY - I_{xy}X)}{(I_xI_y - I_{xy}{}^2)} \tag{12}$$

For clarification or easing calculation process, if the statement next to M_x moment is considered as coefficient of Mx, then inverse of that coefficient can be described as “W” (section modulus), thus, Equation 13 is obtained. If “W” is substituted in Equation 12, then, Equation 14 is obtained.

$$W = \frac{I_x I_y - I_{xy}^{\ 2}}{I_y Y - I_{xy} X} \quad (13)$$

$$\sigma_z = \frac{M_x}{W} \quad (14)$$

When those moments of inertia i.e. Ix, Iy, Ixy, and X and Y value are determined as described above, and substituted in section modulus (Equation 13), then “W” section modulus is obtained. The value of W can be substituted in Equation 14, thus, the highest stress acted on kerbstone section namely σ_z is obtained. The resultant value is also called flexural strength of kerbstone.

When the neutral axis is considered, bending moment about y axis was zero, this can be substituted in neutral axis Equation 3, then, simplified form of neutral axis can be obtained as presented below (Equation 15).

$$Y = \frac{X I_{xy}}{I_y} \quad (15)$$

It can be seen from Equation 15 that slope of the neutral axis is ratio of product moment of inertia to moment of inertia about y axis for entire section namely I_{xy}/Iy. Definition of slope is that tangent of the angle between neutral axis and x axis, thus, that angle ϕ can be determined using Equation 16.

$$\phi = \arctan\left(\frac{I_{xy}}{I_y}\right) \quad (16)$$

By using Equation 14 and 16, flexural tensile strength of kerbstone and angle between neutral axis and “x” axis can be determined.

2.2. Materials Used and Experimental Study

In real application and production, kerbstone dimensions are about 100 cm in length, and 30 and 40 cm in width and height. Thus, weight of a kerb stone becomes heavy to handle it, i.e. weight of a kerbstone heavier than 100 kg or more. In this study, producing small samples for laboratory were considered. Prismatic mortar samples with square section were produced. Unsymmetrical section was obtained by cutting a triangle from the corner of a square section. Mortar samples regardless of its cross-section were tested under tree point bending test. Then, bending strength were calculated using breaking load, span between supports and section properties. In calculation of bending strength Equation 1 and Equation 2 were employed. In Equation 1, moment of inertia value (I) was taken as Ix calculated in this study, and “y” value was substituted by “y_{koor}” value. It is known that strength of a material is constant, thus, the results obtained from unsymmetrical section and symmetrical section have to be equal. Therefore, it is expected that Equation 1 would be failed, and Equation 2 would satisfy the results. Mortar sample was prepared with 40x40 mm section and with 160 mm length. In preparation of mortar sample, a standard mixture was used. In the mixture, 450 gr cement, 1350 gr standard Rilem sand, and 225 water used to prepare three 40x40x160 mm3 sized prismatic specimens. In total, 18 prismatic specimens were prepared, 9 of them were subjected to saw cutting to obtain unsymmetrical section that looks like kerb stone section. Other 9 of them were used with square symmetrical section. For each sample, bending testing was carried out. Picture of mortar samples with symmetrical and unsymmetrical section were presented in the following figures (Figure 4 and 5) [6].

2.2.1 Flexural tensile strength measurements on symmetrical sectioned specimens

In the following, the specimens with square sections used for experiment are presented in Figure 4. The result of breaking force of the section is also measured for each specimen. The results of measurement obtained from flexural tensile strength testing were presented in Table 1, with the dimensions of each section.

Figure 4. Specimens for symmetrical section

Table 1. Breaking load for each square sectioned samples

Exp. Number	a mm	b mm	c mm	d mm	e mm	L mm	P Newton
1	0	40	40	0	40	100	3520
2	0	40	40	0	40	100	3330
3	0	40	40	0	40	100	3460
4	0	40	40	0	40	100	3410
5	0	40	40	0	40	100	3550
6	0	40	40	0	40	100	3360
7	0	40	40	0	40	100	3290
8	0	40	40	0	40	100	3650
9	0	40	40	0	40	100	3220

2.2.2 Flexural tensile strength measurements on unsymmetrical sectioned specimens

In the following, the specimens with unsymmetrical sections used for flexural tensile strength are presented in Figure 5. Dimensions of section of each specimen are marked on specimen. The result of breaking force of the specimens is also measured for each specimen. The results of measurement obtained from flexural tensile strength testing were presented in Table 2, with the dimensions of each section.

