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ALGEBRAIC HYPERSTRUCTURES OF SOFT SETS ASSOCIATED TO N-ARY POLYGROUPS

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Abstract. This paper concerns a relationship between soft sets and n-ary polygroups. We consider the notion of an n-ary polygroup as a generalization of a polygroup and apply the notion of soft sets to n-ary polygroups. Some related notions are defined and several basic properties are discussed by using the soft set theory. Furthermore, we propose the homomorphism of n-ary polygroups and investigate the properties which are preserved under the homomorphism.

1. Introduction

The theory of algebraic hyperstructures introduced by Marty in 1934 [13] is a natural generalization of the theory of classical algebraic structures. It has been applied in many areas [3], such as geomety, lattices, fuzzy sets and rough sets, automata, cryptography, combinatorics, artifical intelligence, probabilities and so on. The concept of n-ary hypergroups is introduced by Davvaz and Vougiouklis [4] for the first time. n-ary polygroups defined by Ghadiri and Waphare in [8] are suitable generalizations of polygroups and a special case of n-ary hypergroups.

To solve complicated problems in economics, engineering, environmental science, medical science and social sciences, methods in algebraic hyperstructures may not be successfully used because of various uncertainties arising in these problems. Several theories such as probability theory, fuzzy set theory [19], vague set theory [7], rough set theory [16, 17] and interval mathematics [9] were established to model uncertainties appearing in the above fields. Molodtsov [14] introduced the concept of soft sets, which can be seen as a mathematical tool for dealing with uncertainties. Furthermore, he demonstrated that soft set theory has potential applications in many directions, including function smoothness, Riemann integration, Perron integration, probability theory, measurement theory, game theory and operations research [14, 15].

Inspired by the study of algebraic structures of soft sets, our aim in this paper is

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to initiate the study on the connection between soft sets and n-ary polygroups. We apply the notion of soft sets to n-ary polygroups. Some related notions are defined and several basic properties are discussed by using the soft set theory. Furthermore, the concept of a soft homomorphism of n-ary polygroups is introduced and it is investigated which properties are preserved under this homomorphism.

2. Preliminaries of n-ary polygroup and soft set

In this section, we review some notions and results concerning n-ary polygroups and soft sets.

Let H be a non-empty set and f be a mapping $f : H\times H \to \wp^*(H)$, where $\wp^*(H)$ is the set of all non-empty subsets of H. Then f is called a binary hyperoperation on H.

Definition 2.1. ([2]) A polygroup is a system $\wp =< P, \circ, e, ^{-1} >$, where $e \in P$, $^{-1}$ is a unitary operation on $P, \circ$ maps $P \times P$ into the non-empty subsets of P and the following axioms hold for all x, y, z in P:
(1) $(x \circ y) \circ z = x \circ (y \circ z)$,
(2) $e \circ x = x \circ e = x$,
(3) $x \in y \circ z$ implies $y \in x \circ z^{-1}$ and $z \in y^{-1} \circ x$.

The following elemantary facts concerning polygroups follow easily from the axioms: $e \in x \circ x^{-1} \cap x^{-1} \circ x, e^{-1} = e, \left(x^{-1}\right)^{-1} = x$ and $(x \circ y)^{-1} = y^{-1} \circ x^{-1}$, where $A^{-1} = \left\{a^{-1}|a \in A\right\}$.

Example 2.1. Suppose H is a subgroup of a group G. Define a system

$$G//H = \left\langle \{HgH|g \in G\}, *, H, ^{-1}\right\rangle$$

where $(HgH)^{-1} = Hg^{-1}H$ and $(Hg_1H) * (Hg_2H) = \{Hg_1hg_2H|h \in H\}$. The algebra of double cosets $G//H$ is a polygroup introduced in Dresher and Ore [5].

We denote by H^n, the cartesian product $H\times ... \times H$ where H appears n times. An element of H^n will be denoted by $(x_1, ..., x_n)$ where $x_i \in H$ for any i with $1 \le i \le n$. In general, a mapping $f : H^n \to \wp^*(H)$ is called an n-ary hyperoperation and n is called the arity of the hyperoperation f.

