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On elementary soft compact topological spaces

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ABSTRACT

This paper is a work on elementary soft (ϵ-soft) compact spaces. We first define the cofinite ϵ-soft compact space and prove that the image of an ϵ-soft compact space under a soft continuous mapping is ϵ-soft compact space. We then examine the relationship between ϵ-soft compact space and classical compact space and give an illustrative example.

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1 Introduction

In 1999, Molodtsov [1] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties. He showed several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Interest in the soft sets and their applications has been continued to grow rapidly afterwards. Maji et al. [2, 3] studied soft set theory in detail and applied it to decision making problems. In the line of reduction and addition of parameters of soft sets, some works have been done by Chen et al. [4], Pei and Miao [5], Kong et al. [6], Zou and Xiao [7]. Shabir and Naz [8] introduced the notion of soft topological space. Many authors studied on soft topological spaces and considered the concept of soft point [9-15]. Then, Das and Samanta introduced notions of soft element, soft reel set and number [16] and soft complex set and number [19] over soft sets. Samanta et al. and several authors examined some mathematical structures such as soft metric, soft vector, soft norm, etc. by using the notion of soft element [18-20]. Also, works on fixed point theory have been ongoing over the soft sets, the soft metrics and soft cone metrics [21-26]. In recent years some authors studied on ϵ-soft topological spaces by using elementary operations on soft sets [27-31].

In this paper we study ϵ-soft compact spaces. ϵ-soft compact spaces and many of their properties are studied in [30, 31]. Here we proved the following features related to compactness:

1. Let $\mathcal{T} = \left\{\Phi, U_i \in S(\tilde{X}) : U_i^{C} \in S(\tilde{X}), \bigcap U_i^{C} \in S(\tilde{X}), U_i^{\mathbb{C}} \text{ is finite}\right\} \subset S(\tilde{X})$ be a family of soft sets over X with parameter set A. Then $\left(\tilde{X}, \mathcal{T}, A\right)$ is an ϵ-soft compact topological space.

2. Let $\left(\tilde{X}, \mathcal{T}, A\right)$ and $\left(\tilde{Y}, \mathcal{T}^*, A\right)$ be two ϵ-soft topological spaces and $F \in S(\tilde{X})$ be an ϵ-soft compact set. If $f : SE\left(\tilde{X}\right) \to SE\left(\tilde{Y}\right)$ is a soft continuous mapping, then $SS\left(f(SE(F)\right) \in S(\tilde{Y})$ is ϵ-soft compact.

3. Let $\left(\tilde{X}, \mathcal{T}, A\right)$ be a ϵ-soft compact space. If $U \tilde{\cap} V \in S(\tilde{X})$ for every $U, V \in \mathcal{T}$, Then for every $\lambda \in A$;

i. $(X,\mathcal{T}_\lambda)$ is a compact space.

ii. For every ϵ-soft compact set $F\in S(\tilde{X})$, $F(\lambda)\subset X$ is a compact set.

Also, we give an example that explains the relationship between ϵ-soft compact space and classical compact space.

2 Preliminaries

Definition 2.1. [1] Let A be a set of parameters and E be an initial universe. Let $\mathcal{P}(E)$ denote the power set of E. A pair (F,A) is called a soft set over E, where F is a mapping given by $F:A\to\mathcal{P}(E)$. In other words, a soft set over E is a parametrized family of subsets of the universe E. For $\lambda\in A$, $F(\lambda)$ may be considered as the set of λ-approximate elements of the soft set (F,A) and denoted by F for short.

Definition 2.2. [2] Let F and G be two soft sets over a common universe E.

