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ON STRONGLY θ-$\beta^* g$-CONTINUOUS MULTIFUNCTIONS

AHU AÇIKGÖZ AND SEDA GÖKTEPE

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Abstract. The purpose of this paper is to define a new class of multifunctions namely strongly θ-$\beta^* g$-continuous multifunctions and to improve some characterizations concerning upper and lower strongly θ-$\beta^* g$-continuous multifunctions.

1. Introduction

Continuity of functions is one of the most important topics in several branches of mathematics. A good number of papers dealing with continuity of functions have appeared and a great number of them have been extended to the setting of multifunctions ([21], [15], [16], [22]). Multifunctions have many applications in mathematical programming, probability, statistics, different inclusions, fixed-point theorems and economics ([9], [10], [11], [4], [5], [6], [7], [8], [12], [14], [17]). This shows that both functions and multifunctions are important tools for studying properties of spaces and for forming new spaces from previously existing ones. The aim of this paper is to introduce the concept of upper and lower strongly θ-$\beta^* g$-continuity for multifunctions.

2. Preliminaries

Throught the paper (X, τ) and (Y, σ) will denote topological spaces. If $A \subset X$, we use the notation Cl(A) and Int(A) for the closure and the interior of the set A, respectively. A subset A of a space (X, τ) is called a β^*-set [20] if $A = U \cap V$, where U is open and $int(V) = Cl(int(V))$. A subset of a space (X, τ) is called a $\beta^* g$-closed set [1] if $Cl(A) \subset U$ whenever $A \subset U$ and U is a β^*-set. A subset A of a space (X, τ) is called $\beta^* g$-open if the complement of A is $\beta^* g$-closed. For a subset A of a space (X, τ), the intersection of all $\beta^* g$-closed subsets of (X, τ) that contain A is the $\beta^* g$-closure of a set A, written $\beta^* g$-cl(A). For a subset A of a space (X, τ), the union of all $\beta^* g$-open sets of (X, τ) contained in A is called $\beta^* g$-interior of A

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and denoted by β^*g-int(A). The family of all β^*g-open (resp.β^*g-closed) sets in a space (X,τ) is denoted by β^*gO(X)(resp.β^*gC(X)). The family of all β^*g-open sets of X containing a point $x \in X$ is denoted by β^*gO(X,x). Multifunctions are denoted by capital letters F, G, H etc. By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y and assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [2], [3] we denote the upper and lower inverse of a set B of Y by $F^+(B) = \{x \in X : F(x) \subseteq B\}$ and $F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}$, respectively. For each $A \subset X$, $F(A) = \bigcup_{x\in A} F(x)$. F is defined to be a surjection if $F(X) = Y$ or equivalently if for each $y \in Y$ there exists a point $x \in X$ such that $y \in F(x)$. A multifunction $F : (X,\tau) \to (Y,\sigma)$ is said to be upper (lower) β^*g-continuous multifunction if $F^+(V)(F^-(V)) \in \beta^*$gO(X) for every $V \in \sigma$.

3. Some properties of upper and lower strongly θ-β^*g-continuous multifunctions

Lemma 3.1. *Let (X,τ) be a topological space and $A \subset X$. Then $x \in \beta^*g$-cl(A) if and only if $U \cap A \neq \emptyset$ for each β^*g-open set U that contains $x \in X$* [1].

Definition 3.1. Let (X,τ) be a topological space , $A \subset X$ and $x \in X$. If $cl(U) \cap A \neq \emptyset$ for each open set U containing x, then x is defined as a θ-closure of A . The set of all θ-closure of A is called θ-closure of A and denoted by θ-cl(A). If $A = \theta - cl(A)$, then A is called θ-closed set [19].

Definition 3.2. Let (X,τ_1) and (Y,τ_2) are topological spaces and $F : (X,\tau_1) \to (Y,\tau_2)$ be a multifunction. F is defined to be upper and lower β^*g-continuous if for each $x \in X$ and for each open set V of Y such that $F(x) \subset V$ $(F(x) \cap V \neq \emptyset\ )$ there exists a β^*g-open subset $U \subset X$ such that $U \subset F^+(V)$ $(U \subset F^-(V))$.

