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ON HERMITE-HADAMARD-TYPE INEQUALITIES FOR STRONGLY-LOG CONVEX STOCHASTIC PROCESSES

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ABSTRACT
In the present the work we introduce strongly logarithmic convex stochastic processes. Also, we obtain Hermite-Hadamard type integral inequalities for these processes.

ÖZET
Bu çalışmada, güçlü logaritmik konveks stokastik süreci tanıtılmaktadır. Ayrıca, bu aüreçler için Hermite-Hadamard tipi integral eşitsizliklerini elde edilmektedir.

Keywords: Hermite-Hadamar'd inequalities,strongly convex with modulus $c > 0$, log-convex function, stochastic process

Anahtar Kelimeler: Hermite-Hadamard eşitsizliği, $c > 0$ modüllü güçlü konveks, log-konveks fonksiyon, skotastik süreç.

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1. INTRODUCTION

In recent years, inequalities are playing a very significant role in all fields of mathematics, and present a very active and attractive field of research. One of the significant inequalities is well known the Hermite-Hadamard integral inequlity.

A function $f : I \to \mathbb{R}$, where $I \subseteq \mathbb{R}$ is an interval, is said to be a convex function on I if the inequality

$$f(tx+(1-t)y) \le tf(x)+(1-t)f(y) \qquad (1)$$

holds for all $x, y \in I$ and $t \in [0,1]$. If the reversed inequality in (1) holds, then f is concave. For some recent results related to this classic result, see the books [3, 4, 5, 6] and the papers [14, 15, 16, 17, 18, 19, 20, 21] where further references are given.

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function defined on the interval I of real numbers and $a < b$. The following double inequality

$$f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)dx\leq\frac{f(a)+f(b)}{2} \tag{2}$$

is well known in the literature as Hadamard's inequality. Both inequalities hold in the reversed direction if f is concave.

Recently, log-convex functions have gained much interest in mathematics and its sub-areas such as optimization theory. A function $f:I\to(0,\infty)$ is said to be *log*-convex (or multiplicatively convex) if $\log(f)$ is convex or namely the following inequality

$$f(tx+(1-t)y)\leq f(x)]^t[f(y)]^{(1-t)} \tag{3}$$

holds for all $x,y\in I$ and $t\in[0,1]$. Moreover, any *log*-convex function is a convex function since the inequality

$$f(x)]^t[f(y)]^{(1-t)}\leq tf(x)+(1-t)f(y) \tag{4}$$

holds for all $x,y\in I$ and $t\in[0,1]$. (Pecaric, 1992)
Recall that a function $f:I\to\mathrm{R}$ is called strongly convex with modulus $c>0$, if

$$f(tx+(1-t)y)\leq tf(x)+(1-t)f(y)-ct(1-t)(x-y)^2$$

for all $x,y\in I$ and $t\in(0,1)$. (Polyak, 1966)

Recall also that a function $f:I\to(0,\infty)$ is called strongly log-convex function with modulus $c>0$ if

$$f(tx+(1-t)y)\leq f(x)]^t[f(y)]^{(1-t)}-ct(1-t)(x-y)^2$$

for all $x,y\in I$ and $t\in(0,1)$. (Sarikaya, 2014)

2 PRELEMINARIES

Let $(\Omega,,P)$ be an arbitrary probability space. A function $X:\Omega\to\mathrm{R}$ is called a random variable if it is measurable. Let $(\Omega,,P)$ be an arbitrary probability space and let $T\subset\mathrm{R}$ be time. A collection of random variables $X(t,w)$, $t\in T$ with values in R is called a stochastic process. If $X(t,w)$ takes values in $S=\mathrm{R}^d$, it is called a vector-valued stochastic process. If the time T can be a discrete subset of R, then $X(t,w)$ is called a discrete time stochastic process. If time is an interval, R^+ or R, it is called a stochastic process with constinuous time. For any fixed $\omega\in\Omega$, one can regard $X(t,w)$ as a function of t. It is called a sample function of the stochastic process. In the case of a vector valued process, it is a sample path, a curve in R^d. Throughout the paper, we restrict our attention stochastic processes with continuous time, i.e. , index set $T=[0,\infty)$.