Let f be an n-ary hyperoperation on H and

$$f(A_1, ..., A_n) = \cup \{f(x_1, ..., x_n)|x_i \in A_i, i = 1, ..., n\}.$$

We shall use the following abbreviated notation: The sequence $x_i, x_{i+1}, ..., x_j$ will be denoted by x_i^j. For $j < i, x_i^j$ is the empty set. Thus

$$f(x_1, ..., x_i, y_{i+1}, ..., y_j, z_{j+1}, ..., z_n)$$

will be written as $f\left(x_1^i, y_{i+1}^j, z_{j+1}^n\right)$. If $m = k(n-1) + 1$, then the m-ary hyperoperation g given by

$$g\left(x_1^{k(n-1)+1}\right) = \underbrace{f(f(..., f(f}_{k}\left(x_1^n\right), x_{n+1}^{2n-1}), ...), x_{(k-1)(n-1)+2}^{k(n-1)+1})$$

will be denoted by $f_{(k)}$.

Definition 2.2. ([8]) An n-ary polygroup is a multivalued system $< P, f, e, ^{-1} >$, where $e \in P$, $^{-1}$ is a unitary operation on P, f is an n-ary hyperoperation on P and the following axioms hold for all $i, j \in 1, ..., n, x_1, ..., x_{2n-1}, x \in P$:

(1) $f\left(x_1^{i-1}, f\left(x_i^{n+i-1}\right), x_{n+i}^{2n-1}\right) = f\left(x_1^{j-1}, f\left(x_j^{n+j-1}\right), x_{n+j}^{2n-1}\right)$,
(2) e is a unique element such that $f(e, ..., e, x, e, ..., e) = x$,
(3) $x \in f(x_1^n)$ implies $x_i \in f\left(x_{i-1}^{-1}, ..., x_1^{-1}, x, x_n^{-1}, ..., x_{i+1}^{-1}\right)$.

It is clear that every 2-ary polygroup is a polygroup.

A non-empty subset S of an n-ary polygroup P is an n-ary subpolygroup if $< S, f, e, ^{-1} >$ is an n-ary polygroup, i.e., if it is closed under the hyperoperation f, $e \in S$ and $x \in S$ implies $x^{-1} \in S$.

Now, we review some notions concerning soft sets. The definitions may be found in references [14, 12, 1, 18] . Let U be an initial universe set and E be a set of parameters. $P(U)$ denotes the power set of U and $A \subseteq E$.

Definition 2.3. ([14]). A pair (η, A) is called a soft set over U, where η is a set-valued function $\eta : A \to P(U)$ can be defined as $\eta(x) = \{y \in U \mid (x, y) \in R\}$ for all $x \in A$ and R will refer to an arbitrary binary relation between an element of A and an element of U, that is, R is a subset of $A \times U$. In fact, a soft set over U is a parameterized family of subsets of the universe U. For a soft set (η, A), the set $Supp(\eta, A) = \{x \in A \mid \eta(x) \neq \emptyset\}$ is called the *support* of the soft set (η, A) and the soft set (η, A) is called a *non-null* if $Supp(\eta, A) \neq \emptyset$ [6].

Definition 2.4. ([10]). Let (η, A) and (γ, B) be soft sets over a common universe U.

(i) (η, A) is said to be a soft subset of (γ, B), denoted $(\eta, A)\widetilde{\subseteq}(\gamma, B)$, if $A \subseteq B$ and $\eta(a) \subseteq \gamma(a)$ for all $a \in A$.
(ii) (η, A) and (γ, B) are said to be soft equal, denoted $(\eta, A) = (\gamma, B)$, if $(\eta, A)\widetilde{\subseteq}(\gamma, B)$ and $(\gamma, B)\widetilde{\subseteq}(\eta, A)$.

Definition 2.5. ([14, 12]).

(i) The *restricted-intersection*(or bi-intersection) of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A) \cap (\gamma, B)$, where $C = A \cap B \neq \emptyset$ and $\delta(c) = \eta(c) \cap \gamma(c)$ for all $c \in C$.
(ii) The *extended intersection* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\cap}(\gamma, B)$, where $C = A \cup B$ and for all $c \in C$,

$$\delta(c) = \begin{cases} \eta(c) & \text{if} \quad c \in A \backslash B \\ \gamma(c) & \text{if} \quad c \in B \backslash A \\ \eta(c) \cap \gamma(c) & \text{if} \quad c \in A \cap B. \end{cases}$$

Definition 2.6. ([12]).