Definition 3.3. Let (X,τ) be a topological space , $A \subset X$ and $x \in X$. If β^*g-$cl(U) \cap A \neq \emptyset$ for each β^*g-open set U containing x, then x is defined as a β^*g-θ-closure of A. The set of all β^*g-θ-closure of A is called β^*g-θ-closure of A and denoted by β^*g-θ-cl(A). If $A = \beta^*g$-θ-cl(A) , then A is called β^*g-θ-closed set. A subset A of a space (X,τ) is called β^*g-θ-closed if the complement of A is β^*g-θ-open.

Remark 3.1. The family of all β^*g-θ-open (resp.β^*g-θ-closed) sets in a space (X,τ) is denoted by β^*g-$\theta O(X)$(resp.β^*g-$\theta C(X)$). The family of all β^*g-θ-open sets of X containing a point $x \in X$ is denoted by β^*g-$\theta O(X,x)$.

Definition 3.4. A topological space X said to be β^*g-closed space if β^*g-closures of every β^*g-open cover of X has a finite subcover covering X.

Definition 3.5. A topological space X said to be β^*g-regular space if there exist $U, V \in \beta^*g$O(X) such that $x \in U$, $F \subset V$ and $U \cap V = \emptyset$ for every β^*g-closed set F and every $x \in (X - F)$.

Lemma 3.2. *A subset U in (X,τ) is β^*g-θ-open if and only if for each $x \in U$ there exists a β^*g-open set W such that $x \in W$ and $\beta^*g{-}cl(W) \subset U$.*

Definition 3.6. A multifunction $F : (X,\tau) \to (Y,\sigma)$ is defined to be:

- **a)** Upper strongly θ-β^*g-continuous if for each $x \in X$ and each open set $V \subset Y$ such that $F(x) \subset V$, then there exists $U \in \beta^*g$O(X,x) such that $F(u) \subset V$ for each $u \in \beta^*g$-cl(U).

b) Lower strongly θ-β^*g-continuous if for each $x \in X$ and each open set $V \subset Y$ such that $F(x) \cap V \neq \emptyset$, then there exists $U \in \beta^*g$O(X,x) such that $F(u) \cap V \neq \emptyset$ for each $u \in \beta^*g$-cl(U).

c) Strongly θ-β^*g-continuous if it is both upper and lower strongly θ-β^*g-continuous.

Now, we give characterizations of upper and lower strongly θ-β^*g-continuous multifunctions, respectively.

Theorem 3.1. *The following conditions are equivalent for a multifunction $F : (X,\tau_1) \to (Y,\tau_2)$*

a) *F is upper strongly θ-β^*g-continuous,*

b) *There exists $U \in \beta^*g$-θO(X,x) such that $F(U) \subset V$, for each $x \in X$ and each open set $V \subset Y$ such that $F(x) \subset V$.*

c) *$F^+(V) \subset X$ is β^*g-θ-open set for each open set V in Y.*

d) *$F^-(B) \subset X$ is β^*g-θ-closed set for each closed set B in Y.*

e) *β^*g-θ-cl($F^-(A)) \subset F^-(cl(A))$ for each subset A in Y.*

Proof. $(a) \Rightarrow (c)$ Let V be an open set and $x \in F^+(V)$. For each $u \in \beta^*g$-cl(U) there exists $U \in \beta^*g$O(X,x) such that $F(U) \subset V$. Then, $x \in \beta^*\text{g}-cl(U) \subset F^+(V)$. This shows that $F^+(V) \subset X$ is a β^*g-θ-open set.

$(c) \Rightarrow (d)$ For each $B \subset Y$, $F^+(Y-B) = X - F^-(B)$. Hence, the proof is obvious.

$(d) \Rightarrow (e)$ Let A be a subset of Y. Then cl(A) is a closed set in Y and $F^-(cl(A))$ is β^*g-θ-closed set in X. Hence, β^*g-θ-cl$(F^-(A)) \subset (F^-(cl(A))$.

