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On solving SDEs with linear coefficients and application to stochastic epidemic models

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Abstract

Stochastic Differential Equations (SDEs) are extensively utilized to model numerous physical quantities from different fields. In particular, linear SDEs are used in epidemic modeling. It is crucial to ensure the positivity of several quantities in an epidemic model. Numerous articles on this topic proves the positivity of SDEs solutions using probabilistic tools, such as in Theorem 3.1 of [10]. In this work, we suggest an alternative way to show the positivity of the solutions. The proposed approach is based on finding solutions to linear SDEs using Itô formula. We comment on several examples of stochastic epidemic models existing in the literature.

Keywords: Epidemic models, Stochastic differential equations, Stochastic epidemic models.
2010 MSC: 60H10, 65C20, 92D30.

1. Introduction

Nowadays, we have many types of interesting differential equations utilized for modelling problems arising from different fields of science. For instance, one can find works on fractional differential equations such as in [6], [3], [2], and/or their applications to study the speed of cancer see for example [4] or to investigate the effect of quarantine or vaccination during pandemic situations as in [13]. Stochastic differential equations constitute since decades a very active trend of research when modeling real life problems such as in [8] for COVID-19 or modelling SARS-COV2 as in [7].

In this paper, the interest is on Stochastic epidemic models that utilize a system of linear SDEs. There

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are plenty of papers on stochastic epidemic models. Yet, as far as we could tell, these papers usually discuss the existence of a global positive solution, but they do not investigate finding a solution of the SDE. In this work, we propose an alternate method to demonstrate the positivity of the solutions of linear SDEs of the form:

$$dX(t) = [\alpha(t) + \beta(t)X(t)]dt + [\gamma(t) + \theta(t)X(t)]dB(t),$$

where B is a standard Brownian motion. The processes $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $\theta(t)$ are all square integrable adapted processes not dependent on X. The reader can find a solution to the above SDE in the book of [9]. In this short communication, we provide also detailed solutions of a system of linear SDEs using Itô formula. Moreover, we use the solution to show when it is positive, and we apply all this to a variety of existing stochastic epidemic models. This can be seen as a direct alternative to what is existing in the literature, where research articles on this topic do not solve the SDEs but prove the existence and positivity of the solution in a relatively long proof.

This paper is structured as follows: Theoretical solutions of linear SDEs is presented in Section 2. Section 3 offers applications to a variety of stochastic epidemic models. Finally, concluding remarks are discussed in Section 4.

2. SDEs with linear coefficients

Let $(\Omega, F_T, (F_t)_{t\in[0,T]}, P)$ be a filtered probability space and let $W := (W(t))_{t\in[0,T]}$ be a m-dimensional Brownian motion. We assume that the filtration $(F_t)_{t\in[0,T]}$ is the natural filtration generated by the Brownian motion W. Consider the n-dimensional stochastic process $X := (X(t))_{t\in[0,T]}$ which satisfies the below system of Stochastic Differential Equations (SDEs)

$$\begin{aligned}
dX_1(t) &= [\alpha_1(t) + \beta_1(t)X_1(t)]dt + \sum_{j=1}^{m}[\gamma_{1j}(t) + \theta_{1j}(t)X_1(t)]\rho_{1j}dW_j(t) \\
&\;\;. \qquad\qquad . \qquad\qquad\qquad . \\
dX_k(t) &= [\alpha_k(t) + \beta_k(t)X_k(t)]dt + \sum_{j=1}^{m}[\gamma_{kj}(t) + \theta_{kj}(t)X_k(t)]\rho_{kj}dW_j(t) \qquad (1) \\
&\;\;. \qquad\qquad . \qquad\qquad\qquad . \\
dX_n(t) &= [\alpha_n(t) + \beta_n(t)X_n(t)]dt + \sum_{j=1}^{m}[\gamma_{nj}(t) + \theta_{nj}(t)X_n(t)]\rho_{nj}dW_j(t),
\end{aligned}$$