(i) The *restricted-union* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A) \cup (\gamma, B)$, where $C = A \cap B \neq \emptyset$ and $\delta(c) = \eta(c) \cup \gamma(c)$ for all $c \in C$.
(ii) The *extended-union* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\cup}(\gamma, B)$, where $C = A \cup B$

and for all $c \in C$,

$$\delta(c) = \begin{cases} \eta(c) & \text{if} \quad c \in A \backslash B \\ \gamma(c) & \text{if} \quad c \in B \backslash A \\ \eta(c) \cup \gamma(c) & \text{if} \quad c \in A \cap B. \end{cases}$$

Definition 2.7. ([14, 1]).

(i) The $\wedge$-*intersection* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\wedge}(\gamma, B)$ where $C = A \times B$ and $\delta(a, b) = \eta(a) \cap \gamma(b)$ for all $(a, b) \in C$.

(ii) The $\vee$- *union* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\vee}(\gamma, B)$, where $C = A \times B$ and $\delta(a, b) = \eta(a) \cup \gamma(b)$ for all $(a, b) \in C$.

Definition 2.8. ([1]). Let (η, A) and (γ, B) be two soft sets over U and V, respectively. The *cartesian product* of two soft sets (η, A) and (γ, B) is defined as the soft set $(\delta, A \times B) = (\eta, A)\widetilde{\times}(\gamma, B)$, where $\delta(x, y) = \eta(x) \times \gamma(y)$ for all $(x, y) \in A \times B$.

3. Soft n-ary polygroups

In this section, we give the algebraic structure of soft n-ary polygroups and discuss some properties.

Definition 3.1. Let (η, A) be a soft set over P. Then (η, A) is called a *soft n-ary polygroup* over P if $\eta(x)$ is an *n-ary subpolygroup* of P for all $x \in A$; for our convenience, the empty set $\emptyset$ is regarded as an n-ary subpolygroup of P.

Example 3.1. Let $P = \{e, x, y\}$ be a set endowed with a 3-ary hyperoperation f as follows:
$f(e, e, e) = e$, $f(x, x, e) = \{e, y\}$, $f(y, e, e) = y$
$f(e, e, x) = x$, $f(x, x, x) = \{x, y\}$, $f(y, e, x) = \{x, y\}$
$f(e, e, y) = y$, $f(x, x, y) = P$, $f(y, e, y) = \{e, x\}$
$f(e, x, e) = x$, $f(x, e, e) = x$, $f(y, x, e) = \{x, \} y$
$f(e, x, x) = \{e, y\}$, $f(x, e, x) = \{e, y\}$, $f(y, x, x) = P$
$f(e, x, y) = \{x, y\}$, $f(x, e, y) = \{x, y\}$, $f(y, x, y) = P$
$f(e, y, e) = y$, $f(x, y, e) = \{x, y\}$, $f(y, y, e) = \{e, x\}$
$f(e, y, x) = \{x, y\}$, $f(x, y, x) = P$, $f(y, y, x) = P$
$f(e, y, y) = \{e, x\}$, $f(x, y, y) = P$, $f(y, y, y) = \{x, y\}$

It is easy to see that $< P, f, e,^{-1} >$ is a 3-ary polygroup. Let (η, A) be a soft set over P, where $A = \{1, 2, 3\}$ and $\eta : A \to P(P)$ be a set-valued function defined by

$$\eta(x) = \{y \in P \mid (x, y) \in R\}$$

for all $x \in A$, where $R = \{(1, e), (3, e), (3, x), (3, y)\}$. Then $\eta(1) = \{e\}$, $\eta(2) = \emptyset$ and $\eta(3) = P$ are 3-ary subpolygroups of P. Therefore (η, A) is a soft 3-ary polygroup over P.