1. F is said to be null soft set, denoted by Φ, if for all $\lambda\in A$, $F(\lambda)=\varnothing$. F is said to an absolute soft set denoted by $\tilde{E}$, if for all $\lambda\in A$, $F(\lambda)=E$.
2. F is said to be a soft subset of G if for all $\lambda\in A$, $F(\lambda)\subseteq G(\lambda)$ and it is denoted by $F\tilde{\subseteq}G$. F is said to be a soft upper set of G if G is a soft subset of F. We denote it by $F\tilde{\supseteq}G$. F and G is said to be equal if F is a soft subset of G and G is a soft subset of F.
3. The intersection H of F and G over E is defined as $H(\lambda)=F(\lambda)\cap G(\lambda)$ for all $\lambda\in A$. We write $H=F\tilde{\cap}G$.
4. The union H of F and G over E is defined as $H(\lambda)=F(\lambda)\cup G(\lambda)$ for all $\lambda\in A$. We write $H=F\tilde{\cup}G$.
5. The product H of F and G over E is defined as $H(\lambda)=F(\lambda)\times G(\lambda)$ for all $\lambda\in A$. We write $H=F\tilde{\times}G$.
6. The difference (H,A) of (F,A) and H of F and G over E is defined as $H(\lambda)=F(\lambda)\setminus G(\lambda)$ for all $\lambda\in A$. We write $H=F\tilde{\setminus}G$.
7. The complement F^c of F is defined as $(F,A)^c=(F^c,A)$, where $F^c:A\to\mathcal{P}(E)$ is a mapping given by $F^c(\lambda)=X\setminus F(\lambda)$ for all $\lambda\in A$. Clearly, we have $\tilde{E}^c=\Phi$ and $\Phi^c=\tilde{E}$

Definition 2.3. [16, 19] Let A be a non-empty parameter set and E be a non-empty set. Then a function $\epsilon:A\to E$ is said to be a soft element of E. A soft element ϵ of E is said to belongs to a soft set F of E which is denoted by $\epsilon\tilde{\in}F$ if $\epsilon(\lambda)\in F(\lambda)$, $\forall\lambda\in A$. Thus for a soft set F of E with respect to the index set A, we have $F(\lambda)=\{\epsilon(\lambda):\epsilon\tilde{\in}F\}$, $\lambda\in A$. In that case, ϵ is also said to be a soft element of the soft set F. Thus every singleton soft set (a soft set F of E for which $F(\lambda)$ is a singleton set, $\forall\lambda\in A$) can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall\lambda\in A$.

Throughout this paper, we consider the null soft set Φ and those soft sets F over E for which $F(\lambda)\neq\varnothing$, $\forall\lambda\in A$. We denote this collection by $S(\tilde{E})$. Thus for a soft set $F(\neq\Phi)\in S(\tilde{E})$, $F(\lambda)\neq\varnothing$ for all $\lambda\in A$. For any soft set $F\in S(\tilde{E})$, the collection of all soft elements of F is denoted by $SE(F)$. Also, we use the notation $\tilde{x},\tilde{y},\tilde{z}$ to denote soft elements of a soft set

Proposition 2.4. [19] The following statements about soft sets are satisfied.

1. For any soft sets $F,G\in S(\tilde{E})$, we have $F\tilde{\subset}G$ if and only if every soft element of F is also a soft elements of G.

2. Any collection of soft elements of a soft set can generate a soft subset of that soft set. The soft set constructed from a collection $\mathcal{B}$ of soft elements is denoted by $SS(\mathcal{B})$.
3. For any soft set $F \in S(\tilde{E})$, $SS(SE(F)) = F$; whereas for a collection $\mathcal{B}$ of soft elements, $SE(SS(\mathcal{B})) \supset \mathcal{B}$.

Remark 2.5. [19] $\tilde{x} \tilde{\in} F \tilde{\cup} G$ does not necessarily imply that either $\tilde{x} \tilde{\in} F$ or $\tilde{x} \tilde{\in} G$. Also, the intersection of two soft sets or complement of a soft set of $S(\tilde{E})$ is not necessarily a member of $S(\tilde{E})$

Definition 2.6. [19] Let $F, G \in S(\tilde{E})$ be two soft sets.

1. $F \Cup G$ denotes the e-union of F and G, that is defined by $F \Cup G = SS(\mathcal{B})$, where $\mathcal{B} = \left\{\tilde{x} \tilde{\in} \tilde{E} : \tilde{x} \tilde{\in} F \text{ or } \tilde{x} \tilde{\in} G\right\}$, ie, $F \Cup G = SS\left(SE(F) \cup SE(G)\right)$.
2. $F \Cap G$ denotes the e-intersection of F and G, that is defined by $F \Cap G = SS(\mathcal{B})$, where $\mathcal{B} = \left\{\tilde{x} \tilde{\in} \tilde{E} : \tilde{x} \tilde{\in} F \text{ and } \tilde{x} \tilde{\in} G\right\}$, ie, $F \Cap G = SS\left(SE(F) \cap SE(G)\right)$. If the two soft sets have no soft elements in common, then $F \Cap G = \Phi$.
3. $F^{\mathbb{C}}$ denotes the e-complement of F, that is defined by $F^{\mathbb{C}} = SS(\mathcal{B})$, where $\mathcal{B} = \left\{\tilde{x} \tilde{\in} \tilde{E} : \tilde{x} \tilde{\in} F^{C}\right\}$, ie, $F^{\mathbb{C}} = SS\left(SE(F^{\mathbb{C}})\right)$.

