İSTANBUL TİCARET ÜNİVERSİTESİ
FEN BİLİMLERİ DERGİSİ

İstanbul Commerce University Journal of Science

http://dergipark.gov.tr/ticaretfbd

Research Article / Araştırma Makalesi

ALTITUDE-HOLD FLIGHT CONTROL SYSTEM DESIGN FOR LONGITUDINAL MOTION

UZUNLAMASINA HAREKET KAPSAMINDA İRTİFA KİLİTLEME UÇUŞ KONTROL SİSTEMİ TASARIMI

Ahmet Hulusi ÖZ[1] **Muammer KALYON[2]**

Corresponding Author / Sorumlu Yazar
ahmethulusi.oz@tai.com.tr

Received / Geliş Tarihi
28.05.2021

Accepted / Kabul Tarihi
02. 06. 2021

Abstract

This article proposes the conventional implementation of a *"Altitude-Hold Controller"* for a high speed hypothetical aircraft. The static stability, *which is called as Stability Augmentation System (SAS),* is studied for the system. The stability conditions are analyzed and suitable controller design is developed over the longitudinal motion. The controller is designed by linearizing the longitudinal equation of motions and it is studied for performance issue. The controller design is optimized in order to get a good approximation for the overall flight system. The Root-Locus method is used to get the controller coefficients. The controller is divided into two sections which are; the inner loop that deals with the pitching motion parameters and the outer loop that deals with the altitude reference, flight-path angle on the vertical motion of the aircraft. The simulations, analysis and results are developed in MATLAB/Simulink program. The final results are discovered and expressed over the MATLAB/Simulink.

Keywords: Altitude-Hold control, flight mechanics, flight control systems, longitudinal motion control, Pitch control.

Öz

Bu makale, yüksek hızlı varsayımsal bir uçak için "İrtifa-Kilitleme Kontrolcü"sünün geleneksel uygulamasını göstermektedir. Sistem için *"Kararlılık Arttırma Sistemi (SAS)"* olarak adlandırılan statik kararlılık çalışılmıştır. Kararlılık koşulları analiz edilmiş ve uçağın uzunlamasına hareketi üzerinden uygun kontrolcü tasarımı geliştirilmiştir. Kontrolcü, uzunlamasına hareket denklemlerinin lineerleştirilmesi ile birlikte tasarlanmış ve performans sorunu için incelenmiştir. Kontrolcü tasarımı, genel uçuş sistemi için iyi bir yaklaşım elde etmek adına optimize edilmiştir. Kontrolcü katsayılarını elde etmek için Root-Locus yöntemi kullanılmıştır. Kontrolcü iki bölüme ayrılmıştır: yunuslama hareketi parametreleri ve açıları ile ilgilenen iç kontrol döngüsü ve uçağın dikey hareketindeki irtifa referansı ve uçuş-yolu açısı ile ilgilenen dış kontrol döngüsüdür. Simülasyonlar, analizler ve sonuçlar MATLAB/Simulink programında geliştirilmiştir. Nihai sonuçlar keşfedilmiş ve MATLAB/Simulink üzerinden ifade edilmiştir.

Anahtar Kelimeler: İrtifa-Kilitleme kontrolü, Pitch açısı kontrolü, uçuş kontrol sistemleri, uçuş mekaniği, uzunlamasına hareket kontrolü.

This publication was produced from the Master thesis of Ahmet Hulusi ÖZ in the Mechatronics Engineering Program of Istanbul Commerce University, Institute of Science and Technology.

[1]İstanbul Commerce University, Institute of Science and Technology, Department of Mechatronics Engineering, Küçükyalı, İstanbul, Turkey.
ahmethulusi.oz@tai.com.tr, Orcid.org/0000-0003-4374-7932.

[2]İstanbul Commerce University, Engineering Faculty, Department of Mechatronics Engineering, Küçükyalı, İstanbul, Turkey.
mkalyon@ticaret.edu.tr, Orcid.org/0000-0002-8168-2773.

Abbrevations

F_D	: Drag Force
F_Y	: Side Force
F_L	: Lift Force
C_D	: Drag coefficient
C'_D	: Equivalent drag coefficient for level flight
C_{D_0}	: Mean Drag coefficient
$C_{D_{CL}2}$	: Drag coefficient with respect to lift coefficient
$C_{D_{flaps}}$	: Drag coefficient with respect to flaps
$C_{D_{control\,surfaces}}$	: Drag coefficient with respect to control surfaces
C_{Y_β}	: Side-force coefficient with respect to side-slip angle
$C_{Y_{\delta r}}$	: Side-force coefficient with respect to rudder deflection
C_Y	: Mean Side-force coefficient
C_L	: Lift coefficient
C'_L	: Equivalent lift coefficient for level flight
C_{L_0}	: Mean Lift coefficient
C_{L_α}	: Lift coefficient with respect to angle of attack
$C_{L_{\delta e}}$	: Lift coefficient with respect to elevator deflection
LL	: Rolling Moment
MM	: Pitching Moment
NN	: Yawing
C_l	: Rolling moment coefficient
C_m	: Pitching moment coefficient
C_n	: Yawing moment coefficient
C_{l_β}	: Rolling moment coefficient with respect to side-slip angle
C_{l_p}	: Rolling moment coefficient with respect to roll rate
C_{l_r}	: Rolling moment coefficient with respect to yaw rate
$C_{l_{\delta a}}$	: Rolling moment coefficient with respect to aileron deflection
$C_{l_{\delta r}}$	: Rolling moment coefficient with respect to rudder deflection
C_{M_0}	: Mean Pitching moment coefficient
C_{M_α}	: Pitching moment coefficient with respect to angle of attack
$C_{M_{\delta e}}$	: Pitching moment coefficient with respect to elevator deflection
C_{M_q}	: Pitching moment coefficient with respect to pitch rate
$C_{M_{\dot\alpha}}$	: Pitching moment coefficient with respect to angle of attack rate
C_{N_β}	: Yawing moment coefficient with respect to side-slip angle
C_{N_p}	: Yawing moment coefficient with respect to roll rate
C_{N_r}	: Yawing moment coefficient with respect to yaw rate
$C_{N_{\delta a}}$	: Yawing moment coefficient with respect to aileron deflection
$C_{N_{\delta r}}$	: Yawing moment coefficient with respect to rudder deflection
δ_a	: Aileron deflection
δ_e	: Elevator deflection
δ_r	: Rudder deflection
$RCAM$	: Research Civilization Aircraft Model
$\boldsymbol{m}$	: Mass of the aircraft
$\boldsymbol{V_p}$	: Velocity of the aircraft
$\boldsymbol{g}$	: Gravity acceleration

1. INTRODUCTION

The Altitude-Hold control system is one of the most important control segment for the aircrafts. It forces the overall system on a designed altitude which is a significant flight. If the aircraft is supersonic, which is like in this study, this control system is getting important. Therefore, over the years, so many control methods were developed and studied to overcome this problem. The common method is dealing with the characteristic of the longitudinal motion of the aircraft via classical methods. Some of these methods are Eigenvalue assignment which is known as pole placement method, Root-Locus method and Routh-Hurwitz Criteria. The common feature of these methods is that they have to be all built with linear mathematical background. So that, the system has to be linearized to design and develope a proper controller.

