Mathematical Sciences And Applications E-Notes
Volume 1 No. 2 pp. 28–34 (2013) ©MSAEN

# INEXTENSIBLE FLOWS OF CURVES IN 4-DIMENSIONAL GALILEAN SPACE $G_4$

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(Communicated by Bayram ŞAHİN)

Abstract. In this paper we study inextensible flows of curves in 4-dimensional Galilean space. We give necessary and sufficient conditions for inextensible flows are expressed as a partial differential equation involving the curvature in 4-dimensional Galilean space.

## 1. INTRODUCTION

It is well known that many nonlinear phenomena in physics, chemistry and biology are described by dynamics of shapes, such as curves and surfaces. The evolution of curve and surface has significant applications in computer vision and image processing. The time evolution of a curve or surface generated by its corresponding flow in $\mathbb{R}^3$ is said to be inextensible if, in the former case, its arclength is preserved, and in the latter case, if its intrinsic curvature is preserved. Physically, the inextensible curve flows give rise to motions in which no strain energy is induced. The swinging motion of a cord of fixed length, for example, or of a piece of paper carried by the wind, can be described by inextensible curve and surface flows. Such motions arise quite naturally in a wide range of the physical applications. The inextensible curve and surface flows also arise in the context of many problems in computer vision [6], [8], computer animation [9] and even structural mechanics [4].

The distinction between heat flows and inextensible flows of planar curves were elaborated in detail, and some examples of the latter were given by [3]. Also, a general formulation for inextensible flows of curves and developable surfaces in $\mathbb{R}^3$ are exposed by [2]. Latifi et al. [5] studied inextensible flows of curves in Minkowski 3-space. Moreover Öğrenmiş et al. [1] studied inextensible curves in the Galilean space $G_3$ and Ergüt et al.[7] studied characterization of inextensible flows of spacelike curves with Sabban Frame in $S_1^2$.

In this paper we study inextensible flows of curves in 4-dimensional Galilean space. We give necessary and sufficient conditions for inextensible flows are expressed as a partial differential equation involving the curvature in 4-dimensional Galilean space.

*Date*: Received: February 6, 2013; Revised: April 17, 2013; Accepted: May 9, 2013.
2000 *Mathematics Subject Classification.* 53A35,53A04,53A05.
*Key words and phrases.* Inextensible flows, Galilean Space.

## 2. PRELIMINARIES

In Affine coordinates the Galilean scalar product between two points

$$P_i = (p_{i1}, p_{i2}, p_{i3}, p_{i4}),\ i = 1, 2$$

is defined by

$$g(P_1, P_2) = \begin{cases} |p_{21} - p_{11}|, & if \quad p_{21} \neq p_{11}, \\ \sqrt{\left|(p_{22} - p_{12})^2 + (p_{23} - p_{13})^2 + (p_{24} - p_{14})^2\right|}, & if \quad p_{21} = p_{11}. \end{cases}$$

We define the Galilean cross product in $G_4$ for the vectors $\overrightarrow{u} = (u_1, u_2, u_3, u_4)$, $\overrightarrow{v} = (v_1, v_2, v_3, v_4)$ and $\overrightarrow{w} = (w_1, w_2, w_3, w_4)$ as follows:

$$\overrightarrow{u} \wedge \overrightarrow{v} \wedge \overrightarrow{w} = - \begin{vmatrix} 0 & e_2 & e_3 & e_4 \\ u_1 & u_2 & u_3 & u_4 \\ v_1 & v_2 & v_3 & v_4 \\ w_1 & w_2 & w_3 & w_4 \end{vmatrix},$$

where $e_i$ , $1 \leq i \leq 4$, are the standart basis vectors.