Remark 2.7. [19]. It can be easily verified that $F \Cup G$, $F \Cap G$, and $F^{\mathbb{C}}$ are members of $S(\tilde{E})$, if $F, G \in S(\tilde{E})$.

Definition 2. 8. [27, 28] Let $\mathcal{T} \subset S\left(\tilde{X}\right)$ be a family of soft sets over *X* with parameter set *A*. $\mathcal{T}$ is a topology on $\tilde{X}$ according to the e-operations if it meets the following conditions.

1. $\Phi, \tilde{X} \in \mathcal{T}$,
2. If $\{U_i\}_{i\in I} \subset \mathcal{T}$, then $\Cup_{i\in I} U_i \in \mathcal{T}$,
3. If $\{U_i\}_{i=1}^{n} \subset \mathcal{T}$, then $\Cap_{i=1}^{n} U_i \in \mathcal{T}$.

This topology is called e-soft topology, and the triple $\left(\tilde{X}, \mathcal{T}, A\right)$ is called a ϵ-soft topological space. The members of $\mathcal{T}$ are called soft open sets.

Proposition 2.9. [28] Let $\left(\tilde{X}, \mathcal{T}, A\right)$ be a ϵ-soft topological space. If $U \tilde{\cap} V \in S(\tilde{X})$ for every $U, V \in \mathcal{T}$, then for every $\lambda \in A$, $\mathcal{T}_\lambda = \{U(\lambda) : U \in \mathcal{T}\}$ is a topology on *X*.

Definition 2.10. [28] Let $f : SE\left(\tilde{X}\right) \to SE\left(\tilde{Y}\right)$ be a soft mapping. If for every $\lambda \in A$, $\tilde{x} \tilde{\in} \tilde{X}$ and $x \in X$, $\{f(\tilde{x})(\lambda) : \tilde{x}(\lambda) = x\}$ is a unit set, $f_\lambda : X \to Y$ given by $f_\lambda(\tilde{x}(\lambda)) = f(\tilde{x})(\lambda)$ is a mapping from *X* to *Y*. Then $f : SE\left(\tilde{X}\right) \to SE\left(\tilde{Y}\right)$ is called a soft function.

Definition 2.11. [28] Let $\left(\tilde{X}, \mathcal{T}, A\right)$ and $\left(\tilde{Y}, \mathcal{T}^*, A\right)$ be two ϵ-soft topological spaces. A soft mapping $f : SE\left(\tilde{X}\right) \to SE\left(\tilde{Y}\right)$ is called soft continuous at $\tilde{x} \tilde{\in} \tilde{X}$ if for any soft neighborhood N' of $f(\tilde{x})$ there exists N of $(\tilde{x})$

such that $f\left(SE(N)\right)\subset f\left(SE(N')\right)$. f is called soft continuous over $\left(\tilde{X},\mathcal{T},A\right)$, if it is soft continuous at every $\tilde{x}\,\tilde{\in}\,\tilde{X}$.

Theorem 2.12. [28] Let $\left(\tilde{X},\mathcal{T},A\right)$ and $\left(\tilde{Y},\mathcal{T}^{*},A\right)$ be two ϵ-soft topological spaces and $f: SE\left(\tilde{X}\right)\to SE\left(\tilde{Y}\right)$ be a soft mapping. f is soft continuous if and only if for every $U\in\mathcal{T}^{*}$, $SS\left(f^{-1}(SE(U))\right)\in\mathcal{T}$.

3. ϵ-soft compactness

Definition 3.1. [30] Let $\left(\tilde{X},\mathcal{T},A\right)$ be an ϵ-soft topological space, $F\in S(\tilde{X})$ and $\{U_i : i\in I\}$ be a family of soft open sets in $\left(\tilde{X},\mathcal{T},A\right)$.

i. $\{U_i : i\in I\}$ is called an ϵ-soft open cover of $\left(\tilde{X},\mathcal{T},A\right)$ if $\tilde{X}=\Cup_{i\in I}U_i$

ii. $\{U_i : i\in I\}$ is called an ϵ-soft open cover of F if $F\,\tilde{\subseteq}\,\Cup_{i\in I}U_i$.

The following definition is an adaptation of the definition for soft e-quasi compact space in [30].