Supersonic longitudinal flight control system is studied for the modern civil supersonic transport aircraft. Since the supersonic regime is increasing complex input command set and more nonlinearity for the aircraft, the stability augmentation system is also not easy to control (Steer, 2004). In this study, the set of input command response for one phase of flight, which is longitudinal flight, and handling qualities is revised.

Nonlinear Dynamic Inversion control method is implemented to a supersonic aircraft for the longitudinal motion to get handling qualities in steady conditions (Steer, 2001). The NDI control system is based on the pitch rate criteria and pitch attitude, and also the normal acceleration commands. Hence, the pitching motion response and behaviour is analyzed for a longitudinal flight control system in this study.

In another way, angle of attack can be used as an input command for the longitudinal control of a supersonic aircrafts (Lee, 2020). In order to increase the aerodynamic performance of the aircraft, static stability is analyzed. The longitudinal control law is based on dynamic inversion and proportional and integral control methods in this study.

In latest years of 70’s, there are some flight experience beyond the altitude hold and mach hold autopilots on YF-12 aircraft at mach number 3 (Gilyard, 1978). The main reason is to obtain the maximum range at high altitude and high mach number. The controller is designed as two sections, which are high-pass filtered pitch rate feedback for inner loop with altitude rate proportional and integral gains and auto-throttle control for mach number conditions

2. AIRCRAFT PLANT MODEL

Hypothetical supersonic aircraft is evaluated for the airframe model, since the behaviour of the aircraft will be shown in this study. The controller is also designed from this plant model. The mathematical representation of the plant model is explained.

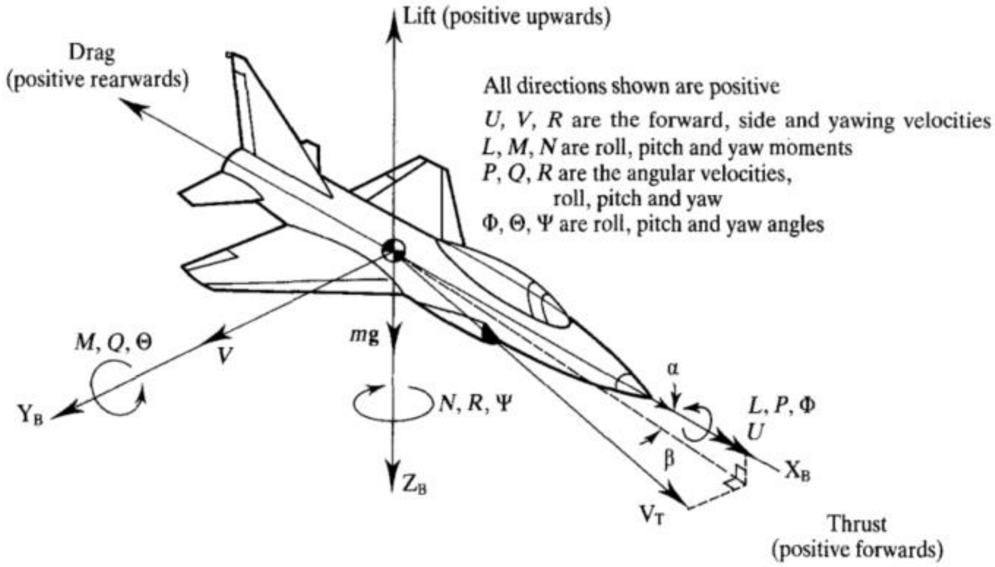

Figure 1: Body Axis System of Aircraft (McLean, 1990)

The following definitions contain the states of the dynamics for aircraft (McLean, 1990). These parameters are used for the feeding the control system and analyzing the behaviour of the aircraft. Therefore, it is useful to write the aerodynamic forces and moments and also the angular rates. $[u, v, w, p, q, r, \Phi, \theta, \psi]^T$ are the states which will be derived from the state equations given below, and the expressions are velocity on x, y and z-axis, angular roll, pitch and yaw rates, phi, theta, psi Euler-Angles, respectively. Aerodynamic forces $F_D, F_Y \; and \; F_L$, drag, side and lift forces (Blakelock, 1991), are given below:

$$\boldsymbol{F_D} = \frac{1}{2}\rho V_p^2 C_D \text{ , where, } C_D = C_{D_0} + C_{D_{C_L^2}} C_L^2 + C_{D_{flaps}} + C_{D_{control\,surfaces}}$$

$$\boldsymbol{F_Y} = \frac{1}{2}\rho V_p^2 C_Y \text{ , where, } C_Y = C_{Y_\beta}\beta + C_{Y_{\delta_r}}\delta_r \tag{1}$$

$$\boldsymbol{F_L} = \frac{1}{2}\rho V_p^2 C_L \text{ , where, } C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e + C_{L_{flaps}} + C_{L_{control\,surfaces}}$$

Aerodynamic moments LL, MM, NN, rolling, pitching and yawing moments, are given below:

$$\boldsymbol{LL} = \frac{1}{2}\rho V_p^2 S b C_l \text{ , where, } C_l = C_{l_\beta}\beta + C_{l_p}\bar{p} + C_{l_r}\bar{r} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r$$

$$\boldsymbol{MM} = \frac{1}{2}\rho V_p^2 S c C_M \text{ , where, } C_M = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{\delta_e}}\delta_e + C_{M_q}\bar{q} + C_{M_{\dot{\alpha}}}\dot{\alpha_{bar}} \tag{2}$$

$$\boldsymbol{NN} = \frac{1}{2}\rho V_p^2 S b C_N \text{ , where, } C_N = C_{N_\beta}\beta + C_{N_p}\bar{p} + C_{N_r}\bar{r} + C_{N_{\delta_a}}\delta_a + C_{N_{\delta_r}}\delta_r$$

Where, $\bar{p} = \frac{b}{2V_p}(pcos\alpha + rsin\alpha)$, $\bar{r} = \frac{b}{2V_p}(-psin\alpha + rcos\alpha)$

$\bar{q} = \frac{c}{2V_p}q$, $\dot{\alpha_{bar}} = \frac{c}{2V_p}\dot{\alpha}$

Angular rate definitions are given below:

$$\dot{p} = \frac{M_x}{I_{xx}} + \left(I_{yy} - I_{zz}\right)\frac{qr}{I_{xx}} + \frac{I_{xz}}{I_{xx}}(pq - \dot{r})$$

$$\dot{q} = \frac{M_y}{I_{yy}} + (I_{zz} - I_{xx})\frac{rp}{I_{yy}} + \frac{I_{xz}}{I_{yy}}(r^2 - p^2) \tag{3}$$

$$\dot{r} = \frac{M_z}{I_{zz}} + \left(I_{xx} - I_{yy}\right)\frac{pq}{I_{zz}} + \frac{I_{xz}}{I_{zz}}(\dot{p} - qr)$$

And, Euler-Angles are given below:

$$\frac{d}{dt}\begin{bmatrix}\Phi\\ \theta\\ \psi\end{bmatrix} = \begin{bmatrix}\dot{\Phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & tan\theta\sin\Phi & \cos\Phi tan\theta\\ 0 & \cos\Phi & -\sin\Phi\\ 0 & \frac{\sin\Phi}{cos\theta} & \frac{\cos\Phi}{cos\theta}\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \tag{4}$$

For the Altitude-Hold control system, the longitudinal motion of the aircraft is analyzed. Therefore, the overall longitudinal airframe structure is linearized and studied, since the method used in this study is based on linear control strategy. When the linearization is applied to the airframe, the longitudinal equations can be expressed with State-Space, transfer functions or total linear airframe.