The scalar product of two vectors $\overrightarrow{U} = (u_1, u_2, u_3, u_4)$ and $\overrightarrow{V} = (v_1, v_2, v_3, v_4)$ in $G_4$ is defined by

$$\left\langle \overrightarrow{U}, \overrightarrow{V} \right\rangle_{G_4} = \begin{cases} u_1 v_1, & \text{if } u_1 \neq 0 \text{ or } v_1 \neq 0, \\ u_2 v_2 + u_3 v_3 + u_4 v_4 & \text{if } u_1 = 0 \text{ and } v_1 = 0. \end{cases}$$

The norm of vector $\overrightarrow{U} = (u_1, u_2, u_3, u_4)$ is defined by

$$\left\| \overrightarrow{U} \right\|_{G_4} = \sqrt{\left| \left\langle \overrightarrow{U}, \overrightarrow{U} \right\rangle_{G_4} \right|}.$$

Let $\alpha : I \subset R \longrightarrow G_4$ , $\alpha(s) = (s, y(s), z(s), w(s))$ be a curve parametrized by arclength $s$ in $G_4$. Here we denote differentiation with respect to $s$ by a dash. The first vector of the Frenet-Serret frame, that is the tangent vector of $\alpha$ is defined by

$$\mathbf{t} = \alpha'(s) = (1, y'(s), z'(s), w'(s)).$$

Since $t$ is a unit vector, so we can express

$$\langle \mathbf{t}, \mathbf{t} \rangle_{G_4} = 1. \tag{2.1}$$

Differentiating the equation (2.1) with respect to $s$, we have

$$\langle \mathbf{t}', \mathbf{t} \rangle_{G_4} = 0.$$

The vector function $\mathbf{t}'$ gives us the rotation measurement of the curve $\alpha$. The real valued function

$$\kappa(s) = \|\mathbf{t}'(s)\| = \sqrt{(y''(s))^2 + (z''(s))^2 + (w''(s))^2}$$

is called the first curvature of the curve $\alpha$. We assume that, $\kappa(s) \neq 0$, for all $s \in I$. Similar to space $G_3$, we define the principal vector

$$\mathbf{n}(s) = \frac{\mathbf{t}'(s)}{\kappa(s)}$$

in another words

$$\mathbf{n}(s) = \frac{1}{\kappa(s)}(0, y''(s), z''(s), w''(s)) \tag{2.2}$$

By the aid of the differentiation of the principal normal vector given in (2.2), we define the second curvature function as

$$\tau(s) = \|\mathbf{n}'(s)\|_{G_4}. \tag{2.3}$$

This real valued function is called torsion of the curve $\alpha$. The third vector field, namely binormal vector field of the curve $\alpha$ is defined by

$$\mathbf{b}(s) = \frac{1}{\tau(s)}\left(0, \left(\frac{y''(s)}{\kappa(s)}\right)', \left(\frac{z''(s)}{\kappa(s)}\right)', \left(\frac{w''(s)}{\kappa(s)}\right)'\right). \tag{2.4}$$

Thus the vector $\mathbf{b}(s)$ is both perpendicular to $\mathbf{t}$ and $\mathbf{n}$. The fourth unit vector is defined by

$$\mathbf{e}(s) = \mu \mathbf{t}(s) \Lambda \mathbf{n}(s) \Lambda \mathbf{b}(s). \tag{2.5}$$

Here the coefficient $\mu$ is taken $\pm 1$ to make $+1$ determinant of the matrix $[\mathbf{t}, \mathbf{n}, \mathbf{b}, \mathbf{e}]$.

We define the third curvature of the curve $\alpha$ by the Galilean inner product

$$\sigma = \langle \mathbf{b}', \mathbf{e} \rangle_{G_4}. \tag{2.6}$$

Here, as well known, the set $\{\mathbf{t}, \mathbf{n}, \mathbf{b}, \mathbf{e}, \kappa, \tau, \sigma\}$ is called the Frenet-Serret apparatus of the curve $\alpha$. We know that the vectors $\{\mathbf{t}, \mathbf{n}, \mathbf{b}, \mathbf{e}\}$ are mutually orthogonal vectors satisfying

$$\langle \mathbf{t}, \mathbf{t} \rangle_{G_4} = \langle \mathbf{n}, \mathbf{n} \rangle_{G_4} = \langle \mathbf{b}, \mathbf{b} \rangle_{G_4} = \langle \mathbf{e}, \mathbf{e} \rangle_{G_4} = 1, \tag{2.7}$$

$$\langle \mathbf{t}, \mathbf{n} \rangle_{G_4} = \langle \mathbf{t}, \mathbf{b} \rangle_{G_4} = \langle \mathbf{t}, \mathbf{e} \rangle_{G_4} = \langle \mathbf{n}, \mathbf{b} \rangle_{G_4} = \langle \mathbf{n}, \mathbf{e} \rangle_{G_4} = \langle \mathbf{b}, \mathbf{e} \rangle_{G_4} = 0.$$