where $X(0) = (X_1(0), \dots, X_n(0))$ is a given positive real vector and $t \in [0, T]$. We assume that $\alpha_k(t) := \alpha(t, \hat{X}^{-k}(t))$, $\beta_k(t) := \beta(t, \hat{X}^{-k}(t))$, $\gamma_k(t) := \gamma(t, \hat{X}^{-k}(t))$, and $\theta_k(t) := \theta(t, \hat{X}^{-k}(t))$ where

$$\hat{X}^{-k}(t) := (X_1(t), \dots, X_{k-1}(t), X_{k+1}(t), \dots, X_n(t))$$

is the $n-1$-dimension vector obtained from $X(t)$ by removing the k^{th} component for any $k = 1, \dots, n$. In other words, for each SDE of the above system, the coefficients are linear.

To achieve this objective, we need to employ the Itô formula provided in the next lemma.

Lemma 2.1. *Consider a process $Y = (Y_1, \dots, Y_n)$, where for $k \in \{1, \dots, n\}$, Y_k is driven by the stochastic differential equation*

$$dY_k(t) = u_k(t)dt + \sum_{j=1}^{m} v_{kj}(t)dW_j(t), \quad Y_k(0) \in \mathbb{R}_+,$$

where $v_{kj} = (v_{1j}, \ldots, v_{nj})$, $j = 1, \ldots, m$, as well $u = (u_1, \ldots, u_n)$ denotes n-dimensional square-integrable adapted processes. Given a function $g(y)$ twice continuously differentiable from $\mathbb{R}^n$ to $\mathbb{R}$, then we have

$$dg(Y(t)) = \sum_{k=1}^{n} \frac{\partial g}{\partial y_k}(Y(t))dY_k(t) + \sum_{k,l=1}^{n} \frac{1}{2}\frac{\partial^2 g}{\partial y^k y^l}(Y(t))d\left\langle Y_k(t), Y_l(t)\right\rangle, \tag{2}$$

where $d\left\langle W_k(t), W_l(t)\right\rangle = \delta_{kl}dt$ and $d\left\langle t, W_k(t)\right\rangle = d\left\langle W_k(t), t\right\rangle = d\left\langle t, t\right\rangle = 0$.

Note that if is a function f twice continuously differentiable from $\mathbb{R}$ to $\mathbb{R}$, then the formula (2) can be reduced to

$$\begin{aligned} df(Y_k(t)) = \quad & \left[f^{'}(Y_k(t))u_k(t) + \frac{1}{2}f^{''}(Y_k(t)) \sum_{j=1}^{m} v_{kj}^2(t) \right] dt \\ & + f^{'}(Y_k(t)) \sum_{j=1}^{m} v_{kj}(t)dW_j(t). \end{aligned} \tag{3}$$

We also need the processes $(\zeta_k(t))_{t\in[0,T]}$, $k \in \{1, \ldots, n\}$ defined by the SDE

$$d\zeta_k(t) = a_k(t)\zeta_k(t)dt + \sum_{j=1}^{m} b_{kj}(t)\zeta_k(t)dW_j(t), \qquad \zeta_k(0) = 1, \tag{4}$$

where $(a_k(t))_{t\in[0,T]}$ and $(b_{kj}(t))_{t\in[0,T]}$ are two stochastic processes that do not depend on ζ_k. The solution of (4) can be obtained by applying Itô formula (3) to $\ln \zeta_k(t)$; i.e. one obtains

$$\zeta_k(t) = \exp\left[\int_0^t \left(a_k(u) - \frac{1}{2}\sum_{j=1}^{m} b_{kj}^2(u) \right) du + \sum_{j=1}^{m} \int_0^t b_{kj}(u)dW_j(u) \right], \tag{5}$$

where $t \in [0, T]$. Now, we can provide the main result of this paper.