On the translational equation for the symmetric and longitudinal flight, the wind and stability axis are coincided. There are total forces, which are aerodynamic, propulsive and gravitational forces has to be considered deriving the above perturbation equations acting along the x,y and z-axis. These are expressed below:

If $G_x, G_y\ and\ G_z$ are the gravitational terms and $P_x, P_y\ and\ P_z$ are the propulsive terms, the combination of these equations along the stability axes are:

$$\begin{aligned} P_{x_s} + G_{x_s} &= (P_x - mgsin\theta)cos\alpha + (P_z + mgcos\theta cos\Phi)sin\alpha \\ P_{y_s} + G_{y_s} &= mgcos\theta sin\Phi \\ P_{z_s} + G_{z_s} &= (P_x - mgsin\theta)sin\alpha + (P_z + mgcos\theta cos\Phi)cos\alpha \end{aligned} \tag{5}$$

2.1. Linearization of Longitudinal Equations

Linearization to the airframe is applied by using Taylor Series expansion method. The reference or trim conditions is added up the linearized small perturbations. Therefore, the desired, reference or operating points are considered and implemented to the airframe with linearization. The trim conditions are chose to get fully longitudinal motion over the total equations of motions. So that, the lateral effects are assumed as there are no influence to the linearized system. Then, the linearized airframe is simplified and the total longitudinal motion equations are derived easily. The longitudinal motion state equations, initial states and linearization assumptions are expressed below:

Lateral effects assumptions ➔ $\beta = p = r = \Phi = 0$

Inital states and operating/trim points ➔ $V_p = V_{p_0} = constant$

$$\begin{aligned} \alpha &= \alpha_0 = constant \\ q &= q_0 = 0 \\ \theta &= \theta_0 = constant \end{aligned}$$

If the angle of attack operating points is chose so that the $\alpha_0 = 0$, the derivation of the linearized longitudinal motion state is simplified further:

$V_p = V_{p_0} + \delta v_p$
$\alpha = \alpha_0 + \delta\alpha; \quad \alpha = \delta\alpha$
$q = q_0 + \delta q; \quad q = \delta q$
$\theta = \theta_0 + \delta\theta; \quad \theta = \delta\theta$

The velocity term, V_p *and* α, are expressed below according to above equations:

$$\dot{V_p} = \sum \frac{F_{x_W}}{m} = \frac{F_D + P_{x_S} + G_{x_S}}{m} \tag{6}$$
$$\dot{\alpha} = q + \sum \frac{F_{z_W}}{mV_p} = q + \frac{F_L + P_{z_S} + G_{z_S}}{mV_p} \tag{7}$$

The pitching motion terms, q *and* θ, are expressed below according to above equations:

$$\dot{q} = \frac{MM}{I_{yy}} \tag{8}$$
$$\dot{\theta} = q \tag{9}$$

If the expressions like $\cos\alpha \cong 1,\ \sin\alpha \cong \alpha \quad and \quad C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e$ are considered as these values for the equations (6)-to-(9) , the linearization can be simplified and expressed in a general form by using Taylor Series Expansion. Considering initial states and trim points, the equation (6) is linearized like below in order to get perturbation expression of velocity term and angle of attack:

$$\dot{\delta v_p} = \frac{-0.5\rho V_{p_0}^2 S}{m}\left(C_{D_0} + C_{D_{CL^2}} C_{L_0}^2\right) + \frac{-0.5\rho V_{p_0}^2 S}{m}\left(2C_{D_{CL^2}} C_{L_0} C_{L_\alpha}\right)\delta\alpha + \frac{-0.5\rho V_{p_0} S}{m} 2\left(C_{D_0} + C_{D_{CL^2}} C_{L_0}^2\right)\delta v_p - g(\delta\theta - \delta\alpha) + \frac{P_x}{m} + \frac{P_z}{m}\delta\alpha \tag{10}$$

The equation (7) is linearized like below according to the initial states and trim points:

$$\dot{\delta\alpha} = \delta q + \frac{-0.5\rho V_{p_0} S C_{L_0}}{m} + \frac{-0.5\rho S C_{L_0}}{m}\delta v_p + \frac{-0.5\rho V_{p_0} S C_{L_\alpha}}{m}\delta\alpha + \frac{-0.5\rho V_{p_0} S C_{L_{\delta e}}}{m}\delta e + \frac{g}{V_{p_0}} - \frac{g}{V_{p_0}^2}\delta v_p - \frac{P_x}{mV_{p_0}}\delta\alpha + \frac{P_z}{mV_{p_0}} - \frac{P_z}{mV_{p_0}^2}\delta v_p \tag{11}$$

The equations (10) and (11) can be simplified so that the aircraft is in a level flight. In this form of flight, $\delta e = 0$. Therefore, those equations are simplified by considering following expressions:

$$\frac{\rho V_{p_0}^2 S}{2}\left(C_{D_0} + C_{D_{CL^2}} C_{L_0}^2\right) + P_x = 0 \tag{12}$$

$$\frac{\rho V_{p_0}^2 S}{2} C_{L_0} = mg + P_z \ \ (There\ is\ no\ propulsion\ force\ in\ z - direction, P_z = 0) \tag{13}$$

According to equations (12) and (13), the aerodynamic coefficients can be simplified like below, if it is considered that the aircraft is in a level flight:

$C_L' = C_{L_0}$
$C_D' = \left(C_{D_0} + C_{D_{CL^2}} C_{L_0}^2\right)$

In conclusion, the pitching moment equation is expressed and linearized below:

$$\dot{\delta q} = \frac{0.5\rho(V_{p_0})^2 Sc}{I_{yy}} C_{M_0} + \frac{0.5\rho V_{p_0} Sc}{I_{yy}} 2C_{M_0}\delta v_p + \frac{0.5\rho(V_{p_0})^2 Sc}{I_{yy}} C_{M_\alpha}\delta\alpha + \frac{0.5\rho(V_{p_0})^2 Sc}{I_{yy}} C_{M_{\delta e}}\delta e + \frac{0.5\rho(V_{p_0})^2 Sc^2}{I_{yy}} C_{M_q}\delta q + \frac{0.5\rho(V_{p_0})^2 Sc^2}{I_{yy}} C_{M_{\dot{\alpha}}}\dot{\delta \alpha} + \frac{T_y}{I_{yy}} \tag{14}$$

Finally, the total linearized longitudinal motion state equations can be wrote like below in a general and simplified form:

$$\dot{\delta v_p} = a_{11}\delta v_p + a_{12}\delta\alpha - a_{14}\delta\theta \tag{15}$$
$$\dot{\delta\alpha} = a_{21}\delta v_p + a_{22}\delta\alpha + a_{23}\delta q + b_2\delta e \tag{16}$$
$$\dot{\delta q} = a_{31}\delta v_p + a_{32}\delta\alpha + a_{33}\delta q - c_{32}\dot{\delta\alpha} + b_3\delta e \tag{17}$$
$$\dot{\delta\theta} = \delta q \tag{18}$$