Example 3.2. Consider the polygroup $P = S_3 // < (132) >$. Then
$\{H\sigma H | \sigma \in S_3\} = \{H, A\}$ where $H =< (132) >= \{(1), (123), (132)\}$ and
$A = \{(1), (12), (13), (23)\}$. Let (η, A) be a soft set over P, where $A = \{a, b, c, d\}$ and $\eta : A \to P(P)$ be a set-valued function defined by

$$\eta(x) = \{y \in P \mid (x, y) \in R\}$$

for all $x \in A$, where $R = \{(a,H),(b,A),(c,H),(c,A),(d,H)\}$. Then $\eta(a) = \eta(d) = \{H\}$, $\eta(b) = \{A\}$ and $\eta(c) = \{H,A\}$. Since $\eta(b) = \{A\}$ is not a subpolygroup of $S_3// < (132) >$. Therefore (η, A) is not a soft polygroup over $S_3// < (132) >$.

Theorem 3.1. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P.*

(i) *The extended intersection $(\eta, A)\widetilde{\cap}(\gamma, B)$ is a soft n-ary polygroup over P.*
(ii) *The restricted intersection $(\eta, A) \cap (\gamma, B)$, with $A \cap B \neq \emptyset$, is a soft n-ary polygroup over P.*

Proof. (i) By definition 2.5 (ii), we can write $(\eta, A)\widetilde{\cap}(\gamma, B) = (\delta, C)$, where $C = A \cup B$ and for all $c \in C$,

$$\delta(c) = \begin{cases} \eta(c) & \text{if} \quad c \in A\backslash B \\ \gamma(c) & \text{if} \quad c \in B\backslash A \\ \eta(c) \cap \gamma(c) & \text{if} \quad c \in A \cap B. \end{cases}$$

For all $x \in C$, if $x \in A\backslash B$, then $\delta(x) = \eta(x)$ is an n-ary subpolygroup of P, since (η, A) is a soft n-ary polygroup over P; if $x \in B\backslash A$, then $\delta(x) = \gamma(x)$ is an n-ary subpolygroup of P, since (γ, A) is a soft n-ary polygroup over P; if $x \in A \cap B$, then $\delta(x) = \eta(x) \cap \gamma(x)$ is an n-ary subpolygroup of P, since the intersection of any two n-ary subpolygroups of P is also an n-ary subpolygroup of P. Thus we conclude that $\delta(x)$ is an n-ary subpolygroup of P for all $x \in C$. It follows that $(\delta, C) = (\eta, A)\widetilde{\cap}(\gamma, B)$ is a soft n-ary polygroup over P. (ii)According to Definition 2.5(i), by assumption and the proof (i), it follows that $(\eta, A) \cap (\gamma, B)$ is a soft n-ary polygroup over P. □

Corollary 3.1. *Let (η, A) and (γ, A) be two soft n-ary polygroups over P. Then the extended intersection $(\eta, A)\widetilde{\cap}(\gamma, A)$ is a soft n-ary polygroups over P.*

Theorem 3.2. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P.*

(i) *If $A \cap B = \emptyset$, then the extended union $(\eta, A)\widetilde{\cup}(\gamma, B)$ is a soft n-ary polygroup over P.*
(ii) *If $\eta(x) \subseteq \gamma(x)$ or $\gamma(x) \subseteq \eta(x)$ for all $x \in A \cap B$, then the restricted union $(\eta, A) \cup (\gamma, B)$ is a soft n-ary polygroup over P.*

Proof. (i) By Definition 2.6 (ii), we can write $(\eta, A)\widetilde{\cup}(\gamma, B) = (\delta, C)$, where $C = A \cup B$. Since $A \cap B = \emptyset$, so for all $x \in C$, either $x \in A\backslash B$ or $x \in B\backslash A$. If $x \in A\backslash B$, then $\delta(x) = \eta(x)$ is an n-ary subpolygroup of P, since (η, A) is a soft n-ary polygroup over P; if $x \in B\backslash A$, then $\delta(x) = \gamma(x)$ is an n-ary subpolygroup of P, since (γ, A) is a soft n-ary polygroup over P. Therefore $(\delta, C) = (\eta, A)\widetilde{\cup}(\gamma, B)$ is a soft n-ary polygroup over P. (ii)According to Definition 2.6 (i), by assumption and the proof (i), it follows that $(\eta, A) \cup (\gamma, B)$ is a soft n-ary polygroup over P. □

Theorem 3.3. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P. Then the $\wedge$-intersection $(\eta, A)\widetilde{\wedge}(\gamma, B)$ is a soft n-ary polygroup over P.*