For the curve $\alpha$ in $G_4$, we have following the Frenet-Serret equations

$$\mathbf{t}' = \kappa(s)\mathbf{n}(s), \ \mathbf{n}' = \tau(s)\mathbf{b}(s), \ \mathbf{b}' = -\tau(s)\mathbf{n}(s) + \sigma(s)\mathbf{e}(s), \ \mathbf{e}' = -\sigma(s)\mathbf{b}(s), \tag{2.8}$$

[10].

## 3. INEXTENSIBLE FLOWS OF CURVES IN 4D GALILEAN SPACE

Throughout this paper, we assume that $\alpha(u,t)$ is a one parameter family of smooth curves in 4-dimensional Galilean space $G_4$. The arclength of $\gamma$ is given by

$$s(u) = \int_0^u \left|\frac{\partial \gamma}{\partial u}\right| du, \tag{3.1}$$

where

$$\left|\frac{\partial \gamma}{\partial u}\right| = \left|\left\langle \frac{\partial \gamma}{\partial u}, \frac{\partial \gamma}{\partial u} \right\rangle\right|^{\frac{1}{2}}. \tag{3.2}$$

The operator $\frac{\partial}{\partial s}$ is given in terms of $u$ by

$$\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u},$$

where $v = \left|\frac{\partial \gamma}{\partial u}\right|$ and the arclength parameter is $ds = vdu$.

Any flow of $\gamma$ can be represented as

$$\frac{\partial \gamma}{\partial t} = f_1 \mathbf{t} + f_2 \mathbf{n} + f_3 \mathbf{b} + f_4 \mathbf{e} \tag{3.3}$$

Letting the arclength variation be

$$s(u,t) = \int_0^u v du \ .$$

In the 4-dimensional Galilean space $G_4$ the requirement that the curve not be subject to any elongation or compression can be expressed by the condition

$$\frac{\partial}{\partial t} s(u,t) = \int_0^u \frac{\partial v}{\partial t} du = 0 \,, \tag{3.4}$$

for all $u \in [0,l]$.

**Definition 3.1.** A curve evolution $\gamma(u,t)$and its flow $\frac{\partial \gamma}{\partial t}$ in 4D Galilean space $G_4$ are said to be inextensible if

$$\frac{\partial}{\partial t}\left|\frac{\partial \gamma}{\partial u}\right| = 0.$$

**Lemma 3.1.** *Let* $\frac{\partial \gamma}{\partial t} = f_1\mathbf{t} + f_2\mathbf{n} + f_3\mathbf{b} + f_4\mathbf{e}$ *be a smooth flow of the curve* $\gamma$ *in* $G_4$*. The flow is inextensible if and only if*

$$\frac{\partial v}{\partial t} = \frac{\partial f_1}{\partial u} \ . \tag{3.5}$$

*Proof. Suppose that* $\frac{\partial \gamma}{\partial t}$ *be a smooth flow of the curve* $\gamma$ *in* $G_4$*. Using definition of* $\gamma$ *, we have*

$$v^2 = \left\langle \frac{\partial \gamma}{\partial u}, \frac{\partial \gamma}{\partial u} \right\rangle \ . \tag{3.6}$$

So, by differentiating of the formula (3.6), we get

$$v\frac{\partial v}{\partial t} = \left\langle \frac{\partial \gamma}{\partial u}, \frac{\partial}{\partial u}\left(f_1\mathbf{t} + f_2\mathbf{n} + f_3\mathbf{b} + f_4\mathbf{e}\right) \right\rangle .$$

By the formula of the Frenet, we have

$$\frac{\partial v}{\partial t} = \left\langle \mathbf{t}, \frac{\partial f_1}{\partial u}\mathbf{t} + \left(f_1 v\kappa + \frac{\partial f_2}{\partial u} - f_3 v\tau\right)\mathbf{n} + \left(f_2 v\tau + \frac{\partial f_3}{\partial u} - f_4 v\sigma\right)\mathbf{b} + \left(f_3 v\sigma + \frac{\partial f_4}{\partial u}\right)\mathbf{e} \right\rangle .$$

Making necessary calculations from above equation, we have (3.5), which proves the lemma.