$(e) \Rightarrow (a)$ Let $x \in X$ and $V \subset Y$ be an open set such that $F(x) \subset V$. Since $(Y-V) \subset Y$ is closed, β^*g-θ-cl$(F^-(Y-V)) \subset F^-(Y-V)$. Hence, $F^-(Y-V) \subset X$ is β^*g-θ-closed. Since $F^-(Y-V) = X - F^+(V)$ is satisfied, $F^+(V) \subset X$ is β^*g-θ-open. Then, there exists $U \in \beta^*g$O(X,x) such that β^*g-$cl(U) \subset F^+(V)$. This is the definition of upper strongly θ-β^*g-continuity of F.

$(b) \Rightarrow (c)$ Let an open set $V \subset Y$ and $x \in F^+(V)$. Then, there exists $U \in \beta^*g$-θO(X,x) such that $x \in U \subset F^+(V)$. Since U is β^*g-θ-open in X, there exists $G \in \beta^*g$O(X,x) such that $x \in \beta^*\text{g}-cl(G) \subset U \subset F^+(V)$. Hence, $F^+(V)$ is a β^*g-θ-open set in X.

$(c) \Rightarrow (b)$ Let $x \in X$ and an open set $V \subset Y$ such that $F(x) \subset V$. $F^+(V) \subset X$ is β^*g-θ-open set and $x \in F^+(V)$. Use $U = F^+(V)$, then $F(U) \subset V$. □

Theorem 3.2. *The following conditions are equivalent for a multifunction $F : (X,\tau_1) \to (Y,\tau_2)$*

a) *F is lower strongly θ-β^*g-continuous,*

b) *There exists $U \in \beta^*g$-θO(X,x) such that $U \subset F^-(V)$, for each $x \in X$ and each open set $V \subset Y$ such that $x \in F^-(V)$.*

c) *$F^-(V) \subset X$ is β^*g-θ-open set for each open set V in Y.*

d) *$F^+(B) \subset X$ is β^*g-θ-closed set for each closed set B in Y.*

e) *β^*g-θ-cl($F^+(A)) \subset F^+(cl(A)$ for each subset A in Y.*

Proof. Similar to that of Theorem 3.1. □

Remark 3.2. Upper (lower) strongly θ-β^*g-continuous $\Rightarrow$ Upper (lower) β^*g-continuous

Example 3.1. Let $X = \{1,2,3\}$, $\tau = \{X, \emptyset, \{1\}, \{1,2\}, \{1,3\}\}$ and
$Y = \{a,b,c\}$, $\sigma = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$.

Let a multifunction $F : (X, \tau) \to (X, \sigma)$ such that $F(x) = \{a, b\}$ for $x = 1$, $F(x) = \{c\}$ for $x = 2$, $F(x) = \{b, c\}$ for $x = 3$.
Then $F^+(\{a\}) = \emptyset$, $F^+(\{b\}) = \emptyset$, $F^+(\{a, b\}) = \{1\}$. While $\{1\} \in \beta^*g$O(X), $\{1\} \notin \beta^*g$-θO(X). Therefore , F is upper β^*g-continuous , but not strongly β^*g-θ-continuous.

Theorem 3.3. *Let (X, τ_1), (Y, τ_2) and (Z, τ_3) be any topological spaces and $F_1 : X \to Y$ and $F_2 : Y \to Z$ are multifunctions. If $F_2 : (X, \tau_1) \to (Y, \tau_2)$ is upper (lower) strongly θ-β^*g-continuous and $F_2 : (Y, \tau_2) \to (Z, \tau_3)$ is upper (lower) continuous, then $F : F_2 \circ F_1 : X \to Z$ is upper (lower) strongly θ-β^*g-continuous multifunction.*

Proof. We shall only prove the case where F is upper strongly θ-β^*g-continuous. Let $G \subset Z$ be an open set. From the definition of $F_2 \circ F_1$,$F^+(G) = (F_2 \circ F_1)^+(G) = F_1^+(F_2^+(G))$. Since F_2 is upper continuous multifunction, $F_2^+(G) \subset Y$ is open set. Since F_1 is upper strongly θ-β^*g-continuous , $F_1^+(F_2^+(G))$ is β^*g-θ-open set. Hence, $F = F_2 \circ F_1$ is upper strongly θ-β^*g-continuous. □

Definition 3.7. For multifunctions $F_1 : X_1 \to Y_1$ and $F_2 : X_2 \to Y_2$, the product multifunction is defined as $F_1 \times F_2 : X_1 \times X_2 \to Y_1 \times Y_2$, for $x_1 \in X_1$ and $x_2 \in X_2$, $(F_1 \times F_2)\ (x_1, x_2) = F_1(x_1) \times F_2(x_2)$.