Proof. By Definition 2.7 (i), we can write $(\eta, A)\widetilde{\wedge}(\gamma, B) = (\delta, C)$, where $C = A \times B$ and $\delta(x, y) = \eta(x) \cap \gamma(y)$ for all $(x, y) \in C$. Since (η, A) and (γ, B) are soft n-ary polygroups over P, we deduce that $\eta(x)$ and $\gamma(y)$ are both n-ary subpolygroups of P and their intersection is also an n-ary subpolygroup of P. So $\delta(x, y) = \eta(x) \cap \gamma(y)$ is an n-ary subpolygroup P for all $(x, y) \in C$. Hence the $\wedge$-intersection $(\eta, A)\widetilde{\wedge}(\gamma, B)$ is a soft n-ary polygroup over P.□

Theorem 3.4. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P. If $\eta(a) \subseteq \gamma(b)$ or $\gamma(b) \subseteq \eta(a)$ for all $(a, b) \in A \times B$, then the $\vee$-union $(\eta, A)\widetilde{\vee}(\gamma, B)$ is a soft n-ary polygroup over P.*

Proof. Assume that (η, A) and (γ, B) be two soft n-ary polygroups over P. By Definition 2.7 (ii), we can write $(\eta, A)\widetilde{\vee}(\gamma, B) = (\delta, C)$, where $C = A \times B$ and $\delta(x, y) = \eta(x) \cup \gamma(y)$ for all $(x, y) \in C$. Since (η, A) and (γ, B) are soft n-ary polygroups over P, we deduce that the set $\eta(x)$ and $\gamma(y)$ are both n-ary subpolygroups of P. By assumption, $\delta(x, y) = \eta(x) \cup \gamma(y)$ is an n-ary subpolygroup of P for all $(x, y) \in C$. Hence the $\vee$-union $(\eta, A)\widetilde{\vee}(\gamma, B)$ is a soft n-ary polygroup over P.□

Theorem 3.5. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P_1 and P_2, respectively. Then the cartesian product $(\eta, A)\widetilde{\times}(\gamma, B)$ is a soft n-ary polygroup over $P_1 \times P_2$.*

Proof. By Definition 2.8., we can write $(\eta, A)\widetilde{\times}(\gamma, B) = (\delta, C)$, where $C = A \times B$ and $\delta(x, y) = \eta(x) \times \gamma(y)$. Since (η, A) and (γ, B) are soft n-ary polygroups over P, we deduce that the set $\eta(x)$ and $\gamma(y)$ are both n-ary subpolygroups of P_1 and P_2, respectively. It follows that the cartesian product $\eta(x) \times \gamma(y)$ is an n-ary subpolygroup of $P_1 \times P_2$ for all $(x, y) \in C$. That is $\delta(x, y)$ is an n-ary subpolygroup of P for all $(x, y) \in C$. Hence the cartesian product $(\eta, A)\widetilde{\times}(\gamma, B)$ is a soft n-ary polygroup over $P_1 \times P_2$. □

Definition 3.2. Let (η, A) be a soft n-ary polygroup over P.

(i) (η, A) is called a *trivial* soft n-ary polygroup over P if $\eta(x) = \{e\}$ for all $x \in A$.
(ii) (η, A) is called a *whole* soft n-ary polygroup over P if $\eta(x) = P$ for all $x \in A$.

Proposition 3.1. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P.*

(i) *If (η, A) and (γ, B) are trivial soft n-ary polygroups over P, then the restricted intersection $(\eta, A) \cap (\gamma, B)$ is a trivial soft n-ary polygroup over P.*
(ii) *If (η, A) and (γ, B) are whole soft n-ary polygroups over P, then the restricted intersection $(\eta, A) \cap (\gamma, B)$ is a whole soft n-ary polygroup over P.*
(iii) *If (η, A) is a trivial soft n-ary polygroup over P and (γ, B) is a whole soft n-ary polygroup over P, then the restricted intersection $(\eta, A) \cap (\gamma, B)$ is a trivial soft n-ary polygroup over P.*

Proposition 3.2. *Let (η, A) and (γ, B) be soft n-ary polygroups over P_1 and P_2, respectively.*