Let (Ω,A,P) be a probability space and $T\subset\mathrm{R}$ be an interval. We say that a stochastic process $X:T\times\Omega\to\mathrm{R}$ is *convex if*

$$X(\lambda u+(1-\lambda)v,\cdot)\leq\lambda X(u,\cdot)+(1-\lambda)X(v,\cdot)$$

for all $u,v\in T$ *and* $\lambda\in[0,1]$. This class of stochastic process are denoted by C .[13]

Let (Ω,A,P) be a probability space and $T\subset \mathbb{R}$ be an interval. We say that a stochastic process $X:T\times\Omega\to[0,\infty)$ is *log – convex if*

$$X\left(\lambda u+(1-\lambda)v,\cdot\right)\le\left[X\left(u,\cdot\right)\right]^{\lambda}\left[X\left(v,\cdot\right)\right]^{1-\lambda} \tag{5}$$

for all $u,v\in T$ and $\lambda\in[0,1]$. *log – convex stochastic processes have been introduced by* Tomar et al. in (Tomar, 2014) and they proved following theorem in this article.

Let us denote by $A(a,b)$ the aritmetic mean of the nonnegative real numbers, and by $G(a,b)$ the geometric mean of the same numbers.

Theorem 1 *Let* $X:T\times\Omega\to(0,\infty)$ *be a log – convex stochastic process on* $T\times\Omega$ *and* $u,v\in T$ *with* $u<v$. *Then, one has the inequalities:*

$$\begin{aligned}
&X\left(\frac{u+v}{2},\cdot\right)\le\exp\left[\frac{1}{v-u}\int_u^v\ln\left[X\left(t,\cdot\right)\right]dt\right] \qquad (6)\\
&\le\frac{1}{v-u}\int_u^v G\left(X\left(t,\cdot\right),X\left(u+v-t,\cdot\right)\right)dt\\
&\le\frac{1}{v-u}\int_u^v X\left(t,\cdot\right)dt\\
&\le L\left(X\left(u,\cdot\right),X\left(v,\cdot\right)\right),
\end{aligned}$$

where $L(p,q)$ is the logaritmic mean of strictly positive real numbers p,q, i.e.,

$$L(p,q)=\frac{p-q}{\ln p-\ln q}\quad \text{if}\quad p\ne q\quad\text{and}\quad L(p,p)=p.$$

Also, note that the related results for convex stochastic processes and various types of convex stochastic processes can be seen in (Skowronski, 1995), (Skowronski, 1995), (Nikodem, 1980), (Bekar, 2014,) (Maden, 2014), (Set, 2014), (Kotrys, 2014).

The main subject of this paper is to introduce strongly-log-convex stochastic processes with modulus $c>0$ and to give Hermite-Hadamard type inequalities for these processes, such as in (6).

3 HERMITE-HADAMAD TYPE INEQUALITIES FOR STRONGLY LOG-CONVEX STOCHASTİC PROCESSES

Definition 1 *Let* (Ω,A,P) *be a probability space and* $T\subset\mathbb{R}$ *be an interval. We say that a stochastic process* $X:T\times\Omega\to[0,\infty)$ *is strongly log – convex with modulus* $c>0$ *if*

$$X\left(\lambda u+(1-\lambda)v,\cdot\right)\le\left[X\left(u,\cdot\right)\right]^{\lambda}\left[X\left(v,\cdot\right)\right]^{1-\lambda}-c\lambda(1-\lambda)(v-u)^2 \tag{7}$$

for all $u,v\in T$ and $\lambda\in(0,1)$.

The following result offers the Hermite-Hadamard type inequalities for strongly $log-$convex stochastic process.