Definition 3.2. Let $\left(\tilde{X},\mathcal{T},A\right)$ be a ϵ-soft topological space and $F\in S(\tilde{X})$.

i. $\left(\tilde{X},\mathcal{T},A\right)$ is called an ϵ-soft compact space if every ϵ-soft open cover of $\tilde{X}$ has a finite ϵ-soft open sub-cover.

ii. $F\in S(\tilde{X})$ is called an ϵ-soft compact set if every ϵ-soft open cover of F has a finite ϵ-soft open sub-cover.

Proposition 3.3. Let $\mathcal{T}=\left\{\Phi,\, U_i\in S(\tilde{X}) :\ U_i^{C}\in S(\tilde{X}),\, \bigcap U_i^{C}\in S(\tilde{X}),\, U_i^{\mathbb{C}}\ \text{is finite}\right\}\subset S(\tilde{X})$ be a family of soft sets over X with parameter set A. Then $\left(\tilde{X},\mathcal{T},A\right)$ is an ϵ-soft compact topological space.

Proof. Let's first show that $\left(\tilde{X},\mathcal{T},A\right)$ is an ϵ-soft topological space. For this three axioms of Definition 2.8 must be provided.

1. Since $\tilde{X}^{C}=\Phi\in S(\tilde{X})$ and $\tilde{X}^{\mathbb{C}}=SS(SE(\tilde{X}^{C}))=\Phi$ is finite, $\Phi,\tilde{X}\in\mathcal{T}$.

2. If $\{U_i\}_{i\in I}\subset\mathcal{T}$, Then $U_i^{C}\in S(\tilde{X})$ and $U_i^{\mathbb{C}}$ is finite. So $\tilde{\bigcup}_{i\in I}U_i^{C}\in S(\tilde{X})$ and $\Cup_{i\in I}U_i^{\mathbb{C}}$ is finite. Thus $\Cup_{i\in I}U_i\in\mathcal{T}$.

3. Let $U,V\in\mathcal{T}$. Then $U^{C},V^{C}\in S(\tilde{X})$, $U^{\mathbb{C}}$ and $V^{\mathbb{C}}$ are finite and $U^{C}\,\tilde{\cap}\,V^{C}\in S(\tilde{X})$. Then $U^{\mathbb{C}}\Cap V^{\mathbb{C}}$ is finite. So $U^{\mathbb{C}}\Cap V^{\mathbb{C}}\in\mathcal{T}$.

Now let's show that $\left(\tilde{X},\mathcal{T},A\right)$ is ϵ-soft compact. Let $\{U_i : i\in I\}$ be an ϵ-soft open cover of $\left(\tilde{X},\mathcal{T},A\right)$. Let's take a soft open set U_j in the family $\{U_i : i\in I\}$. Then $U_j^{C}\in S(\tilde{X})$ and $U_j^{\mathbb{C}}$ is finite. Say $U_j^{\mathbb{C}}=\{\tilde{u}_1,\tilde{u}_2,...,\tilde{u}_n\}$. Then, there are a finite number of the soft open sets $U_1,U_2,...,U_n$ such that $u_1\in U_1, u_2\in U_2,...,u_n\in U_n$. Thus, we obtain $\tilde{X}=U_j\Cup U_j^{\mathbb{C}}=U_j\Cup\left(\Cup_{i=1}^{n}U_i\right)$. Hence, the space $\tilde{X}$ is ϵ-soft compact as it is covered by a finite number of soft open sets.

One of the most important properties of compact spaces is that compactness is a topological property. This is a result of the following theorem.

Theorem 3.4. Let $\left(\tilde{X},\mathcal{T},A\right)$ and $\left(\tilde{Y},\mathcal{T}^*,A\right)$ be two ϵ-soft topological spaces and $F\in S(\tilde{X})$ be an ϵ-soft compact set. If $f:SE\left(\tilde{X}\right)\to SE\left(\tilde{Y}\right)$ is a soft continuous mapping, then $SS\left(f(SE(F)\right)\in S(\tilde{Y})$ is ϵ-soft compact.