(i) *If (η, A) and (γ, B) are trivial soft n-ary polygroups over P_1 and P_2, respectively. Then $(\eta, A)\widetilde{\times}(\gamma, B)$ are trivial soft n-ary polygroups over $P_1 \times P_2$.*
(i) *If (η, A) and (γ, B) are whole soft n-ary polygroups over P_1 and P_2, respectively. Then $(\eta, A)\widetilde{\times}(\gamma, B)$ are whole soft n-ary polygroups over $P_1 \times P_2$.*

Definition 3.3. Let (η, A) and (γ, B) be two soft sets over the n-ary polygroups P_1 and P_2, respectively; $f : P_1 \to P_2$, $g : A \to B$ be two functions. Then the pair (f, g) is called a soft function from (η, A) to (γ, B), denoted by $(f, g) : (\eta, A) \to (\gamma, B)$

if $f(\eta(x)) = \gamma(g(x))$ for all $x \in A$. If f, g are injective(resp. surjective, bijective), then (f, g) is called injective(resp. surjective, bijective).

Definition 3.4. Let (η, A) and (γ, B) two soft sets over the n-ary polygroups P_1 and P_2, respectively; (f, g) be a soft function from (η, A) to (γ, B).

(i) The *image* of (η, A) under the soft function (f, g), denoted by $(f, g)(\eta, A) = (f(\eta), B)$, is the soft set over P_2 defined by

$$f(\eta)(x) = \begin{cases} \cup_{g(x)=y} f(\eta(x)) & \text{if } \; y \in Img \\ \emptyset & \text{if } \; otherwise \end{cases}$$

for all $y \in B$.

(ii) The *pre-image* of (γ, B) under the soft function (f, g), denoted by $(f, g)^{-1}(\gamma, B) = (f^{-1}(\gamma), B)$, is the soft set over P_1 defined by $f^{-1}(\gamma)(y) = f^{-1}(\gamma(y))$ for all $y \in B$.

It is clear that $(f, g)(\eta, A)$ is a soft subset of (γ, B) and (η, A) is a soft subset $(f, g)^{-1}(\gamma, B)$.

Definition 3.5. ([8]). Let $< P_1, f, e_1, ^{-1} >$ and $< P_2, g, e_2, ^{-1} >$ be two n-ary polygroups. A mapping φ from P_1 to P_2 is said to be strong homomorphism if for every $a_1, a_2, ..., a_n \in P_1$,

$$\varphi(e_1) = e_2 \text{ and } \varphi(f(a_1^n)) = g(\varphi(a_1), ..., \varphi(a_n)).$$

A strong homomorphism φ is said to be an isomorphism if f is one to one and onto. Two n-ary polygroups P_1 and P_2 are said to be isomorphic if there exists an isomorphism from P_1 onto P_2. In this case, we write $P_1 \cong P_2$. Moreover if φ is a strong homomorphism from P_1 to P_2, then the kernel of φ is the set $Ker(\varphi) = \{x \in P_1 | \varphi(x) = e_2\}$.

Definition 3.6. Let (η, A) and (γ, B) be two soft n-ary polygroups over P_1 and P_2, respectively and (f, g) be a soft function from (η, A) to (γ, B). If f is a strong homomorphism from P_1 to P_2, then (f, g) is called a *soft n-ary polygroup homomorphism* and we say that (η, A) is soft homomorphic to (γ, B) under the soft homomorphism (f, g). In this definition, if f is an isomorphism from P_1 to P_2 and g is a surjective mapping from A to B, then we say that (f, g) is a soft n-ary polygroup isomorphism and that (η, A) is soft isomorphic to (γ, B) under the soft homomorphism (f, g), which is denoted by $(\eta, A) \simeq (\gamma, B)$.

Proposition 3.3. *The relation $\simeq$ is an equivalence relation on soft n-ary polygroups.*

Theorem 3.6. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P_1 and P_2, respectively and (f, g) be a soft n-ary polygroup homomorphism from (η, A) to (γ, B).*

(i) *If g is injective, f is surjective and (η, A) is a soft n-ary polygroup over P_1, then $(f, g)(\eta, A)$ is a soft n-ary polygroup over P_2.*

(ii) *If (γ, B) is a soft n-ary polygroup over P_2, then $(f, g)^{-1}(\gamma, B)$ is a soft n-ary polygroup over P_1.*