Theorem 2.2. *Let $k \in \{1, 2, \ldots, n\}$. The solution of X_k the k^{th} process of the system (1) is given by*

$$\begin{aligned} X_k(t) \quad = \quad \zeta_k(t) & \left[X_k(0) + \int_0^t [\alpha_k(u) - \sum_{j=1}^{m} \theta_{kj}(u)\gamma_{kj}(u)\rho_{kj}^2]\zeta_k^{-1}(u)du \right. \\ & \left. + \sum_{j=1}^{m} \int_0^t \gamma_{kj}(t)\rho_{kj}\zeta^{-1}(u)dW_j(u) \right]. \end{aligned} \tag{6}$$

where $t \in [0, T]$, and $\zeta_k(t)$ is as in equation (5).

Proof. Let $k \in \{1, 2, \ldots, n\}$ and assume that the solution of $X_k(t)$ can be written as

$$X_k(t) := Z_k(t)\zeta(t), \quad t \in [0, T], \quad \text{with} \quad X_k(0) = Z_k(0), \tag{7}$$

where $Z_k(t)$ is a stochastic process to be determined. From (7) we can write $Z_k(t) = X_k(t)\zeta^{-1}(t)$. By Itô formula and (4), $d\zeta_k^{-1}(t)$ can be expressed as

$$d\zeta_k^{-1}(t) = \left(-a_k(t) + \sum_{j=1}^{m} b_{kj}^2(t) \right) \zeta_k^{-1}(t)dt - \sum_{j=1}^{m} b_{kj}(t)\zeta_k^{-1}(t)dW_j(t), \tag{8}$$

with $\zeta_k^{-1}(0) = 1$. Then, $dZ_k(t)$ can be calculated using the below integration by parts for stochastic processes

$$dZ_k(t) = d(\zeta_k^{-1}(t)X_k(t)) = \zeta_k^{-1}(t)dX_k(t) + X_k(t)d\zeta_k^{-1}(t) + [d\zeta_k^{-1}(t), dX_k(t)].$$

Using (1) and (8) the above expression can be written as

$$\begin{aligned} dZ_k(t) &= \zeta_k^{-1}(t)\Bigg([\alpha_k(t)+\beta_k(t)X_k(t)]dt + \sum_{j=1}^m [\theta_{kj}(t)X_k(t) \\ &\quad +\gamma_{kj}(t)]\rho_{kj}dW_j(t)) + \zeta_k^{-1}(t)\Bigg\{X_k(t)\Bigg((-a_k(t)+\sum_{j=1}^m b_{kj}^2(t))dt \\ &\quad -\sum_{j=1}^m b_{kj}(t)dW_j(t)\Bigg) - \sum_{j=1}^m b_{kj}(t)[\gamma_{kj}(t)+\theta_{kj}(t)X_k(t)]\rho_{kj}dt\Bigg\} \\ &= \Bigg(\sum_{j=1}^m \gamma_{kj}(t)\rho_{kj}\zeta^{-1}(t) + [\sum_{j=1}^m \theta_{kj}(t)\rho_{kj} - b_{kj}(t)]Z_k(t)\Bigg)dW_j(t). \\ &\quad + \Bigg([\alpha_k(t) - \sum_{j=1}^m b_{kj}(t)\gamma_{kj}(t)\rho_{kj}]\zeta_k^{-1}(t) + [\beta_k(t) - a_k(t) \\ &\quad + \sum_{j=1}^m b_{kj}(t)(b_{kj}(t)-\theta_{kj}(t)\rho_{kj})\Bigg] Z_k(t)\Bigg) dt. \end{aligned} \tag{9}$$

Assume that a_k and b_{kj} have the values

$$a_k(t) = \beta_k(t), \qquad \text{and} \qquad b_{kj}(t) = \theta_{kj}(t)\rho_{kj}, \quad \text{for } j \in \{1,\dots,m\}. \tag{10}$$