Lemma 3.3. *For $F_1 : (X_1, \tau_1) \to (Y_1, \sigma_1)$ and $F_2 : (X_2, \tau_2) \to (Y_2, \sigma_2)$ multifunctions and $A \subset X$, $B \subset Z$, then the following properties are hold:*

a) $(F_1 \times F_2)^+(A \times B) = F_1^+(A) \times F_2^+(B)$
b) $(F_1 \times F_2)^-(A \times B) = F_1^-(A) \times F_2^-(B))$

Theorem 3.4. *Let $F_1 : (X_1, \tau_1) \to (Y_1, \sigma_1)$ and $F_2 : (X_2, \tau_2) \to (Y_2, \sigma_2)$ are upper (lower) strongly θ-β^*g-continuous multifunctions, then the product multifunction $F_1 \times F_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is upper (lower) strongly θ-β^*g-continuous.*

Proof. We shall only prove the case where F is upper strongly θ-β^*g-continuous. Let $(x_1, x_2) \in X_1 \times X_2$ and let $V \subset Y_1 \times Y_2$ be an open set such that $F_1(x_1) \times F_2(x_2) \subset V$. Then there exists $U_1 \subset Y_1$ and $U_2 \subset Y_2$ open sets such that $F_1(x_1) \times F_2(x_2) \subset U_1 \times U_2 \subset V$. Since F_1 and F_2 are upper strongly θ-β^*g-continuous multifunctions, there exists $A \in \beta^*g$-θO(X_1, x_1) and $B \in \beta^*g$-θO(X_2, x_2) such that $F_1(A) \subset U_1$ and $F_2(B) \subset U_2$. Also, $A \times B \subset F_1^+(U_1) \times F_2^+(U_2) = (F_1 \times F_2)^+(U_1 \times U_2) \subset (F_1 \times F_2)^+(V)$. Hence, $A \times B \in \beta^*g$-θO$(X_1 \times X_2)$,$(x_1, x_2))$ and $(F_1 \times F_2)(A \times B) \subset V$ are satisfied. This is definition of upper strongly θ-β^*g-continuous multifunction $F_1 \times F_2$. □

Definition 3.8. For a given multifunction $F : X \to Y$, the graph multifunction $G_F : X \to X \times\ Y$ is defined as $G_F(x) = \{x\} \times F(x)$ for every $x \in X$.

Lemma 3.4. [13]*For a multifunction $F : X \to Y$, the following holds:*
$G_F^+(A \times B) = A \cap F^+(B)$ and $G_F^-(A \times B) = A \cap F^-(B)$ where $A \subseteq X$ and $B \subseteq Y$.

Theorem 3.5. *Let $F : (X, \tau_1) \to (Y, \tau_2)$ be a multifunction. If the graph multifuncion G_F is upper (lower) strongly θ-β^*g-continuous, then F is upper (lower) strongly θ-β^*g-continuous.*

Proof. Let $x \in X$ and V be an open set in Y. $X \times V$ is open in $X \times Y$ and $\{x\} \times F(x) \subset X \times V$,$G_F(x) \subset X \times V$. From the upper strongly θ-β^*g-continuity of G_F,there exists a β^*g-θ-open set U such that $x \in U$ and $U \subset G_F^+(X \times V)$. By using Lemma 3.4 , we obtain $U \subset F^+(V)$. Consequently, F is upper strongly θ-β^*g-continuous. □

Definition 3.9. A topological space (X, τ) is said to be β^*g-Hausdorff if for each pair of distinct points x and y of X, there exist disjoint β^*g-open subsets U and V of X containing x and y, respectively.

Definition 3.10. Let $F : (X, \tau_1) \to (Y, \tau_2)$ be a multifunction. If there exist β^*g-open set U containing x and β^*g-open set V containing y such that $(\beta^*g - cl(U) \times V) \cap G_F = \emptyset$ for each $(x, y) \notin G_F$, $G_F \subset X \times Y$ is strongly β^*g-closed.