Proof.(i) Since (η, A) is a soft n-ary polygroup over P_1, it is clear that $(f(\eta), B)$ is a soft set over P_2. Let $y \in Img$, then there exist a $x \in A$ such that $g(x) = y$ and $\eta(x)$ is an n-ary subpolygroup of P_1 and its onto homomorphic image $f(\eta(x))$ is an n-ary subpolygroup of P_2. Since g is injective, by Definition 3.4., we get

$f(\eta)(y) = f(\eta(x))$, and so, $f(\eta)(y)$ is an n-ary subpolygroup of P_2. Therefore $(f(\eta), B)$ is a soft n-ary polygroup over P_2.
(ii) Assume that (γ, B) is a soft n-ary polygroup over P_2. Then $\gamma(y)$ is an n-ary subpolygroup P_2 for all $y \in B$ and its homomorphic inverse image $f^{-1}(\gamma(y))$ is also an n-ary subpolygroup of P_1. Hence $f^{-1}(\gamma(y))$ is an n-ary subpolygroup of P_1 for all $y \in B$. That is, $(f,g)^{-1}(\gamma), B)$ is a soft n-ary polygroup over P_1. □

Theorem 3.7. *Let (η, A) and (γ, B) be two soft n-ary polygroups over P_1 and P_2, respectively and (f, g) be a soft n-ary polygroup homomorphism from (η, A) to (γ, B).*

(i) *If $\eta(x) = ker(f)$ for all $x \in A$, then $(f,g)(\eta, A)$ is a trivial soft n-ary polygroup over P_2.*
(ii) *If f is onto and (η, A) is whole, then $(f,g)(\eta, A)$ is a whole soft n-ary polygroup over P_2.*
(iii) *If $\gamma(y) = f(P_1)$ for all $y \in B$, then $(f,g)^{-1}(\gamma, B)$ is a whole soft n-ary polygroup over P_1.*
(iv) *If f is injective and (γ, B) is trivial, then $(f,g)^{-1}(\gamma, B)$ is a trivial soft n-ary polygroup over P_1.*

Proof. (i) Assume that $\eta(x) = ker(f)$ for all $x \in A$. Then, $f(\eta)(x) = f(\eta(x)) = \{0_{P_2}\}$ for all $x \in A$. Hence $(f,g)(\eta, A)$ is a soft n-ary polygroup over P_2 by Theorem 3.6. and Definition 3.2.
(ii) Suppose that f is onto and (η, A) is whole. Then, $\eta(x) = P_1$ for all $x \in A$, and so $f(\eta)(x) = f(\eta(x)) = f(P_1) = P_2$ for all $x \in A$. It follows from Theorem 3.6. and Definition 3.2. that $(f,g)(\eta, A)$ is a whole soft n-ary polygroup over P_2.
(iii) Assume that $\gamma(y) = f(P_1)$ for all $y \in B$. Then, $f^{-1}(\gamma)(y) = f^{-1}(\gamma(y)) = f^{-1}(f(P_1)) = P_1$ for all $y \in B$. Hence $(f^{-1}(\gamma), B)$ is a whole soft n-ary polygroup over P_1 by Theorem 3.6. and Definition 3.2.
(iv) Suppose that f is injective and (γ, B) is trivial. Then, $\gamma(y) = \{0\}$ for all $y \in B$, and so $f^{-1}(\gamma)(y) = f^{-1}(\gamma(y)) = f^{-1}(\{0\}) = ker(f) = \{0_{P_1}\}$ for all $y \in B$. It follows from Theorem 3.6. and Definition 3.2. that $(f^{-1}(\gamma), B)$ is a trivial soft n-ary polygroup over P_1.□

Proposition 3.4. *Let P_1, P_2 and P_3 be n-ary polygroups and (η, A), (γ, B) and (δ, C) soft n-ary polygroups over P_1, P_2, and P_3 respectively. Let the soft function (f, g) from (η, A) to (γ, B) be a soft homomorphism from L_1 to L_2 and the soft function $(f^{'}, g^{'})$ from (γ, B) to (δ, C) be a soft homomorphism from P_2 to P_3. Then the the soft function $(f^{'} \circ f, g^{'} \circ g)$ from (η, A) to (δ, C) is a soft homomorphism from P_1 to P_3.*