Then $dZ_k(t)$ reduces to

$$dZ_k(t) = [\alpha_k(t) - \sum_{j=1}^m \theta_{kj}(t)\gamma_{kj}(t)\rho_{kj}^2]\zeta_k^{-1}(t)dt + \sum_{j=1}^m \gamma_{kj}(t)\rho_{kj}\zeta^{-1}(t)dW_j(t),$$

and thus

$$\begin{aligned} Z_k(t) &= Z_k(0) + \int_0^t [\alpha_k(u) - \sum_{j=1}^m \theta_{kj}(u)\gamma_{kj}(u)\rho_{kj}^2]\zeta_k^{-1}(u)du \\ &\quad + \sum_{j=1}^m \int_0^t \gamma_{kj}(t)\rho_{kj}\zeta^{-1}(u)dW_j(u). \end{aligned}$$

Using the above equation and (7), we obtain the theoretical solution of $X_k(t)$ as expressed in (6) which ends the proof. □

The next proposition provides an important particular case of the Theorem. 2.2 which can be applied to epidemic stochastic models.

Proposition 2.3. *Consider the system (1) and let $\gamma_{kj}(t) = 0$ for any $k \in \{1,\dots,n\}$, $j \in \{1,\dots,m\}$, and $t \in [0,T]$. Then the solution (6) is given by*

$$X_k(t) = \zeta_k(t)\left(X_k(0) + \int_0^t \alpha_k(u)\zeta_k^{-1}(u)du\right). \tag{11}$$

Moreover if $X_k(0) > 0$ and $\alpha_k(t) \geq 0$ then $X_k(t) > 0$.

Proof. To obtain the solution (11) it is sufficient to vanish the terms $\gamma_{kj}(t)$ in (6). The positivity of $X_k(t)$ is guaranteed when $X_k(0) > 0$ and $\alpha_k(t) \geq 0$ since by (5) $\zeta_k^{-1}(t) > 0$ for any $t \in [0,T]$. □

3. Application to the solution of SDEs of stochastic epidemic models

In this section, we provide solutions to a variety of stochastic epidemic models and show that the solutions are positive using Proposition. 2.3. We first consider the below SIRI stochastic model discussed in [10]

$$\begin{aligned} dS(t) &= [\alpha - \lambda S(t)I(t) - \beta S(t)]dt + \sigma_1 S(t)dW_1(t) \\ dI(t) &= [\lambda S(t)I(t) - (\gamma+\beta)I(t) + \eta R(t)]dt + \sigma_2 I(t)dW_2(t) \\ dR(t) &= [\gamma I(t) - (\eta+\beta)R(t)]dt + \sigma_3 R(t)dW_3(t). \end{aligned} \tag{12}$$

To solve (12), we apply Proposition. 2.3 with $k = 3$, $m = 3$, $\rho_{kj} = \delta_{kj}$, and $\gamma_{kj} = 0$ (for $k, j = 1, 2, 3$). In addition, we apply the following assumptions

$$\begin{aligned} &X_1 = S, \quad \alpha_1 = \alpha, \quad a_1 = \beta_1 = -(\lambda I + \beta), \quad b_{1j} = \theta_{1j} = \sigma_1 \quad j = 1,2,3 \\ &X_2 = I, \quad \alpha_2 = \eta R, \quad a_2 = \beta_2 = \lambda S - (\lambda+\beta), \quad b_{2j} = \theta_{2j} = \sigma_2 \quad j = 1,2,3 \\ &X_3 = R, \quad \alpha_3 = \gamma I, \quad a_3 = \beta_3 = -(\eta+\beta), \quad b_{3j} = \theta_{3j} = \sigma_3 \quad j = 1,2,3. \end{aligned} \tag{13}$$

Using equations (5), (10), and (13), one may explicitly express $\zeta_k(t)$, $k = 1, 2, 3$. Consequently, with the implementation of (11), we obtain the following exact solutions for the SIRI model (12):

$$\begin{aligned} S(t) &= S(0)\zeta_1(t) + \alpha \int_0^t \zeta_1(t-u)du, \\ I(t) &= I(0)\zeta_2(t) + \eta \int_0^t R(u)\zeta_2(t-u)du, \\ R(t) &= R(0)\zeta_3(t) + \gamma \int_0^t I(t)\zeta_3(t-u)du. \end{aligned} \tag{14}$$

It is a trivial task to show that S, I, and R given in (14) are positive and unique. This is a clear proof of the importance of Proposition 2.3 which provides a simple and straightforward alternative to the long discussion in section 3 of [10]. It should be mentioned herein that authors in [10] proved the uniqueness and positivity of the solutions of the SDEs of the SIRI model (12) by using a totally different theory.