Theorem 2 *If a stochastic process* $X:T\times\Omega\to(0,\infty)$ *be a strongly* $log-$*convex with modulus* $c>0$ *and integrable on* $T\times\Omega$*, we have*

$$\begin{aligned}
&X\left(\frac{u+v}{2},\cdot\right)+c\frac{(v-u)^2}{12} \\
&\le\frac{1}{v-u}\int_u^v G\left(X\left(t,\cdot\right),X\left(u+v-t,\cdot\right)\right)dt \\
&\le\frac{1}{v-u}\int_u^v X\left(t,\cdot\right)dt \\
&\le L\left(X(u,\cdot),X(v,\cdot)\right)-c\frac{(v-u)^2}{6} \\
&\le A(X(u,\cdot),X(v,\cdot))-c\frac{(v-u)^2}{6}
\end{aligned} \tag{8}$$

for all $u,v\in I$ with $u<v$.

Proof. From (7) and aritmatic-geometric mean, we have

$$\begin{aligned}
&X\left(\alpha s+(1-\alpha)z,\cdot\right)\le\left[X\left(s,\cdot\right)\right]^{\alpha}\left[X\left(z,\cdot\right)\right]^{1-\alpha}-c\alpha(1-\alpha)(v-u)^2 \\
&\le\alpha X\left(s,\cdot\right)+(1-\alpha)X\left(z,\cdot\right)-c\alpha(1-\alpha)(v-u)^2.
\end{aligned} \tag{9}$$

If we take $\alpha=\frac{1}{2}$ in (9), we have

$$\begin{aligned}
&X\left(\frac{s+z}{2},\cdot\right)\le\sqrt{X\left(s,\cdot\right)X\left(z,\cdot\right)}-c\frac{(z-s)^2}{4} \\
&\le\frac{X\left(s,\cdot\right)+X\left(z,\cdot\right)}{2}-c\frac{(z-s)^2}{4}.
\end{aligned} \tag{10}$$

i.e., $s=\lambda u+(1-\lambda)v,\ z=(1-\lambda)u+\lambda v,$

$$\begin{aligned}
&X\left(\frac{u+v}{2},\cdot\right) \\
&\le\sqrt{X\left(\lambda u+(1-\lambda)v,\cdot\right)X\left((1-\lambda)u+\lambda v,\cdot\right)}-c\frac{(v-u)^2(1-2\lambda)^2}{4} \\
&\le\frac{X\left(\lambda u+(1-\lambda)v,\cdot\right)+X\left((1-\lambda)u+\lambda v,\cdot\right)}{2}-c\frac{(v-u)^2(1-2\lambda)^2}{4}.
\end{aligned} \tag{11}$$

Integrating the inequality (11) on $(0,1)$ over λ, and taking into account,

$$\int_0^1 X\left(\lambda u+(1-\lambda)v,\cdot\right)d\lambda=\int_0^1 X\left((1-\lambda)u+\lambda v,\cdot\right)d\lambda$$

we obtain

$$X\left(\frac{u+v}{2},\cdot\right)\le\frac{1}{v-u}\int_u^v G\left(X(t,\cdot),X(u+v-t,\cdot)\right)dt-c\frac{(v-u)^2}{12} \tag{12}$$
$$\le\frac{1}{v-u}\int_u^v A\left(X(t,\cdot),X(u+v-t,\cdot)\right)dt-c\frac{(v-u)^2}{12}.$$

And so,

$$X\left(\frac{u+v}{2},\cdot\right)+c\frac{(v-u)^2}{12} \tag{13}$$
$$\le\frac{1}{v-u}\int_u^v G\left(X(t,\cdot),X(u+v-t,\cdot)\right)dt$$
$$\le\frac{1}{v-u}\int_u^v X(t,\cdot)dt.$$

Since X is a strongly $log-$convex function on $T\times\Omega$, for $s=u$ and $z=v$, we get

$$X\left(\lambda u+(1-\lambda)v,\cdot\right)\le\left[X(u,\cdot)\right]^{\lambda}\left[X(v,\cdot)\right]^{1-\lambda}-c\lambda(1-\lambda)(v-u)^2 \tag{14}$$
$$\le\lambda X(u,\cdot)+(1-\lambda)X(v,\cdot)-c\lambda(1-\lambda)(v-u)^2.$$