Where,

$a_{11} = -\frac{2gC_D'}{V_{p_0}C_L'}$, $\quad a_{12} = -g(1 - 2C_{D_{C_L^2}}C_{L_\alpha})$, $\quad a_{14} = -g$

$a_{21} = -\frac{2g}{V_{p_0}^2}$, $\quad a_{21} = -\frac{g}{V_{p_0}}[\frac{C_{L_\alpha}}{C_L'} + \frac{C_D'}{C_L'}]$, $\quad a_{23} = 1$

$a_{31} = 0$, $\quad a_{32} = \frac{mgcC_{M_\alpha}}{I_{yy}C_L'}$, $\quad a_{33} = \frac{mgc^2C_{M_q}}{2I_{yy}V_{p_0}C_L'}$

$a_{43} = 1$

$b_2 = \frac{-gC_{L_{\delta e}}}{V_{p_0}C_L'}$, $\quad b_3 = \frac{mgcC_{M_{\delta e}}}{I_{yy}C_L'}$, $\quad c_{23} = \frac{-mgc^2C_{M_{\dot{\alpha}}}}{2I_{yy}V_{p_0}C_L'}$

If the above equations is expressed in a state space form, it is like below:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\delta v_p} \\ \dot{\delta\alpha} \\ \dot{\delta q} \\ \dot{\delta\theta} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & 0 & a_{43} & 0 \end{bmatrix} \begin{bmatrix} \delta v_p \\ \delta\alpha \\ \delta q \\ \delta\theta \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{bmatrix} \delta e \tag{19}$$

3. ALTITUDE-HOLD CONTROL SYSTEM BEYOND LONGITUDINAL MOTION

Since the longitudinal motion state equations are linearized, the further control methods can be applied to the linear airframe. Applying those methods lead to get a proper, optimized control constant coefficient for the system by using suitable control rules such as Root-Locus, Routh-Hurwitz Criteria. However, the first thing is to guarantee the static stability for the aircraft.

The controller which is designed in this study has two sections. The first one is the inner loop which consists of pitching motion expressions. The pitching motion expression is dealing with the pitching angle, θ $(linearized \cong \delta\theta)$, for the system by choosing and controlling the elevator displacement, δe. The second section is the outer loop which contains the altitude, flight-path angle and velocity calculations. The controller for the outer loop is designed when a suitable and optimized inner loop controller is developed.

3.1. Inner Loop Controller Design

The pitching motion expression has to be expressed in linear form in order to design the controller. Therefore, the pitching angle, $\theta\ (linearized \cong\ \delta\theta)$, is used from the linearized state equations with respect to the elevator displacement, δe. After the static stability conditions for this state is guaranteed and analyzed, the proper controller coefficients is derived for the inner loop.

From the equations (15)-to-(18), the following transfer function of pitching angle with respect to elevator displacement can be obtained and expressed in equation (20a):

$$\frac{Y_{\theta\delta e}(s)}{\delta e(s)} = \frac{(b_3 - c_{32}b_2)s^2 + [b_2(c_{32}a_{11} + a_{32}) - b_3(a_{11} + a_{22})]s + b_3(a_{11}a_{22} - a_{21}a_{12}) - b_2a_{32}a_{11}}{Den(s)} \quad (20a)$$

Where,

$$Den(s) = s^4 + (c_{32} - a_{11} - a_{22} - a_{33})s^3 + [a_{11}a_{22} - a_{21}a_{12} + a_{33}(a_{11} + a_{22}) - c_{32}a_{11} - a_{32}]s^2 + [c_{32}a_{21}a_{14} + a_{32}a_{11} - a_{33}(a_{11}a_{22} - a_{21}a_{12})]s + a_{32}a_{21}a_{14} \quad (20b)$$

When the pitching motion analyzed, it can be divided into two sections which are short-period and phugoid motion. At first, it is better to guarantee the stability for short-period approximation, since the roots of the transfer operators are closer to the imaginary axis (unstable region). If the stability is guaranteed in this period of motion, the other periods can follow the reference commands in a stable way. The other reason of using the short-period approximation at first is that the derivation of the velocity does not affect the angle of attack. Therefore, the designing of the controller can be studied with different velocity regimes.

$$\frac{Y_{\theta\delta e}(s)}{\delta e(s)} = \frac{\left(\frac{b_3 - b_2c_{32}}{a_{22}a_{33} - a_{32}}\right)s + \left(\frac{b_2a_{32} - b_3a_{22}}{a_{22}a_{33} - a_{32}}\right)}{s\left[\left(\frac{1}{a_{22}a_{33} - a_{32}}\right)s^2 + \left(\frac{c_{32} - a_{33} - a_{22}}{a_{22}a_{33} - a_{32}}\right)s + 1\right]} = \frac{-0.7206s - 0.5144}{0.03087s^3 + 0.08553s^2 + s} \quad (21)$$

The characteristic equation of the equation (21) has to be analyzed for the static stability in order to decide and understand whether the aircraft is stable in level flight or not. Root-Locus method is used for analyzing the static stability. The Figure 2 shows the expression beyond the pole and zero branches of the pitching motion of the linearized aircraft airframe:

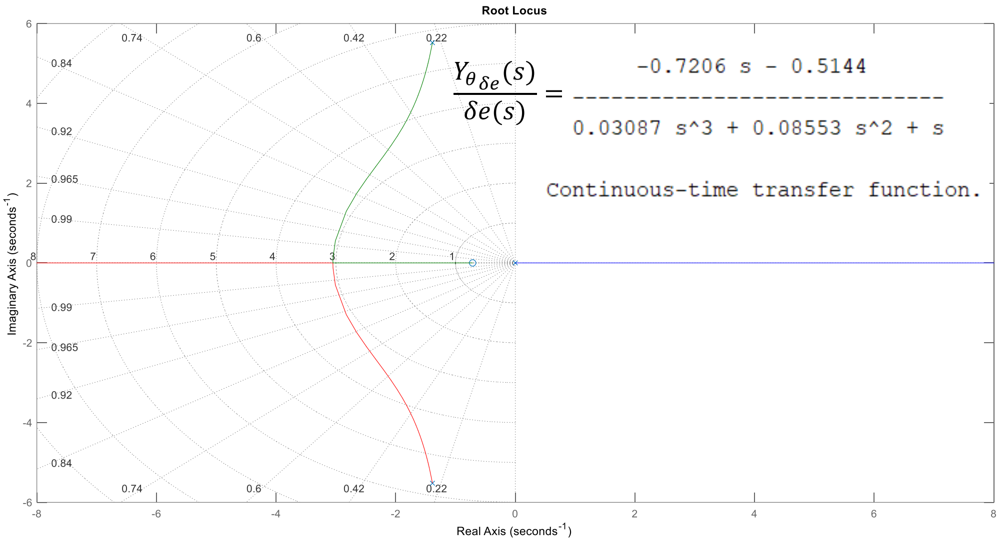

Figure 2:Root-Locus Diagram of Theta over Elevator Displacement

The result for the pithcing angle by feeding a small elevator discplacement motion can be seen in Figure 3:

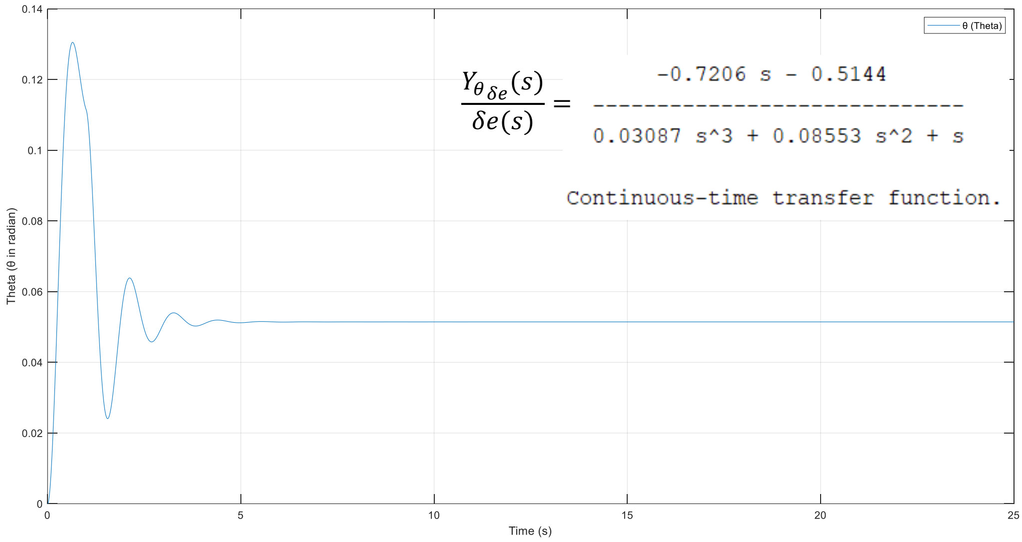

Figure 3: Time response of Pitching Angle Theta

The further dynamic which will be applied to the inner loop pitching motion is the elevator servo dynamics. This dynamic leads to the system in a more reality analysis. The common servo dynamic in terms of transfer function expression is like below:

$$Y_{elevator}(s) = -\frac{10}{s+10} \tag{22}$$

When the equations (21) and (22) are considered as the inner loop plant transfer operator, it is better to analyze the static stability by multiplying of these two equations, since they can be considered as a combined open-loop transfer function.

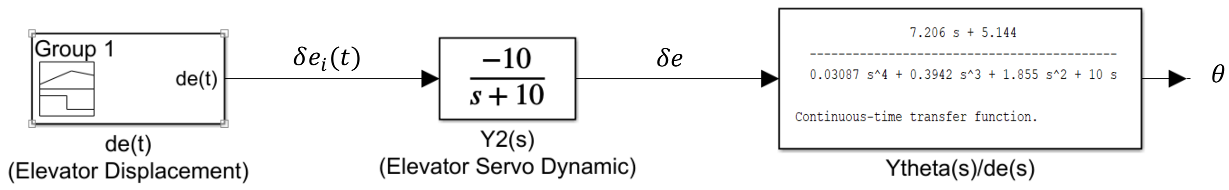

Figure 4: Body Diagram of Elevator Servo and Theta over Elevator Displacement Tr. Function

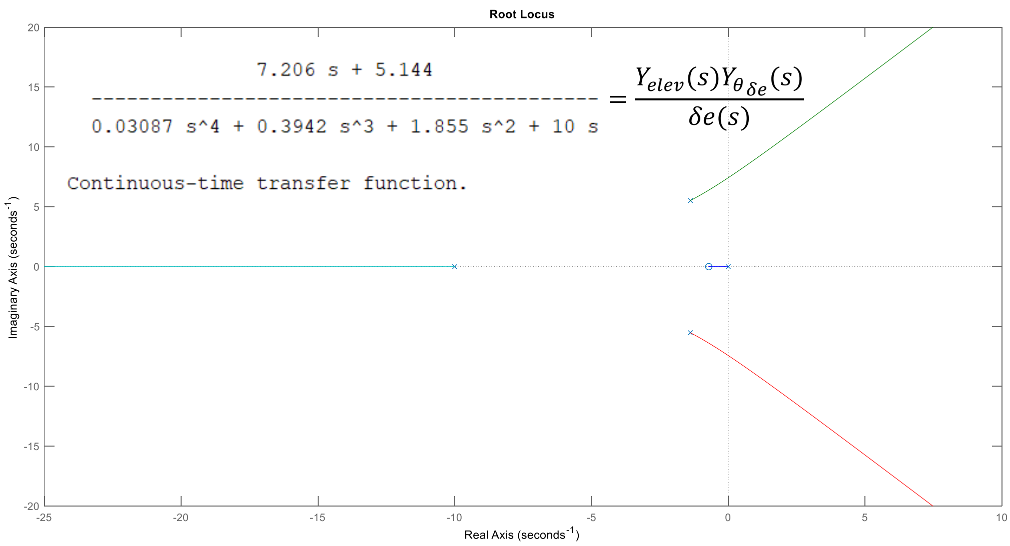

Figure 5:Root-Locus Diagram of Elevator Servo and Theta over Elevator Displacement Tr. Function

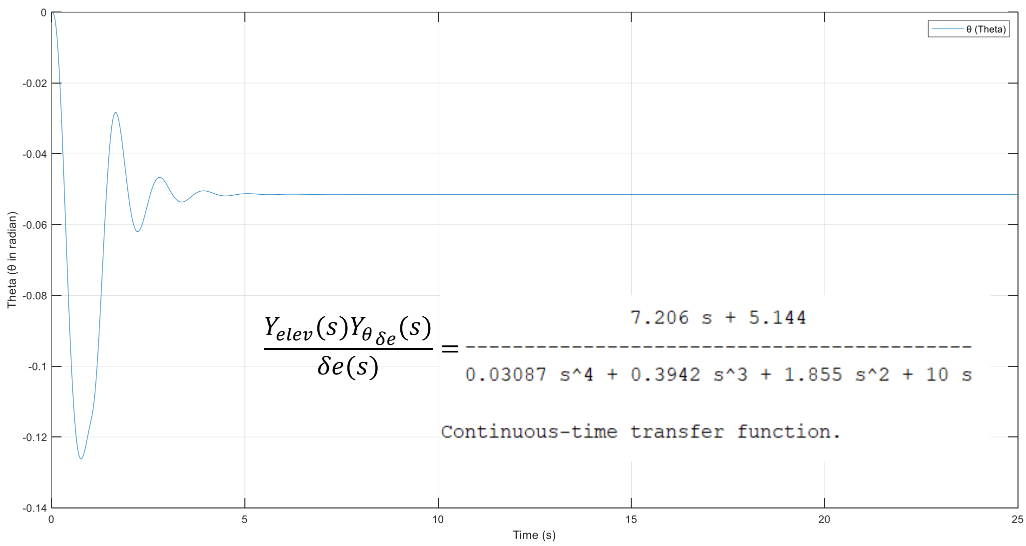

Figure 6: Time response of Elevator Servo and Theta over Elev. Displ.