Tables 1 and 2 show the implementation of our present strategy -using Equations (5) and (11)- to handle several stochastic epidemic models already published in the literature; [12], [1], [14], [5], [11].

Ref.	Model and solution by Proposition 2.3
[12]	$dS_m = [\Gamma_1 - \mu S_m - a_1 S_m I_f]\,dt + \sigma_{S_m} S_m dB_{S_m}(t)$, $dS_f = [\Gamma_2 - \mu S_f - a_2 S_f I_m]\,dt + \sigma_{S_f} S_f dB_{S_f}(t)$, $dI_m = [a_1 S_m I_f - (\mu+\beta_1) I_m - b_1 I_m]\,dt + \sigma_{I_m} I_m dB_{I_m}(t)$, $dI_f = [a_2 S_f I_m - (\mu+\beta_2) I_f - b_2 I_f]\,dt + \sigma_{I_f} I_f dB_{I_f}(t)$, Parameters' values for solutions by (11) and (5), $b_{ij} = 0, i \neq j$ $X_1 = S_m,\ \alpha_1 = \Gamma_1,\ a_1 = -(\mu + a_1 I_f),\ b_{11} = \sigma_{S_m}$, $X_2 = S_f,\ \alpha_2 = \Gamma_2,\ a_1 = -(\mu + a_2 I_m),\ b_{22} = \sigma_{S_f}$, $X_3 = I_m,\ \alpha_3 = a_1 S_m I_f,\ a_3 = -(\mu + \beta_1 + b_1),\ b_{33} = \sigma_{I_m}$ $X_4 = I_f,\ \alpha_4 = a_2 S_f I_m,\ a_4 = -(\mu + \beta_2 - b_2),\ b_{44} = \sigma_{I_f}$

[1]	$\frac{dS}{dt} = (\mu N - \beta IS - (\mu+\omega)S - \emptyset V)dt + \sigma_1 S dB_1,$ $\frac{dV}{dt} = (\omega S - (\mu+\emptyset)V)dt + \sigma_2 V dB_2,$ $\frac{dL}{dt} = (\beta IS - (\mu+\delta+\gamma)L)dt + \sigma_3 L dB_3,$ $\frac{dI}{dt} = (\gamma L - (\mu+\alpha+d)I) + \sigma_4 I dB_4,$ Parameters' values for solutions by (11) and (5), $b_{ij} = 0, i \neq j$ $X_1 = S,\ \alpha_1 = \mu N - \emptyset V,\ a_1 = -(\beta I + \mu + \omega),\ b_{11} = \sigma_1,$ $X_2 = V,\ \alpha_2 = \omega S,\ a_1 = -(\mu + \emptyset)V,\ b_{22} = \sigma_2,$ $X_3 = L,\ \alpha_3 = \beta IS,\ a_3 = -(\mu+\delta+\gamma),\ b_{33} = \sigma_3$ $X_4 = I,\ \alpha_4 = \gamma L,\ a_4 = -(\mu+\alpha+d),\ b_{44} = \sigma_4$
[14]	$dS(t) = (\mu - \beta S(t)I(t) - (\mu+\phi)S(t))dt - \sigma S(t)I(t)dW(t)$ $dI(t) = (\beta S(t)I(t) + \rho\beta V(t)I(t) - (\lambda+\mu)I(t))dt + \sigma(S(t)$ $+\rho V(t))I(t)dW(t)$ $dV(t) = (\phi S(t) - \rho\beta V(t)I(t) - \mu V(t))dt - \rho\sigma V(t)I(t)dW(t)$ $dR(t) = (\lambda I(t) - \mu R(t))dt,$ Parameters' values for solutions by (11) and (5), $b_{ij} = 0, i \neq j$ $X_1 = S,\ \alpha_1 = \mu,\ a_1 = -(\beta I(t) + \mu + \phi),\ b_{11} = -\sigma I(t),$ $X_2 = I,\ \alpha_2 = 0,\ a_2 = \beta S(t) + \rho\beta V(t) - (\lambda+\mu),$ $b_{22} = \sigma(S(t) + \rho V(t))$ $X_3 = V,\ \alpha_3 = \phi S(t),\ a_3 = -(\rho\beta I(t) + \mu),\ b_{33} = -\rho\sigma I(t)$ $X_4 = R,\ \alpha_4 = \lambda I(t),\ a_4 = -\mu,\ b_{44} = 0$