Figure 5. Specimens for unsymmetrical section

Table 2. Breaking load for unsymmetrical sectioned samples

Exp. Number	a mm	b mm	c mm	d mm	e mm	L mm	P Newton
1	20	20	40	20	20	100	2670
2	17	23	40	16	24	100	2720
3	17	23	40	15	25	100	2560
4	18	22	40	17	23	100	2380
5	19	21	40	19	21	100	2370
6	18	22	40	17	23	100	2380
7	17	23	40	17	23	100	2250
8	18	22	40	18	22	100	2400
9	18	22	40	18	22	100	2380

3. Results and Discussion

The data presented in Table 1 and Table 2 was used to calculate the strength of the entire sections using both formulas suggested by TS 436 (Equation 1) and this study (Equation 2). Flexural tensile strengths obtained from both formulas are presented in Table 3 and Table 4, for symmetrical and unsymmetrical sections, respectively.

Averages and standard deviations of the strength obtained are also calculated and presented in Table 3 and Table 4. The angle of neutral axis was also given in Table 3 and 4.

It can be seen from Table 3 that average strength of symmetrical section is 8.06 MPa, this result is obtained from both formulas. For symmetrical section, formulas described in Equation 1 and 2 gave the same results, this is expected, since the bending occurred on symmetrical section is simple. Thus, the strength of mortar materials is 8.06 MPa.

Table 3. Strength of each squared section samples calculated by both formulas, MPa

Exp. Number	σ_{TS436}	σ_{work}	ϕ^{0}	$\frac{\sigma_{work}}{\sigma_{TS436}}$
1	8.25	8.25	0	1
2	7.80	7.80	0	1
3	8.11	8.11	0	1
4	7.99	7.99	0	1
5	8.32	8.32	0	1
6	7.88	7.88	0	1
7	7.11	7.11	0	1
8	8.55	8.55	0	1
9	8.55	8.55	0	1
Average	8.06	8.06	-	1
Standard Deviation	0.45	0.45	-	-

Table 4. Strength of each unsymmetrical section samples calculated by both formulas, MPa

Experiment Number	σ_{TS436}	σ_{work}	ϕ^{0}	$\frac{\sigma_{work}}{\sigma_{TS436}}$
1	7.18	9.30	12.86	1.29
2	7.06	8.49	9.50	1.20
3	6.63	7.88	8.98	1.19
4	6.24	7.66	10.43	1.23
5	6.30	7.98	11.90	1.27
6	6.24	7.66	10.43	1.23
7	5.86	7.11	10.00	1.21
8	6.31	7.83	10.94	1.24
9	6.26	7.76	10.95	1.24
Average	6.45	7.96	10.66	1.23
Standard Deviation	0.43	0.61	1.06	0.03

The average strength for unsymmetrical section calculated using the formula suggested by standard (Equation 1) is 6.45 MPa. The average strength of unsymmetrical section calculated using the formula suggested in this work (Equation 2) is 7.96.

The results of this study have shown that results of symmetrical section (8.06 MPa) the true ones, and results of formula suggested in this work for biaxial bending is near to each other (7.96 MPa). This was expected, since a materials strength does not depend on section geometry. The result obtained from standards formula (Equation 1) deviates in the order of 20% from true result (Equation 2). Therefore, this proves that formula suggested in this study (Equation 2) is more appropriate and realistic in comparison to standards formula (Equation 1) which gave over safe result.

In this study, small number of samples (9 samples) for comparison purposes is used, it is suggested that more experiment should be carried out to verify this results. Additionally, the dimensions of samples used in this study is small, it is suggested that testing may be carried out on real samples.

As a result of this study, it is found that standards formula gave over safe results in calculation of kerbstone section strength. Therefore, it is suggested that the formula used in this work could be used in calculation of kerbstone strength. Another suggestion is that a symmetrical section can be prepared by cutting kerb stone section, and the strength of sample with symmetrical section can be calculated.

4. Conclusion

Determination of flexural tensile strength of kerbstone with unsymmetrical section should be carried out with respect to rule of mechanics of material. The formula suggested by the standard for determination of flexural tensile strength of kerbstone gave over safe results. The relative difference is in the order of 20%, which is significant. Determination of kerbstone flexural strength can be calculated using exact formula, provided by mechanics of materials rule, and computer; and more realistic results can be obtained. The extra stress part of over safe calculation can be reduced appropriately which may result in saving in amounts of cement used in producing kerbstone.

Declaration of Ethical Code

In this study, we undertake that all the rules required to be followed within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" are complied with, and that none of the actions stated under the heading "Actions Against Scientific Research and Publication Ethics" are not carried out.

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