Theorem 3.6. *If $F : (X, \tau_1) \to (Y, \tau_2)$ is upper strongly θ-β^*g-continuous, $F(x)$ is compact for each $x \in X$ and (Y, τ_2) is Hausdorff, then $G_F \subset X \times Y$ is strongly β^*g-closed.*

Proof. Let $(x, y) \in X \times Y - G_F$. Then $y \notin F(x)$.Since Y is Hausdorff, there exist disjoint, open sets $V_z(y), U(z) \subset Y$ such that $z \in U(z)$ and $y \in V_z(y)$ for each $z \in F(x)$. The family of $U(z) : z \in F(x)$ is an open cover of $F(x)$. Since $F(x)$ is compact, there exist a finite number of $z_1, z_2, ..., z_n \in F(x)$ such that $F(x) \subset \cup\{U(z_i) : i = 1, 2, ..., n\}$. Let $U = \{U(z_i) : i = 1, 2, ..., n\}$ and $V = \cap\{v_{z_i}(y) : i = 1, 2, ..., n\}$. Then, there exist disjoint, open sets $U, V \subset Y$ such that $F(x) \subset U$,$y \in V$. Since F is upper strongly θ-β^*g-continuous, there exist β^*g-open set W containing x such that $\beta^*\text{g}-cl(W) \subset F^+(U)$. Hence,$(x, y) \in \beta^*\text{g}-cl(W) \times V \subset (X \times Y) - G_F$. Therefore, $(\beta^*\text{g}-cl(W) \times V) \cap G_F = \emptyset$. This shows that G_F is strongly β^*g-closed. □

Theorem 3.7. *If $F : (X, \tau_1) \to (Y, \tau_2)$ is an upper strongly θ-β^*g-continuous multifunction, $F(x)$ is compact for each $x \in X$ and (Y, τ_2) is a Hausdorff space, then $A = \{(x, y) \in X \times X : F(x) \cap F(y) \neq \emptyset\} \subset X \times X$ is β^*g-θ-closed.*

Proof. Let $(x, y) \in X \times X - A$. Then $F(x) \cap F(y) = \emptyset$. Since $F(x)$ and $F(y)$ are compact and Y is Hausdorff, there exist disjoint, open V_1,$V_2 \subset Y$ sets such that $F(x) \subset V_1$ and $F(y) \subset V_2$. Since F is upper strongly θ-β^*g-continuous, there exist $U_1 \in \beta^*g$O(X,x) and $U_2 \in \beta^*g$O(X,y) such that $x \in \beta^*g - cl(U_1) \subset F^+(V_1)$ and $y \in \beta^*g - cl(U_2) \subset F^+(V_2)$. Also, since $\beta^*g - cl(U_1 \times U_2) \cap A \subset (\beta^*g - cl(U_1) \times \beta^*g - cl(U_2)) \cap A$ and $(\beta^*g - cl(U_1) \times \beta^*g - cl(U_2)) \cap A = \emptyset$, then $\beta^*g - cl(U_1 \times U_2) \cap A = \emptyset$. Since $U_1 \times U_2 \subset X \times X$ is β^*g-open and $(x, y) \in \beta^*g - cl(U_1 \times U_2) \subset X - A$, then $A \subset X \times X$ is $\beta^*g - \theta$-closed set. □

Definition 3.11. Let (X, τ) be a topological space , $A \subset X$ and $F : (X, \tau) \to (A, \tau_A)$ be a multifunction. If $x \in F(x)$ for each $x \in A$, then F is "retraction" from topological space (X, τ) to subspace (A, τ_A) [18].