**Theorem 3.1.** *Let* $\frac{\partial \gamma}{\partial t} = f_1\mathbf{t} + f_2\mathbf{n} + f_3\mathbf{b} + f_4\mathbf{e}$ *be a smooth flow of the curve* $\gamma$ *in* $G_4$*. The flow is inextensible if and only if*

$$\frac{\partial f_1}{\partial u} = 0 \ . \tag{3.7}$$

Proof. From (3.4), we have

$$\frac{\partial}{\partial t} s(u,t) = \int_0^u \frac{\partial v}{\partial t} du = \int_0^u \frac{\partial f_1}{\partial u} = 0 \ . \tag{3.8}$$

Substituting (3.5) in (3.8) complete the proof of the theorem.

We now restrict ourselves to arc length parametrized curves. That is, $v = 1$ and the local coordinate $u$ corresponds to the curve arc length $s$. We require the following lemma.

**Lemma 3.2.** *Let* $\frac{\partial\gamma}{\partial t} = f_1\mathbf{t} + f_2\mathbf{n} + f_3\mathbf{b} + f_4\mathbf{e}$ *be a smooth flow of the curve* $\gamma$ *in* $G_4$*. Then,*

$$\frac{\partial \mathbf{t}}{\partial t} = \left(f_1\kappa + \frac{\partial f_2}{\partial s} - f_3\tau\right)\mathbf{n} + \left(f_2\tau + \frac{\partial f_3}{\partial s} - f_4\sigma\right)\mathbf{b} + \left(\frac{\partial f_4}{\partial s} + f_3\sigma\right)\mathbf{e}\ , \tag{3.9}$$

$$\frac{\partial \mathbf{n}}{\partial t} = \left(-f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau\right)\mathbf{t} + \Psi_1 b + \Psi_2\mathbf{e}\ , \tag{3.10}$$

$$\frac{\partial \mathbf{b}}{\partial t} = \left(-f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma\right)\mathbf{t} - \Psi_1\mathbf{n} + \Psi_3\mathbf{e}\ , \tag{3.11}$$

where $\Psi_1 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{b}\rangle$, $\Psi_2 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{e}\rangle$, $\Psi_3 = \langle \frac{\partial \mathbf{b}}{\partial t}, \mathbf{e}\rangle$ provided that $\left(-f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau\right) = 0$ and $\left(-f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma\right) = 0$ .

Proof. Under the asumption, we have

$$\frac{\partial \mathbf{t}}{\partial t} = \frac{\partial}{\partial t}\frac{\partial\gamma}{\partial s} = \frac{\partial}{\partial s}\left(f_1\mathbf{t} + f_2\mathbf{n} + f_3\mathbf{b} + f_4\mathbf{e}\right).$$

Thus, it is seen that

$$\frac{\partial \mathbf{t}}{\partial t} = \frac{\partial f_1}{\partial s}\mathbf{t} + \left(f_1\kappa + \frac{\partial f_2}{\partial s} - f_3\tau\right)\mathbf{n} + \left(f_2\tau + \frac{\partial f_3}{\partial s} - f_4\sigma\right)\mathbf{b} + \left(\frac{\partial f_4}{\partial s} + f_3\sigma\right)\mathbf{e}. \tag{3.12}$$

On the other hand substituting (3.7) in (3.12), we get

$$\frac{\partial \mathbf{t}}{\partial t} = \left(f_1\kappa + \frac{\partial f_2}{\partial s} - f_3\tau\right)\mathbf{n} + \left(f_2\tau + \frac{\partial f_3}{\partial s} - f_4\sigma\right)\mathbf{b} + \left(\frac{\partial f_4}{\partial s} + f_3\sigma\right)\mathbf{e}\,.$$