According to the Figure 5 and Figure 6, the system more stable if it is compared with the Figure 2 and Figure 3. The way which has to be followed should be adding poles and zeros to the controller, in order to get the pitching angle following the reference pitching angle. The following diagram shows that the inner loop with the controller:

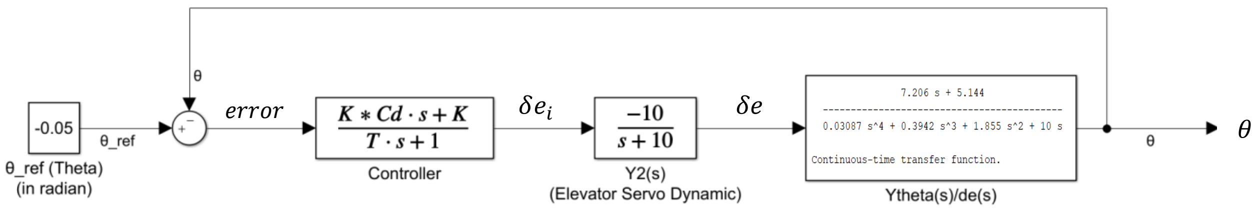

Figure 7: Body Diagram of Inner Loop Control System

If the Figure 5 is analyzed, it can be seen that the place of the controller coefficients can be estimated roughly in Root-Locus diagram. However, to get an optimum solution, Routh-Hurwitz Criteria can be used. The controller transfer operator is below:

$$C(s) = \frac{K(C_d s+1)}{Ts+1} \tag{23}$$

Inserted additional pole to the plant is equal to ➔ $s = -1/\tau$

Inserted additional zero to the plant is equal to ➔ $s = -1/C_d$

If the above expressions are placed to the Root-Locus diagram like below, the approximate control is applied to the inner loop. For the sake of approximation, the point A which is shown in Figure 8 can be chose as an inserted additional zero-branch position for the system.

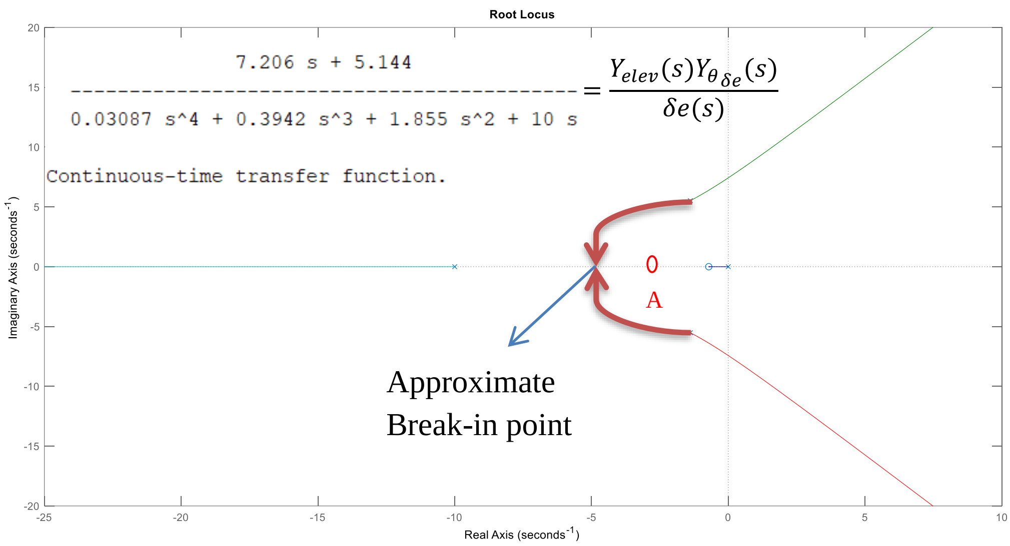

Figure 8: Approximate Zero Branch placement to Root-Locus Diagram

Then, the inserted pole branch is placed a little far away from the pole at negative side, which is shown in Figure 9 with red star. In this case, the integrated pole goes to negative infinity asymptote, and the conjugate pole pair can be forced to diverge from the imaginary axis which is unstable region critical area. So that, two pole branches go to the other asymptotes by staying at the negative side of the Root-Locus diagram. This leads to the system for choosing controller gain without any doubt in terms of stability. The expression of this manipulation is explained graphically below:

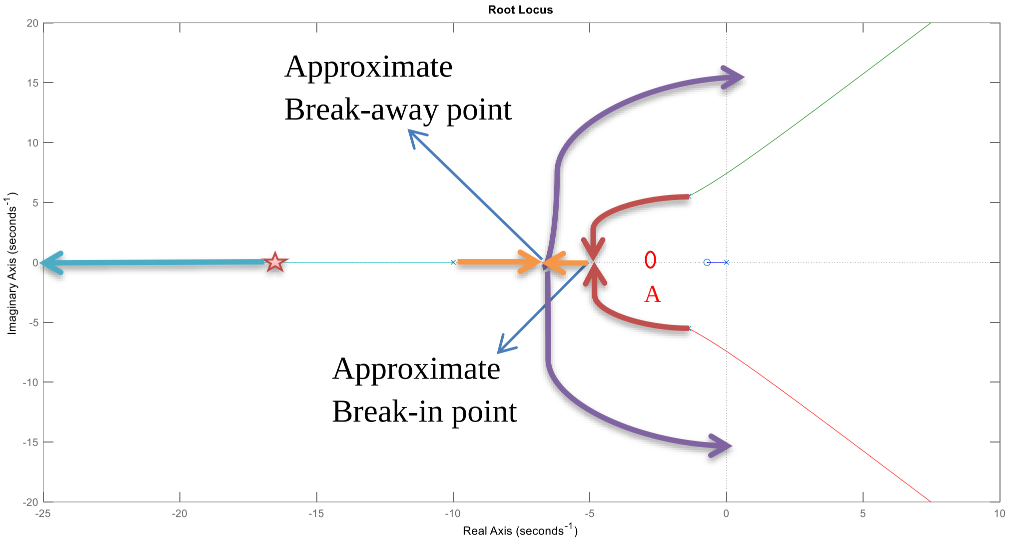

Figure 9: Approximate Pole Branch placement to Root-Locus Diagram

The approximate places of the inserted pole and zero branches are shown at Figure 9. The remain proportional constant can be chose from the lines which are going to the asymptotes, freely in every bound, since the system is stable for each condition of this flight regime. A rough controller design can be analyzed in this way. However, in order to choose a proper and optimum solution for the inner loop. The total closed-loop transfer function is expressed below:

$$G_{inner}(s) = C(s)Y_2(s)Y_{\theta_{\delta e}}(s) = \frac{K(C_d s+1)(7.206s+5.144)}{(\tau s+1)(0.03087s^4+0.3942s^3+1.855s^2+10s)} \tag{24}$$

Implementing the reasonable pole and zero branch positions for the above closed-loop transfer function, $G_{inner}(s)$, the following controller constants can be established:

$\tau \ = 0.05$
$C_d = 0.2$

Above results for the controller coefficients leads to the total system in Root-Locus diagram like below:

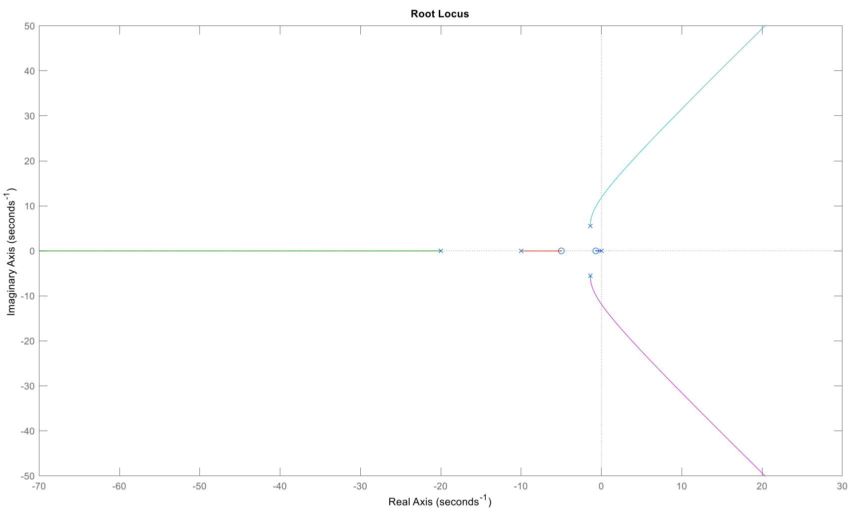

Figure 10: Root-Locus Diagram of Plant with Controller

The Root-Locus diagram in Figure 10, is not comparable with the desired one at Figure 9. This is not a reliable solution for this plant, because the system can behaves unstable motion at some trim and reference conditions. Therefore, the conjugate pole pairs can be forced to the negative right hand side (stable region) by one more inserting same additional pole and zero branches. This leads the systems to an expression like in equation 25:

$$G_{inner}(s) = C(s)Y_2(s)Y_{\theta_{\delta e}}(s) = \frac{K(C_d s+1)^2(7.206s+5.144)}{(\tau s+1)^2(0.03087s^4+0.3942s^3+1.855s^2+10s)} \tag{25}$$

In above equation, the characteristic of the open-loop transfer function is expressed in Figure 11 with Root-Locus diagram:

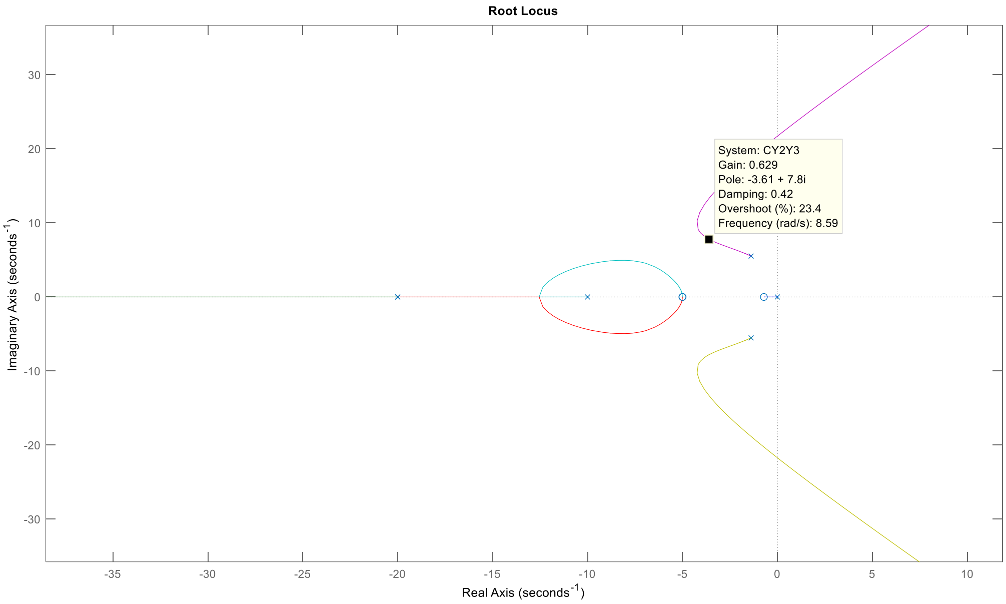

Figure 11: Root-Locus Diagram of Plant with proper Controller

In above diagram, the pole and zero branches behave like the system is in stable for most conditions. The proportional gain of the controller can be chose without any doubt of unstable conditions. It is analyzed and experienced that the K gain satisfy the damping ratio with a reasonable result. In this combination of the "τ and C_d", K (proportional gain) can be chose as 0.629 with respect to 0.42 damping ratio, which is a good approach for a supersonic aircraft.

3.2. Outer Loop Controller Design

Since the inner loop control loop is established in a stable conditions, the outer loop of the altitude hold controller system beyond the longitudinal motion can be studied freely. For the simplicity, the closed-loop transfer function of the inner loop controller can be written as below:

$$G_{inner_{CL}}(s) = \frac{C(s)Y_2(s)Y_{\theta_{\delta e}}(s)}{1+C(s)Y_2(s)Y_{\theta_{\delta e}}(s)} = \frac{0.1813s^3+1.942s^2+5.827s+3.236}{0.0001s^6+0.0041s^5+0.0749s^4+0.786s^3+4.798s^2+15.83s+3.236} \tag{26}$$

The outer loop consists of the following equations:

$$\dot{h} = V_p \sin(\theta - \alpha) = V_p sin\gamma \tag{27}$$

Integrating the equation (27) will lead to get the current altitude value of the system. When taking difference of the current and reference input of the altitude, the error is controlled with a proportional gain and feeding to the reference input of the inner control loop, which is θ_{ref}.

The outer loop controller gain can be found by using the same method of the inner loop control system design. However, the inner loop is stabilizing so fast. Therefore, the outer loop controller gain can be chose as the reducing the rank of the reference altitude input, since it is too big numerically with respect to the other inputs. The controller gain is chose with a constant value of 0.004 for the simplicity.

Hence, the outer loop is designed as tracking the reference altitude input, the characteristic of the controller behaves as holding the altitude at the reference input. The total body diagram of the "Altitude-Hold Control System" is expressed in Figure 12:

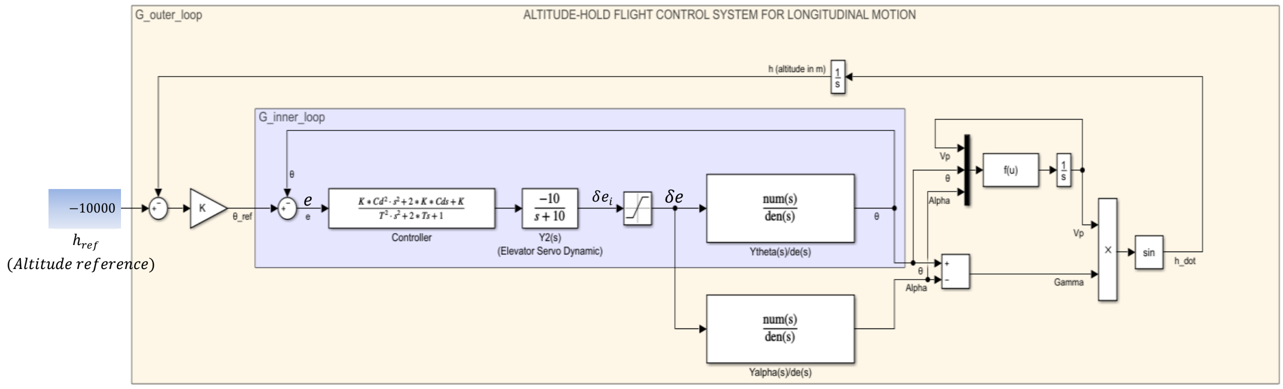

Figure 12: Altitude-Hold Flight Control System Body Diagram

Following example shows that the given reference altitude is stabilized rapidly by analyzing the Figure 13:

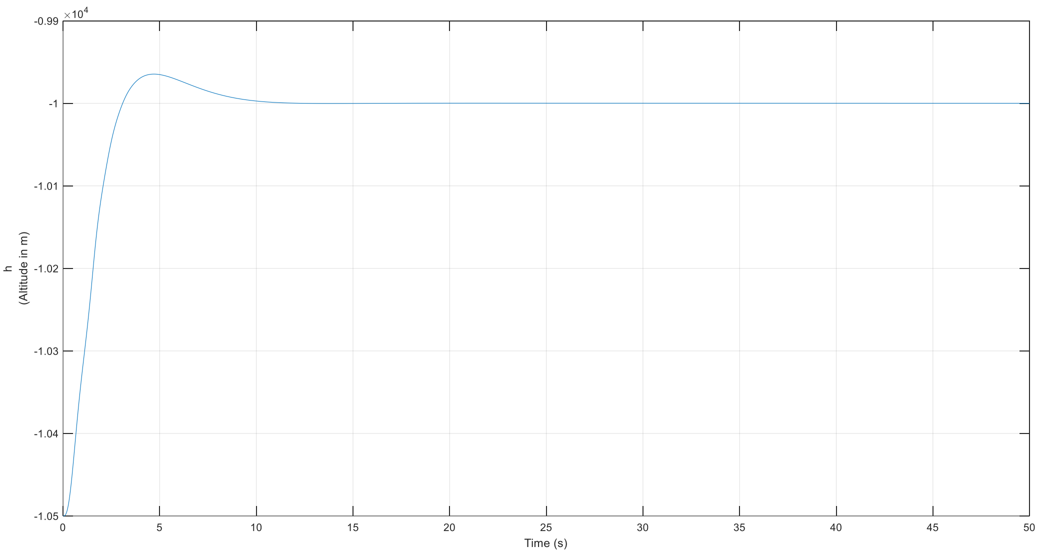

Figure 13:Altitude response of Altitude-Hold Flight Control System

The corresponding pitching motion angles and elevator displacement of the plant can be seen in Figure 14:

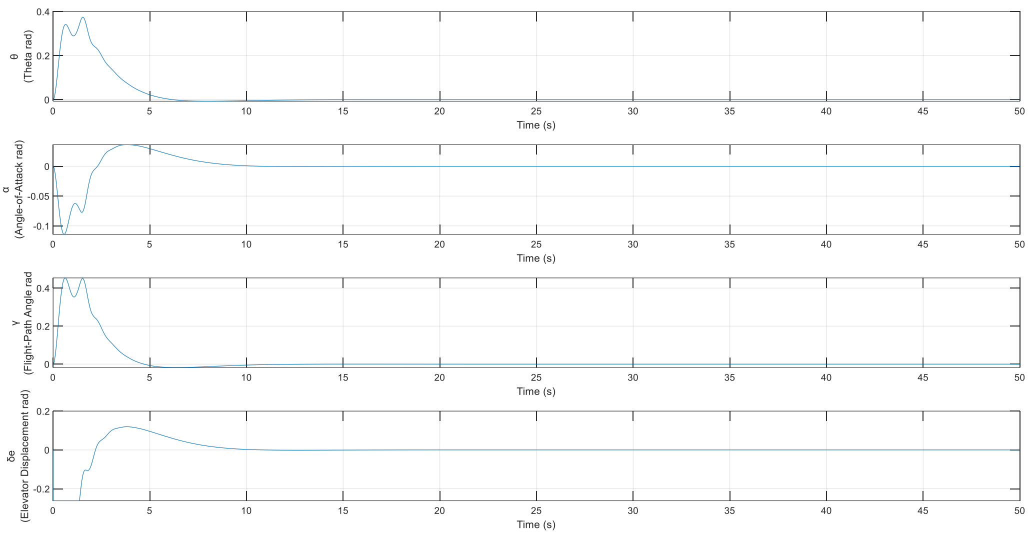

Figure 14: Angles of Pitching Motion and Elev. Displ. responses

4. CONCLUSION

The stabilization of the short-period approximation for supersonic aircraft is completed in this study, at first. Total longitudinal airframe is linearized in order to analyze at steady, equilibrium points and to control with the conventional methods. The design of the controller is based on two stages, which are inner loop and outer loop controllers. The inner loop is dealt with making the longitudinal motion airframe steady at reference pitching motion. The controller design is completed via Root-Locus analysis. At first, a desirable controller is estimated for the current longitudinal motion airframe. Then, the proper controller gains are established by studying several controller designs according the desirable and reliable Root-Locus diagram behaviour. The suitable gains are chose to implement and complete the inner loop controller design.

The outer loop controller design is based on keeping the calculated and measured altitude of the aircraft at the reference altitude input. Therefore, the altitude motion expressions are developed and formed so that the airframe gives the calculated output for feeding back to the reference input section. The evaluating the control variable for the control method is established by choosing the proportional control gain such as the rank of the altitude is acceptable and easy to be calculated by the controller numerically.

In this study, the importance of stabilizing the short-period motion of the aircraft is experienced. The reason of using this motion of study is that it makes the aircraft moves around the critical regions. In terms of Root-Locus diagram, the steady-state response of the aircraft is close to the imaginary axis, which is critical limit of the unstable region. Therefore, the importance of stabilizing this motion leads to guaranteed stable motion of the aircraft at each longitudinal flight regimes. The other reason is that the velocity component of the airframe does not affect to the angle of attack. Hence, the derivation of the velocity does not affect to the longitudinal motion, which means that the analyzing of this motion can be easy to study. It can be worked at every flight velocity regimes.

Authors' Contributions
Contributions of authors to the article are equal.

In this study, Muammer KALYON contributed about idea, critics, analysis, comment and method suggestions. Ahmet Hulusi ÖZ contributed about searching, implementation, simulation, analysis, comment, literature review and writing of the article.

Acknowledgement
Authors thanks to Turkish Aerospace since it provided working environment during the study.

Statement of Interest Conflict
There is no interest confliction between the authors.

Statement of Research and Publication Ethics
Research and publishing ethics are taken into account in this study.

REFERENCES

Blakelock, J.H., (1991), Automatic Control of Aircraft and Missiles, A Wiley-Interscience Publication, 669, Canada.

Gleen, B., (1978), Flight Experience with Altitude Hold and Mach Hold Autopilots on the YF-12 Aircraft at Mach 3, NASA Flight Research Center Edwards, CA, United States.

Lee, D., (2020), "Angle-of-Attack Command Longitudinal Control for Supersonic Advanced Trainer Aircraft", International Journal of Aeronautical and Space Sciences, 22, 120–128.

McLean, D., (1990), Automatic Flight Control Systems, Prentice Hall International, 610, UK.

Steer, A.J., (2001), Flight Control for Advanced Supersonic Transport Aircraft Handling Quality Design, Cranfield University, PhD Thesis,UK.

Steer, A.J., (2004), **"**Supersonic Transport Aircraft Longitudinal Flight Control Law Design", The Aeronautical Journal, 108(1084), 319-329.