Integrating the inequality (14) on $(0,1)$ over λ,

$$\frac{1}{v-u}\int_u^v X(t,\cdot)dt\le X(v,\cdot)\int_0^1\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}d\lambda-c(v-u)^2\int_0^1\lambda(1-\lambda)d\lambda$$
$$\le X(u,\cdot)\int_0^1\lambda d\lambda+X(v,\cdot)\int_0^1(1-\lambda)d\lambda-c(v-u)^2\int_0^1\lambda(1-\lambda)d\lambda,$$

and thereby

$$\frac{1}{v-u}\int_u^v X(t,\cdot)dt \tag{15}$$
$$\le L\left(X(u,\cdot),X(v,\cdot)\right)-c\frac{(v-u)^2}{6}$$
$$\le A(X(u,\cdot),X(v,\cdot))-c\frac{(v-u)^2}{6}$$

So, from (13) and (15), the theorem is proved.

Theorem 3 *If a stochastic process* $X:T\times\Omega\to(0,\infty)$ *be a strongly* $log-$*convex with modulus* $c>0$ *and integrable on* $T\times\Omega$*, we have*

$$\frac{1}{v-u}\int_u^v X(t,\cdot)X(u+v-t,\cdot)dt \tag{16}$$
$$\le X(u,\cdot)X(v,\cdot)+\frac{c^2(v-u)^4}{30}$$
$$-\frac{4c(v-u)^2}{\ln[X(u,\cdot-X(v,\cdot))]^2}\left[A(X(u,\cdot),X(v,\cdot))+L\left(X(u,\cdot),X(v,\cdot)\right)\right]$$
$$\le\frac{2\left[A(X(u,\cdot),X(v,\cdot))\right]^2+\left[G(X(u,\cdot),X(v,\cdot)\right]^2}{3}$$
$$-\frac{cA(X(u,\cdot),X(v,\cdot))(v-u)^2}{3}+\frac{c^2(v-u)^4}{30}$$

for all $u,v\in I$ with $u<v$.

Proof. Since X is strongly log-convex stochastic process with modulus $c>0$, we have that for all $\lambda\in(0,1)$

$$X\left(\lambda u+(1-\lambda)v,\cdot\right)\le\left[X(u,\cdot)\right]^{\lambda}\left[X(v,\cdot)\right]^{1-\lambda}-c\lambda(1-\lambda)(v-u)^2 \tag{17}$$
$$\le\lambda X(u,\cdot)+(1-\lambda)X(v,\cdot)-c\lambda(1-\lambda)(v-u)^2$$

and

$$X\left((1-\lambda)u+\lambda v,\cdot\right)\le\left[X(u,\cdot)\right]^{1-\lambda}\left[X(v,\cdot)\right]^{\lambda}-c\lambda(1-\lambda)(v-u)^2 \tag{18}$$
$$\le(1-\lambda)X(u,\cdot)+\lambda X(v,\cdot)-c\lambda(1-\lambda)(v-u)^2$$

Multiplying both sides of (17) by (18), it follows that

$$X\left(\lambda u+(1-\lambda)v,\cdot\right)X\left((1-\lambda)u+\lambda v,\cdot\right) \tag{19}$$
$$\le X(u,\cdot)X(v,\cdot)+c^2\lambda^2(1-\lambda)^2(v-u)^4$$
$$-c\lambda(1-\lambda)(v-u)^2\left(X(v,\cdot)\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}+X(u,\cdot)\left[\frac{X(v,\cdot)}{X(u,\cdot)}\right]^{\lambda}\right)$$
$$\le\lambda(1-\lambda)\left(\left[X(u,\cdot)\right]^2+\left[X(v,\cdot)\right]^2\right)+\lambda^2(1-\lambda)^2X(u,\cdot)X(v,\cdot)$$
$$-c(v-u)^2\lambda(1-\lambda)\left[X(u,\cdot)+X(v,\cdot)\right]+c^2\lambda^2(1-\lambda)^2(v-u)^4$$