Proof. Let $\left\{U_i\in\mathcal{T}^*:i\in I\right\}$ be any ϵ-soft open cover of $K=SS\left(f(SE(F)\right)$. Then $K\mathrel{\tilde{\subseteq}}\Cup_{i\in I}U_i$. Since each U_i is soft open and f is soft continuous, for every $i\in I$,

$$SS\left(f^{-1}\left(SE(U_1)\right)\right)\in\mathcal{T},\ SS\left(f^{-1}\left(SE(U_2)\right)\right)\in\mathcal{T},\ldots$$

Also,

$$\begin{aligned}F\mathrel{\tilde{\subseteq}}SS\left(f^{-1}\left(SE(K)\right)\right)&\mathrel{\tilde{\subseteq}}SS\left(f^{-1}\left(SE(U_1)\right)\right)\Cup SS\left(f^{-1}\left(SE(U_2)\right)\right)\Cup\ldots\\&=\Cup_{i\in I}SS\left(f^{-1}(SE(U_i))\right)=SS\left(\bigcup_{i\in I}f^{-1}(SE(U_i))\right)\\&=SS\left(f^{-1}\left(\bigcup_{i\in I}SE(U_i)\right)\right)=SS\left(f^{-1}\left(SE\left(\Cup_{i\in I}U_i\right)\right)\right)\end{aligned}$$

Then $\left\{SS\left(f^{-1}\left(SE(U_i)\right)\right):i\in I\right\}$ is an ϵ-soft open cover of F. But F is ϵ-soft compact, so $\left\{SS\left(f^{-1}\left(SE(U_i)\right)\right):i\in I\right\}$ is an ϵ-soft open sub-cover, say

$$\begin{aligned}F\mathrel{\tilde{\subseteq}}SS\left(f^{-1}\left(SE(U_{j1})\right)\right)\Cup SS\left(f^{-1}\left(SE(U_{j2})\right)\right)\Cup\ldots\Cup SS\left(f^{-1}\left(SE(U_{jm})\right)\right)\\=\Cup_{i=1}^{m}SS\left(f^{-1}(SE(U_{ji}))\right)=SS\left(f^{-1}\left(SE\left(\Cup_{i=1}^{m}U_{ji}\right)\right)\right)\end{aligned}$$

Accordingly,

$$\begin{aligned}K=SS\left(f(SE(F)\right)&\mathrel{\tilde{\subseteq}}SS\left(f\left(SE\left(\Cup_{i=1}^{m}SS\left(f^{-1}(SE(U_{ji}))\right)\right)\right)\right)\\&=SS\left(f\left(SE\left(SS\left(\bigcup_{i\in I}^{m}f^{-1}(SE(U_{ji}))\right)\right)\right)\right)\\&=SS\left(f\left(SE\left(SS\left(f^{-1}\left(\bigcup_{i\in I}^{m}SE(U_{ji})\right)\right)\right)\right)\right)\\&=SS\left(SE\left(SS\left(\bigcup_{i\in I}^{m}SE(U_{ji})\right)\right)\right)\\&=SS\left(SE\left(\Cup_{i=1}^{m}U_{ji}\right)\right)\\&=\Cup_{i=1}^{m}U_{ji}\end{aligned}$$

Thus $K=SS\left(f(SE(F)\right)$ is ϵ-soft compact.

Corollary 3.5. Let $(\tilde{X},\mathcal{T},A)$ and $(\tilde{Y},\mathcal{T}^*,A)$ are ϵ-soft topological spaces. If $(\tilde{X},\mathcal{T},A)$ is ϵ-soft compact and $f:SE(\tilde{X})\to SE(\tilde{Y})$ is a soft continuous and onto function, then $(\tilde{Y},\mathcal{T}^*,A)$ is ϵ-soft compact.

The following theorem gives the relationship between ϵ-soft compactness and classical compactness.

Theorem 3.6. Let $(\tilde{X},\mathcal{T},A)$ be a ϵ-soft compact space. If $U\tilde{\cap}V\in S(\tilde{X})$ for every $U,V\in\mathcal{T}$, Then for every $\lambda\in A$;

i. $(X,\mathcal{T}_\lambda)$ is a compact space.

ii. For every ϵ-soft compact set $F\in S(\tilde{X})$, $F(\lambda)\subset X$ is a compact set.