The differentiation of the Frenet frame with respect to $t$ is

$$\begin{aligned}
0 &= \frac{\partial}{\partial t}\langle \mathbf{t}, \mathbf{n}\rangle = f_1\kappa + \frac{\partial f_2}{\partial s} - f_3\tau + \left\langle \mathbf{t}, \frac{\partial \mathbf{n}}{\partial t}\right\rangle, \\
0 &= \frac{\partial}{\partial t}\langle \mathbf{t}, \mathbf{b}\rangle = f_2\tau + \frac{\partial f_3}{\partial s} - f_4\sigma + \left\langle \mathbf{t}, \frac{\partial \mathbf{b}}{\partial t}\right\rangle, \\
0 &= \frac{\partial}{\partial t}\langle \mathbf{t}, \mathbf{e}\rangle = \frac{\partial f_4}{\partial s} + f_3\sigma + \left\langle \mathbf{t}, \frac{\partial \mathbf{e}}{\partial t}\right\rangle\ , \\
0 &= \frac{\partial}{\partial t}\langle \mathbf{n}, \mathbf{b}\rangle = \psi_1 + \left\langle \mathbf{n}, \frac{\partial \mathbf{b}}{\partial t}\right\rangle\ , \\
0 &= \frac{\partial}{\partial t}\langle \mathbf{n}, \mathbf{e}\rangle = \psi_2 + \left\langle \mathbf{n}, \frac{\partial \mathbf{e}}{\partial t}\right\rangle\ , \\
0 &= \frac{\partial}{\partial t}\langle \mathbf{b}, \mathbf{e}\rangle = \psi_3 + \left\langle \mathbf{b}, \frac{\partial \mathbf{e}}{\partial t}\right\rangle\ .
\end{aligned}$$

From the above and using

$$\left\langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{n} \right\rangle = \left\langle \frac{\partial \mathbf{b}}{\partial t}, \mathbf{b} \right\rangle = \left\langle \frac{\partial \mathbf{e}}{\partial t}, \mathbf{e} \right\rangle = 0 \ ,$$

we obtain

$$\begin{aligned} \frac{\partial \mathbf{n}}{\partial t} &= \left( -f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau \right) \mathbf{t} + \Psi_1 \mathbf{b} + \Psi_2 \mathbf{e} \ , \\ \frac{\partial \mathbf{b}}{\partial t} &= \left( -f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma \right) \mathbf{t} - \Psi_1 \mathbf{n} + \Psi_3 \mathbf{e} \ , \\ \frac{\partial \mathbf{e}}{\partial t} &= \left( -\frac{\partial f_4}{\partial s} - f_3\sigma \right) \mathbf{t} - \Psi_2 \mathbf{n} - \Psi_3 \mathbf{b} \ . \end{aligned}$$

where $\Psi_1 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{b} \rangle$ , $\Psi_2 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{e} \rangle$ , $\Psi_3 = \langle \frac{\partial \mathbf{b}}{\partial t}, \mathbf{e} \rangle$ provided that $\left( -f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau \right) = 0$ and $\left( -f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma \right) = 0$. Thus, we obtain the theorem.

**Theorem 3.2.** *Let* $\frac{\partial \gamma}{\partial t} = f_1\mathbf{t} + f_2\mathbf{n} + f_3\mathbf{b} + f_4\mathbf{e}$ *be a smooth flow of the curve* $\gamma$ *in* $G_4$*. Then, the following system of partial differential equations holds:*

$$\begin{aligned} \frac{\partial \kappa}{\partial t} &= 0 \ , \\ \sigma \left( \frac{\partial f_4}{\partial s} + f_3\sigma \right) &= -\frac{\Psi_1}{\Psi_2} \left( \frac{\partial}{\partial s}(f_3\sigma) + \frac{\partial^2 f_4}{\partial s^2} \right) \end{aligned}$$

*where* $\Psi_1 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{b} \rangle$ , $\Psi_2 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{e} \rangle$ *provided that* $\left( -f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau \right) = 0$ *and* $\left( -f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma \right) = 0$.