Theorem 3.8. *Let $F : (X, \tau) \to (X, \tau)$ be an upper strongly θ -β^*g-continuous and (X, τ) be a Hausdorff space. If $F(x)$ is compact for each $x \in X$, then $A = \{x : x \in F(x)\}$ is a β^*g-θ-closed set.*

Proof. Let $x_0 \in \beta^*g$-θ-cl(A). Suppose that $x_0 \notin A$. Then $x_0 \notin F(x_0)$. Since (X, τ) is Hausdorff space and $F(x)$ is compact, then there exist disjoint, open sets U and V such that $x_0 \in U$ and $F(x_0) \subset V$. Since U and V are open sets, $\beta^*g - cl(U) \cap V = \emptyset$. Let $W \in \beta^*gO(X, x_0)$ such that $\beta^*g - cl(W) \subset F^+(W)$. Also, $\beta^*g - cl(U \cap V) \cap A \neq \emptyset$. Let $z \in \beta^*g - cl(U \cap W) \cap A$. Since $z \in A$, $z \in F(z)$ and therefore $z \in \beta^*g - cl(W)$ and $z \in \beta^*g - cl(U)$. Hence $\beta^*g - cl(W)F^+(V)$,that is, it is a contradiction. Then, $x_0 \in A$ and A is a $\beta^*g - \theta$-closed set. □

Corollary 3.1. *Let (X, τ) be a topological space and $A \subset X$. For each $x \in A$, $F : (X, \tau) \to (A, \tau_A)$ upper strongly θ-β^*g-continuous multifunction such that $F(x)$ is compact . If (X, τ) is Hausdorff, $A \subset X$ is β^*g-θ-closed set.*

Definition 3.12. A topological space (X, τ) is said to be β^*g-Urysohn if for each pair of distinct points x and y of X, there exist β^*g-open subsets U and V of X such that $x \in U$, $y \in V$ and $\beta^*g - cl(U) \cap \beta^*g - cl(V) = \emptyset$

Theorem 3.9. *Let $F : (X, \tau_1) \to (Y, \tau_2)$ is an upper strongly θ-β^*g-continuous multifunction and $F(x)$ is compact for each $x \in X$. Also, let $F(x) \cap F(y) = \emptyset$ for each pair of $x, y \in X$ $(x \neq y)$. If Y is a Hausdorff space, (X, τ_1) is β^*g-Urysohn space.*

Proof. Let $F(x) \cap F(y) = \emptyset$ for each pair of $x, y \in X(x \neq y)$. Since Y is Hausdorff, $F(x)$ and $F(y)$ are compact sets, there exist open sets $V_1, V_2 \subset Y$ such that $F(x) \subset V_1$, $F(y) \subset V_2$, $V_1 \cap V_2 = \emptyset$. Since F is upper strongly θ-β^*g-continuous, there exist $U_1 \in \beta^*gO(X, x)$ and $U_2 \in \beta^*gO(X, y)$ such that $x \in \beta^*g - cl(U_1) \subset F^+(V_1)$, $y \in \beta^*g - cl(U_2) \subset F^+(V_2)$. Hence $\beta^*g - cl(U_1) \cap \beta^*g - cl(U_2) = \emptyset$. Therefore, X is β^*g-Urysohn. □

Theorem 3.10. *Let $F : (X, \tau_1) \to (Y, \tau_2)$ is an upper strongly θ-β^*g-continuous and surjective multifunction such that $F(x)$ is compact for each $x \in X$. If X is β^*g-closed space, then (Y, τ_2) is compact space.*

Proof. Let the family of $\{V_\lambda : \lambda \in \Lambda\}$ be an open cover of Y. Since $F(x)$ is a compact set for each $x \in X$, there exist finite $\Lambda(x) \subset \Lambda$ such that $F(x) \subset \cup\{V_\lambda : \lambda \in \Lambda\}$. Let $V(x) = \cup\{V_\lambda : \lambda \in \Lambda\}$. Since F is upper strongly θ-β^*g-continuous, there exist $U(x) \subset X$ β^*g-open set containing x such that β^*g-cl$(U(x)) \subset F^+(V(x))$. Hence, the family of $\{U(x) : x \in X\}$ is a β^*g-open cover of X. Since X is β^*g-closed, there exist a finite number $x_1, x_2, ..., x_n \in X$ such that $X = \cup\{\beta^*\text{g}{-}cl(U(x_i)) : i = 1, 2, ..., n\}$.

$F(X) = Y = F(\cup_{i=1}^n \beta^*g - cl(U(x_i))) = \cup_{i=1}^n F(\beta^*g - cl(U(x_i))) \subset \cup_{i=1}^n V(x_i) = \cup_{i=1}^n \cup_{\lambda \in \Lambda(x_i)} V_\lambda$.