Integrating the inequality (19) with respect to λ over $(0,1)$ and , we obtain

$$\int_0^1 X\left(\lambda u+(1-\lambda)v,\cdot\right)X\left((1-\lambda)u+\lambda v,\cdot\right)d\lambda \tag{20}$$
$$\le\int_0^1 X(u,\cdot)X(v,\cdot)d\lambda+c^2(v-u)^4\int_0^1\lambda^2(1-\lambda)^2d\lambda$$
$$-c(v-u)^2X(v,\cdot)\int_0^1\lambda(1-\lambda)\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}d\lambda$$
$$-c(v-u)^2X(u,\cdot)\int_0^1\lambda(1-\lambda)\left[\frac{X(v,\cdot)}{X(u,\cdot)}\right]^{\lambda}d\lambda$$
$$\le\left(\left[X(u,\cdot)\right]^2+\left[X(v,\cdot)\right]^2\right)\int_0^1\lambda(1-\lambda)d\lambda+X(u,\cdot)X(v,\cdot)\int_0^1\lambda^2(1-\lambda)^2d\lambda$$

$$-c(v-u)^2\left[X(u,\cdot)+X(v,\cdot)\right]\int_0^1\lambda(1-\lambda)d\lambda+c^2(v-u)^4\int_0^1\lambda^2(1-\lambda)^2d\lambda$$

Integrating by parts for I_1 and I_2 integrals, we obtain

$$I_1=\int_0^1\lambda(1-\lambda)\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}d\lambda \tag{21}$$

$$=\lambda(1-\lambda)\frac{1}{\ln\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]}\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}\Bigg|_0^1-\frac{1}{\ln\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]}\int_0^1(1-2\lambda)\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}d\lambda$$

$$=-\frac{1}{\ln\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]}\left[(1-2\lambda)\frac{1}{\ln\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]}\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}\Bigg|_0^1+\frac{2}{\ln\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]}\int_0^1\left[\frac{X(u,\cdot)}{X(v,\cdot)}\right]^{\lambda}d\lambda\right]$$

$$=\frac{1}{X(v,\cdot)}\frac{X(u,\cdot)+X(v,\cdot)}{\left[\ln X(u,\cdot)-\ln X(v,\cdot)\right]^2}+\frac{2X(u,\cdot)-2X(v,\cdot)}{\left[\ln X(u,\cdot)-\ln X(v,\cdot)\right]^2}$$

and similarly we get,

$$I_2=\int_0^1\lambda(1-\lambda)\left[\frac{X(v,\cdot)}{X(u,\cdot)}\right]^{\lambda}d\lambda \tag{22}$$

$$=\frac{1}{X(u,\cdot)}\frac{X(u,\cdot)+X(v,\cdot)}{\left[\ln X(u,\cdot)-\ln X(v,\cdot)\right]^2}+\frac{2X(v,\cdot)-2X(u,\cdot)}{\left[\ln X(u,\cdot)-\ln X(v,\cdot)\right]^2}.$$

And also we get,

$$\left(\left[X(u,\cdot)\right]^2+\left[X(v,\cdot)\right]^2\right)\int_0^1\lambda(1-\lambda)d\lambda+X(u,\cdot)X(v,\cdot)\int_0^1\lambda^2(1-\lambda)^2d\lambda \tag{23}$$

$$-c(v-u)^2\left[X(u,\cdot)+X(v,\cdot)\right]\int_0^1\lambda(1-\lambda)d\lambda+c^2(v-u)^4\int_0^1\lambda^2(1-\lambda)^2d\lambda$$

$$=\frac{\left[X(u,\cdot)\right]^2+\left[X(v,\cdot)\right]^2}{6}+\frac{2X(u,\cdot)X(v,\cdot)}{3}$$

$$-\frac{c(v-u)^2\left[X(u,\cdot)+X(v,\cdot)\right]}{6}+\frac{c^2(v-u)^4}{6}$$

$$=\frac{2\left[A(X(u,\cdot),X(v,\cdot))\right]^2+\left[G(X(u,\cdot),X(v,\cdot)\right]^2}{3}$$

$$-\frac{cA(X(u,\cdot),X(v,\cdot))(v-u)^2}{3}+\frac{c^2(v-u)^4}{30}.$$

Putting (21), (22) and (23), and if we change the variable $t:=\lambda u+(1-\lambda)v,\ \lambda\in(0,1)$, we get the requeired inequality in (16). This proves the theorem.

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