Table 1: Application of Proposition. 2.3 to some existing stochastic epidemic models.

Ref.	Model and solution by Proposition 2.3
[5]	$dS(t) = [\Lambda - \beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \mu S(t)]dt$ $-\sigma(I(t) + \eta_C C(t) + \eta_A A(t))S(t)dB(t),$ $dI(t) = [\beta(I(t) + \eta_C C(t) + \eta_A A(t))S(t) - \xi_3 I(t)$ $+\alpha A(t) + \omega C(t)]dt$ $+\sigma(I(t) + \eta_C C(t) + \eta_A A(t))I(t)dB(t),$ $dC(t) = [\phi I(t) - \xi_2 C(t)]dt$ $dA(t) = [\rho I(t) - \xi_1 A(t)]dt,$ Parameters' values for solutions by (11) and (5), $b_{ij} = 0, i \neq j$ $X_1 = S,\ \alpha_1 = \Lambda,\ a_1 = -(\beta(I(t) + \eta_C C(t) + \eta_A A(t)) + \mu),$ $b_{11} = -\sigma(I(t) + \eta_C C(t) + \eta_A A(t)),$ $X_2 = I,\ \alpha_2 = \beta(\eta_C C(t) + \eta_A A(t))S(t) + \alpha A(t) + \omega C(t),$ $a_2 = \beta S(t) - \xi_3,\ b_{22} = \sigma(I(t) + \eta_C C(t) + \eta_A A(t)),$ $X_3 = C,\ \alpha_3 = \phi I(t),\ a_3 = -\xi_2,\ b_{33} = 0$ $X_4 = A,\ \alpha_4 = \rho I(t),\ a_4 = -\xi_1,\ b_{44} = 0$

The model considered in our paper is slightly modified from the model treated in [5]. Everything is the same except in the equation of $dI(t)$, in the coefficient of $dB(t)$, we replace $S(t)$ by $I(t)$.

	$$\begin{aligned} dS &= \left[\Lambda - \beta\left(I + \eta_C C + \eta_A A\right) S - \mu S\right] dt \\ &\quad + \left(\sigma_{11} + \sigma_{12} S\right) S dB_1(t) \\ dI &= \left[\beta\left(I + \eta_C C + \eta_A A\right) S - \xi_3 I + \omega C + \alpha A\right] dt \\ &\quad + \left(\sigma_{21} + \sigma_{22} I\right) I dB_2(t) \\ dC &= \left[\phi I - \xi_2 C\right] dt + \left(\sigma_{31} + \sigma_{32} C\right) C dB_3(t) \\ dA &= \left[\rho I - \xi_1 A\right] dt + \left(\sigma_{41} + \sigma_{42} A\right) A dB_4(t) \end{aligned}$$
[11]	Parameters' values for solutions by (11) and (5), $b_{ij} = 0, i \neq j$
	$$\begin{aligned} X_1 &= S,\ \alpha_1 = \Lambda,\ a_1 = -(\beta(I + \eta_C C + \eta_A A) + \mu), \\ &\quad b_{11} = \sigma_{11} + \sigma_{12} S, \\ X_2 &= I,\ \alpha_2 = \beta(\eta_C C + \eta_A A)S + \alpha A + \omega C, \\ &\quad a_2 = \beta S(t) - \xi_3,\ b_{22} = \sigma_{21} + \sigma_{22} I, \\ X_3 &= C,\ \alpha_3 = \phi I,\ a_3 = -\xi_2,\ b_{33} = \sigma_{31} + \sigma_{32} C \\ X_4 &= A,\ \alpha_4 = \rho I,\ a_4 = -\xi_1,\ b_{44} = \sigma_{41} + \sigma_{42} A \end{aligned}$$