Therefore, Y is compact. □

References

[1] Acikgoz, A., On β^*g-closed sets and new separation axioms. *European J of Pure and Applied Mathematics.* 4 (2011), No. 1, 20-33.

[2] Banzaru, T., On the upper semicontinuity of the upper topological limit for multifunction nets. *Semin. Mat. Fiz. Inst. Politeh Timisoara.* (1983), 59-64.

[3] Berge, C., Escapes topologiques functions multivoques, Paris, Dunod, (1959).

[4] El Naschie, MS., On the uncertainty of cantorian geometry and the two-slit experiment. *Chaos, Solitons and Fractals.* (1998), No. 3, 517-529.

[5] El Naschie, MS., Quantum gravity from descriptive set theory. *Chaos, Solitons and Fractals.* (2004), No. 19, 1339-1344.

[6] El Naschie, MS., Quantum gravity , clifford algebras , fuzzy set theory and the fundamental constants of nature. *Chaos, Solitons and Fractals.* (2004), No. 20, 437-450.

[7] El Naschie, MS., On a fuzzy Kahler-like manifold which is consistent with the two slit experiment. *Int J Nonlinear Sci Numer Simul Fractals.* (2004), No.6, 95-98.

[8] El Naschie, MS., Topics in the mathematical physics of E-nfinity theory. *Chaos,Solitons and Fractals.* (2006), No. 30, 656-663.

[9] Nasef, A.A., On b-locally closed sets and related topics. *Chaos, Solitons and Fractals.* (2001), No. 12, 1909-1915.
[10] Nasef, A.A., Another weak forms of faint continuity. *Chaos, Solitons and Fractals.* (2001), No. 12, 2219-2225.
[11] Nasef, A.A., Some properties of contra-γ-continuous functions. *Chaos, Solitons and Fractals.* (2005), No. 24, 471-477.
[12] Noiri, T. and Popa, V., Faintly m-continuous functions. *Chaos, Solitons and Fractals.* (2004), No. 19, 1147-1159.
[13] Noiri, T. and Popa, V., Almost weakly continuous multifunctions. *Demonstratio Math.* (1993), No. 2, 363-380.
[14] Park, J.H., Almost p-normal, mildly p-normal spaces and some functions . *Chaos, Solitons and Fractals.* (2003), No. 18, 267-274.
[15] Park, J.H., Lee, B.Y., Son, M.J., On upper and lower δ-precontinuous multifunctions. *Chaos, Solitons and Fractals.* 5 (2004), No. 19, 1231-1237.
[16] Park, J.H., Strongly θ-b-continuous functions . *Acta Mathematica Hungarica.* 4 (2006), No. 110, 347-359.
[17] Park, J.H., Bae, S.W. and Park, Y.B., Almost strongly θ-precontinuous functions. *Chaos, Solitons and Fractals.* (2006), No. 28, 32-41.
[18] Whyburn, G.T., Retracting multifunctions. *Proc. Nat. Acad. Sci. U.S.A. Studies.* (1968), No. 59, 343-348.
[19] Velicko, N.V., H-closed topological spaces. *Amer. Math.Soc.Transl.Studies.* (1968), No. 78, 103-118.
[20] Yuksel, S., and Beceren, Y., A decomposition of continuity. *Selcuk Univ.Fac.of Arts Science J.* 14 (1997), No. 1, 79-83.
[21] Yuksel, S., Simsekler, T.H., and Kut, B., Upper and lower na-continuous multifunctions. *Hacettepe Journal of Mathematics and Statistics* 40 (2011), No. 2, 341-348.
[22] Yuksel, S., Simsekler, T.H., and Bilik, B., Upper and lower pre-strong na continuous multifunctions. *Applied Mathematics and Computation* 218 (2011), No. 3, 1142-1146.

Faculty of Arts and Sciences, Department of Mathematics, Balikesir University, Campus of Cagis, 10145, Balikesir-TURKEY
E-mail address: ahuacikgoz@gmail.com

Faculty of Chemical and Metallurgical Engineering, Department of Mathematical Engineering, Yildiz Technical University, 34220, Istanbul-TURKEY
E-mail address: sgoktepe@yildiz.edu.tr