*Proof. From our assumption provided that* $\left( -f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau \right) = 0$ *and* $\left( -f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma \right) = 0$*, we have*

(3.13)

$$\frac{\partial}{\partial s}\frac{\partial \mathbf{t}}{\partial t} = \frac{\partial}{\partial s}\left[ \left( \frac{\partial f_4}{\partial s} + f_3\sigma \right) \mathbf{e} \right] = \left[ \frac{\partial}{\partial s}(f_3\sigma) + \frac{\partial^2 f_4}{\partial s^2} \right] \mathbf{e} - \left[ \sigma \left( \frac{\partial f_4}{\partial s} + f_3\sigma \right) \right] \mathbf{b}$$

*On the other hand, from the Frenet frame we have*

$$\frac{\partial}{\partial t}\frac{\partial \mathbf{t}}{\partial s} = \frac{\partial}{\partial t}(\kappa \mathbf{n}) = \frac{\partial \kappa}{\partial t}\mathbf{n} + \kappa (\Psi_1 \mathbf{b} + \Psi_2 \mathbf{e}) \tag{3.14}$$

*Hence from (3.13) and (3.14), we get*

$$\begin{aligned} \frac{\partial \kappa}{\partial t} &= 0 \ , \\ \sigma \left( \frac{\partial f_4}{\partial s} + f_3\sigma \right) &= -\frac{\Psi_1}{\Psi_2} \left( \frac{\partial}{\partial s}(f_3\sigma) + \frac{\partial^2 f_4}{\partial s^2} \right) \ . \end{aligned}$$

*Thus we obtain following theorem.*

**Theorem 3.3.** *Let* $\frac{\partial \gamma}{\partial t} = f_1\mathbf{t} + f_2\mathbf{n} + f_3\mathbf{b} + f_4\mathbf{e}$ *be a smooth flow of the curve* $\gamma$ *in* $G_4$*. Then, the following system of partial differential equations holds:*

Then,

$$\begin{aligned} \frac{\partial \tau}{\partial t} &= \frac{\partial \Psi_1}{\partial s} - \Psi_2\sigma \ , \\ \tau &= \frac{1}{\Psi_3} \left( \frac{\partial \Psi_2}{\partial s} + \Psi_1\sigma \right) \ , \end{aligned}$$

where $\Psi_1 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{b}\rangle$ ,$\Psi_2 = \langle \frac{\partial \mathbf{n}}{\partial t}, \mathbf{e}\rangle$ ,$\Psi_3 = \langle \frac{\partial \mathbf{b}}{\partial t}, \mathbf{e}\rangle$ provided that $\left(-f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau\right) = 0$ and $\left(-f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma\right) = 0$.

Proof. Similarly, from Frenet formulas provided that $\left(-f_1\kappa - \frac{\partial f_2}{\partial s} + f_3\tau\right) = 0$ and $\left(-f_2\tau - \frac{\partial f_3}{\partial s} + f_4\sigma\right) = 0$ , we have

$$\frac{\partial}{\partial s}\frac{\partial \mathbf{n}}{\partial t} = \frac{\partial}{\partial s}\left(\Psi_1\mathbf{b} + \Psi_2\mathbf{e}\right) = -\tau\Psi_1\mathbf{n} + \left[\frac{\partial \Psi_1}{\partial s} - \Psi_2\sigma\right] b + \left[\frac{\partial \Psi_2}{\partial s} + \Psi_1\sigma\right]\mathbf{e} \tag{3.15}$$

On the other hand, a straightforward computation gives

$$\frac{\partial}{\partial t}\frac{\partial \mathbf{n}}{\partial s} = \frac{\partial}{\partial t}\left(\tau\mathbf{b}\right) = \frac{\partial \tau}{\partial t}\mathbf{b} + \tau\left(-\Psi_1\mathbf{n} + \Psi_3\mathbf{e}\ \right) \tag{3.16}$$

Hence from (3.15) and (3.16)

$$\begin{aligned} \frac{\partial \tau}{\partial t} &= \frac{\partial \Psi_1}{\partial s} - \Psi_2\sigma \quad , \\ \tau &= \frac{1}{\Psi_3}\left(\frac{\partial \Psi_2}{\partial s} + \Psi_1\sigma\right) \quad . \end{aligned}$$

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