Table 2: Solutions of other recent stochastic epidemic models using Proposition. 2.3.

4. Conclusion

Stochastic Differential Equations are applied in many real-life problems. For example, linear SDEs are extensively utilized when modeling epidemic situations. Showing existence and the positivity of solutions is among the important points to investigate while studying such models. The literature contains a huge number of articles that deal with these issues using probability techniques, but without looking at the SDEs solutions of the model. This research communication suggests investigating the properties of a given stochastic epidemic model by solving the SDEs of the model and show for instance the positivity from the expression of an SDE's solution. Our approach is applied to several existing stochastic epidemic models.

References

[1] K. Abodayeh, M.S. Arif, A. Raza, M. Rafiq, M. Bibi, A. Nazeer, Numerical techniques for stochastic foot and mouth disease epidemic model with the impact of vaccination, Advances in Difference Equations 1 (2020) 1–14.

[2] Q.M. Al-Mdallal, M.A. Hajji, T. Abdeljawad, On the iterative methods for solving fractional initial value problems: new perspective, Journal of Fractional Calculus and Nonlinear Systems 2(1) (2021) 76–81.

[3] A. Atangana, S. Jain, A new numerical approximation of the fractal ordinary differential equation, The European Physical Journal Plus 133:2(2018) 1–15.

[4] A. Atangana, S. Jain, The role of power decay, exponential decay and Mittag-Leffler function's waiting time distribution: application of cancer spread, Physica A: Statistical Mechanics and its Applications 512(2018) 330–351.

[5] J. Djordjevic, C.J. Silva, D.FM Torres, A stochastic SICA epidemic model for HIV transmission, Applied Mathematics Letters 84(2018) 168–175.

[6] S. Jain, Numerical analysis for the fractional diffusion and fractional Buckmaster equation by the two-step Laplace Adam-Bashforth method, The European Physical Journal Plus 133:1(2018) 1–11.

[7] T. Khan, R. Ullah, G. Zaman, Y. El Khatib, Modeling the dynamics of the SARS-CoV-2 virus in a population with asymptomatic and symptomatic infected individuals and vaccination, Physica Scripta 96:10(2021) 104009.

[8] T. Khan, G. Zaman, Y. El Khatib, Modeling the dynamics of novel coronavirus (covid-19) via stochastic epidemic model, Results in Physics 24(2021)104004, 2021.

[9] P.E. Kloeden, E. Platen, Numerical Solution of Stochastic Differential Equations, Springer(1992).

[10] Q. Lei, Z. Yang, Dynamical behaviors of a stochastic SIRI epidemic model, Applicable Analysis 96:16(2017) 2758–2770.

[11] Q. Liu, Dynamics of a stochastic SICA epidemic model for HIV transmission with higher-order perturbation, Stochastic Analysis and Applications (2021) 1–40.

[12] A. Rathinasamy, M. Chinnadurai, S. Athithan, Analysis of exact solution of stochastic sex-structured HIV/AIDS epidemic model with effect of screening of infectives, Mathematics and Computers in Simulation 179 (2021) 213–237.

[13] D.A. Reza, M.N. Billah, S.S. Shanta, Effect of quarantine and vaccination in a pandemic situation: a mathematical modelling approach, Journal of Mathematical Analysis and Modeling 2:3(2021) 77–87.

[14] E. Tornatore, P. Vetro, S.M. Buccellato, SIVR epidemic model with stochastic perturbation, Neural Computing and Applications 24:2(2014) 309–315.