Theorem 3.8. *Let P_1 and P_2 be n-ary polygroups and (η, A), (γ, B) soft sets over P_1 and P_2, respectively. If (η, A) is a soft n-ary polygroup over P_1 and $(\eta, A) \simeq (\gamma, B)$, then (γ, B) is a soft n-ary polygroup over P_2.*

Proof. The proof is obtained by Definition 3.5. and Proposition 3.3. □

Theorem 3.9. *Let $f : P_1 \to P_2$ be an onto homomorphism of n-ary polygroups and (η, A) and (γ, B) be two soft n-ary polygroups over P_1 and P_2, respectively.*

(i) *The soft function (f, I_A) from (η, A) to (δ, A) is a soft homomorphism from P_1 to P_2, where $I_A : A \to A$ is the identity mapping and the set-valued function $\delta : A \to P(P_2)$ is defined by $\delta(x) = f(\eta(x))$ for all $x \in A$.*

(ii) *If $f : P_1 \to P_2$ is an isomorphism, then the soft function (f^{-1}, I_B) from (γ, B) to (ν, B) is a soft homomorphism from P_2 to P_1, where $I_B : B \to B$ is the identity mapping and the set-valued function $\nu : B \to P(P_1)$ is defined by $\nu(x) = f^{-1}(\gamma(x))$ for all $x \in B$.*

Proof. Follows from Definition 3.6. □

References

[1] Ali, M.I., Feng, F., Liu, X., Min, W.K. and Shabira, M., On some new operations in soft set theory, *Computers and Mathematics with Applications* 57 (9) (2009), 1547-1553.

[2] Comer, S. D., Polygroups derived from cogroups, *Journal of Algebra* 89 (1984), 397-405.

[3] Corsini, P. and Leoreanu, V., Applications of Hyperstructure Theory, in: Advances in Mathematics, vol. 5, *Kluwer Academic Publishers* Boston, Mass, 2003.

[4] Davvaz, B. and Vougiouklis, T., n-ary hypergroups, *Iran. J. Sci. Technol.* Trans. A, 30 (2006), 165-174.

[5] Dresher, M. and Ore, O., Theory of multigroups, *American Journal of Mathematics* 60 (1938), 705-733.

[6] Feng, F., Jun, Y.B. and Zhao, X., Soft semirings, *Computers and Mathematics with Applications* 56 (10) (2008), 2621-2628.

[7] Gau, W.L. and Buehrer, D. J., Vague sets, *IEEE Transactions on Systems, Man and Cybernetics* 23(2) (1993), 610-614.

[8] Ghadiri, M. and Waphare, B. N., n-ary polygroups, *Iran. J. Sci. Technol.* Trans. A, 33, No. A2.

[9] Gorzalzany, M. B., A method of inference in approximate reasoning based on interval-valued fuzzy sets, *Fuzzy Sets and Systems* 21 (1987), 1-17.

[10] Jun, Y.B., Lee, K.J. and Khan, A., Soft ordered semigroups, *Mathematical Logic Quarterly* 56 (1) (2010), 42-50.

[11] Kazancı, O., Yılmaz, Ş. and Yamak, S., Soft sets and Soft BCH-algebras, *Hacettepe J. Math. Stst.* 39(2) (2010), 205-217.

[12] Maji, P.K., Biswas, R. and Roy, A.R., Soft set theory, *Computers and Mathematics with Applications* 45 (2003), 555-562.

[13] F. Marty, Sur une generalization de la notion de group, in: *Proc. 8th Congress Math. Scandenaves* Stockholm, (1934) 45-49.

[14] Molodtsov, D., Soft set theory-first results, *Computers and Mathematics with Applications* 37 (1999), 19-31.

[15] Molodtsov, D., The Theory of Soft Sets, *URSS Publishers* Moscow, 2004 (in Russian).

[16] Pawlak, Z., Rough sets, *International Journal of Information and Computer Sciences* 11 (1982), 341-356.

[17] Pawlak, Z., Rough Sets: Theoretical Aspects of Reasoning About Data, *Kluwer Academic Publishers* Boston, MA, 1991.

[18] Xiao, Z., Gong, K., Xia, S.S. and Zou, Y., Exclusive disjunctive soft sets, *Computers and Mathematics with Applications* 59 (2010), 2128-2137.

[19] Zadeh, L. A., Fuzzy sets, *Information and Control* 8 (1965), 338-353.

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