Proof. *i.* By Proposition 2.10, for every $\lambda\in A$, $\mathcal{T}_\lambda=\{U(\lambda):U\in\mathcal{T}\}$ is a topology on *X*. So $(X,\mathcal{T}_\lambda)$ is a topological space. Since $(\tilde{X},\mathcal{T},A)$ is ϵ-soft compact space, every ϵ-soft open cover $\{U_i:i\in I\}$ of $\tilde{X}$ has a finite ϵ-soft open sub-cover $\{U_{ij}:j=1,2,...,m\}$. That is, $\tilde{X}=\Cup_{i\in I}U_i$ implies $\tilde{X}=\Cup_{j=1}^{m}U_{ij}$. Hence, form $\tilde{X}=\Cup_{i\in I}U_i=SS\left(\bigcup_{i\in I}SE(U_i)\right)$ for every $\lambda\in A$, we have

$$X=\tilde{X}(\lambda)=SS\left(\bigcup_{i\in I}SE(U_i)\right)(\lambda)=\bigcup_{i\in I}U_i(\lambda)$$

and

$$X=\tilde{X}(\lambda)=\left(\Cup_{j\in I}^{m}U_{ij}\right)(\lambda)=SS\left(\bigcup_{j=1}^{m}SE(U_{ij})\right)(\lambda)=\bigcup_{j=1}^{m}U_{ij}(\lambda).$$

Thus $(X,\mathcal{T}_\lambda)$ is a compact space.

ii. Since *F* is ϵ-soft compact, every ϵ-soft open cover $\{U_i:i\in I\}$ of *F* has a finite ϵ-soft open sub-cover $\{U_{ij}:j=1,2,...,m\}$ such that $F\tilde{\subseteq}\Cup_{i\in I}U_i$ implies $F\tilde{\subseteq}\Cup_{j=1}^{m}U_{ij}$. Hence, $F\tilde{\subseteq}\Cup_{i\in I}U_i=SS\left(\bigcup_{i\in I}SE(U_i)\right)$ implies, for every $\lambda\in A$,

$$F(\lambda)\tilde{\subseteq}SS\left(\bigcup_{i\in I}SE(U_i)\right)(\lambda)=SS\left(\bigcup_{i\in I}SE(U_i)(\lambda)\right)=\bigcup_{i\in I}U_i(\lambda).$$

Thus $F(\lambda)$ is compact.

Example 3.7. Let $A=\{\lambda,\mu\}$ be a parameters set and $X=\{x,y,z\}$ be a set. Then $\mathcal{T}=\{\Phi,\tilde{X},U_1,U_2,U_3\}$ is a ϵ-soft topology of $\tilde{X}$, where

$$U_1=\{(\lambda,\{x\}),(\mu,\{y\})\}$$
$$U_2=\{(\lambda,\{x,y\}),(\mu,\{x,y\})\}$$
$$U_3=\{(\lambda,\{x,z\}),(\mu,\{y,z\})\}.$$

And $\tilde{X}=\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4,\tilde{x}_5,\tilde{x}_6,\tilde{x}_7,\tilde{x}_8,\tilde{x}_9\}$, where

$$\tilde{x}_1=\{(\lambda,\{x\}),(\mu,\{x\})\}, \qquad \tilde{x}_6=\{(\lambda,\{y\}),(\mu,\{z\})\},$$
$$\tilde{x}_2=\{(\lambda,\{x\}),(\mu,\{y\})\}, \qquad \tilde{x}_7=\{(\lambda,\{z\}),(\mu,\{x\})\},$$
$$\tilde{x}_3=\{(\lambda,\{x\}),(\mu,\{z\})\}, \qquad \tilde{x}_8=\{(\lambda,\{z\}),(\mu,\{y\})\},$$
$$\tilde{x}_4=\{(\lambda,\{y\}),(\mu,\{x\})\}, \qquad \tilde{x}_9=\{(\lambda,\{z\}),(\mu,\{z\})\}.$$
$$\tilde{x}_5=\{(\lambda,\{y\}),(\mu,\{y\})\},$$

Since $\tilde{X}$ is finite, $(\tilde{X},\mathcal{T},A)$ is ϵ-soft compact. Also $(X,\mathcal{T}_\lambda)$ and $(X,\mathcal{T}_\mu)$ are topological spaces, where

$$\mathcal{T}_\lambda=\{\varnothing,X,\{x\},\{x,y\},\{x,z\}\},$$
$$\mathcal{T}_\mu=\{\varnothing,X,\{y\},\{x,y\},\{y,z\}\}.$$

Since X is finite, $(X,\mathcal{T}_\lambda)$ and $(X,\mathcal{T}_\mu)$ are the compact spaces.

4. Conclusion

In this paper, we define cofinite e-soft space as an example of e-soft compact space and prove that the image of an ϵ-soft compact (and an ϵ-soft compact set) space under a soft continuous mapping is ϵ-soft compact space (and is ϵ-soft compact). Also, we examine the relationship between ϵ-soft compact space and classical compact space. We think it will contribute to the studies in this context.

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