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A NOTE ON TRIPLED FIXED POINTS AND FIXED POINTS OF 3-ORDER IN PARTIALLY ORDERED METRIC SPACES

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Abstract. In this paper, reducing tripled fixed point and fixed point of 3-order results in ordered metric spaces to the respective results for mappings with one variable, some recent results established by Karapinar and Sadarangani [E. Karapinar, K. Sadarangani, Triple fixed point theorems for weak $(\psi-\phi)$-contractions, J. Comput. Anal. Appl., 15, 5 (2013), 844–851], as well as Abbas and Berzig [M. Abbas, M. Berzig, Global attractive results on complete ordered metric spaces for third order difference equations, Intern. J. Anal. 2013, Art. ID 486357] are generalized and improved, with much shorter proofs.

1. Introduction and preliminaries

The metric fixed point theory is very important and useful in Mathematics. It can be applied in various areas, for instance in variational inequalities, optimization, approximation theory, etc.

The fixed point theorems in partially ordered metric spaces play a major role in proving the existence and uniqueness of solutions for some differential and integral equations. One of the most interesting fixed point theorems in ordered metric spaces was investigated by Ran and Reurings [21] who applied their result to linear and nonlinear matrix equations. After that, many authors obtained several interesting results in ordered metric spaces (se, e.g., [11]–[19]).

The notion of a tripled fixed point was introduced and studied by Berinde and Borcut in [5]. Fixed point theory in coupled and tripled cases in ordered metric spaces was studied in [2]–[17].

We start by listing some notation and preliminaries that we shall need to express our results. In this paper $(X, d, \preceq)$ denotes a partially ordered metric space where $(X, \preceq)$ is a partially ordered set and (X, d) is a metric space.

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Definition 1.1. [4, 5, 8, 13, 14] Let $(X, \preceq)$ be a partially ordered set and $F : X^3 \to X$ be a mapping.

(1) F is said to have the mixed monotone property if $F(x, y, z)$ is non-decreasing in x and z, and non-increasing in y.
(2) An element $(x, y, z) \in X^3$ is called a tripled fixed point of F if $F(x, y, z) = x$, $F(y, x, y) = y$ and $F(z, y, x) = z$.
(3) [1] An element $(x, y, z) \in X^3$ is a fixed point of 3-order of F if $F(x, y, x) = x$, $F(y, x, y) = y$ and $F(z, y, z) = z$.

Definition 1.2. We say that an ordered metric space $(X, d, \preceq)$ is regular if it has the following properties:

(i) if for a non-decreasing sequence $\{x_n\}$, $d(x_n, x) \to 0$ holds, then $x_n \preceq x$ for all n,

(i) if for a non-increasing sequence $\{y_n\}$, $d(y_n, y) \to 0$ holds, then $y_n \succeq y$ for all n.

The proof of the following lemma is immediate.

Lemma 1.1. *Let $(X, d, \preceq)$ be a partially ordered metric space and let $F : X^3 \to X$.*

(1) *If relation $\sqsubseteq$ is defined on X^3 by*
$$Y \sqsubseteq V \;\Leftrightarrow\; x \preceq u \wedge y \succeq v \wedge z \preceq w, \quad Y = (x, y, z), V = (u, v, w) \in X^3,$$
and $D : X^3 \times X^3 \to \mathbb{R}^+$ is given by
$$D(Y, V) = \max\{d(x, u), d(y, v), d(z, w)\}, \quad Y = (x, y, z), V = (u, v, w) \in X^3,$$
then $(X^3, D, \sqsubseteq)$ is a partially ordered metric spaces. The space (X^3, D) is complete if and only if (X, d) is complete. Also, the space $(X^3, D, \sqsubseteq)$ is regular if and only if $(X, d, \preceq)$ is such.

(2) *If F has the mixed monotone property, then the mapping $T_F : X^3 \to X^3$ given by*
$$T_F(Y) = (F(x, y, z), F(y, x, y), F(z, y, x)), \quad Y = (x, y, z) \in X^3$$
is non-decreasing with respect to $\sqsubseteq$, that is,
$$Y \sqsubseteq V \;\Rightarrow\; T_F(Y) \sqsubseteq T_F(V).$$

(3) *If F has the mixed monotone property, then the mapping $S_F : X^3 \to X^3$ given by*
$$S_F(Y) = (F(x, y, x), F(y, x, y), F(z, y, z)), \quad Y = (x, y, z) \in X^3$$
is also non-decreasing with respect to $\sqsubseteq$.

(4) *The mapping F is continuous if and only if T_F (resp. S_F) is continuous.*

(5) *The mapping F has a tripled fixed point (resp. a fixed point of 3-order) if and only if the mapping T_F (resp. S_F) has a fixed point in X^3.*

In what follows, the classes Φ and Ψ of functions are defined by:

$\phi : [0, +\infty) \to [0, +\infty)$ belongs to Φ if ϕ is non-decreasing and $\lim_{t \to r+} \phi(t) > 0$ for all $r > 0$ and $\lim_{t \to 0+} \phi(t) = 0$;

$\psi : [0, +\infty) \to [0, +\infty)$ belongs to Ψ if:

(i) $\psi(t) = 0$ if and only if $t = 0$;
(ii) ψ is continuous and non-decreasing;
(iii) $\psi(s + t) = \psi(s) + \psi(t)$ for all $s, t \in [0, +\infty)$.

In [14], Karapinar and Sadarangani proved the following results and formulated as Theorem 7, Corollary 8 and Theorem 9, respectively.

Theorem 1.1. *Let $(X, d, \preceq)$ be a complete partially ordered metric space. Let $F : X^3 \to X$ be a mapping having the mixed monotone property. Assume that there exist constants $a, b, c \in [0, 1)$ such that $a + 2b + c < 1$ with*

$$\begin{aligned}(1.1)\qquad \psi(d(F(x,y,z),F(u,v,w))) &\leq \psi(ad(x,u)+bd(y,v)+cd(z,w)) \\ &\quad - \varphi(ad(x,u)+bd(y,v)+cd(z,w))\end{aligned}$$

for all $x \succeq u$, $y \preceq v$, $z \succeq w$, where $\phi \in \Phi$, $\psi \in \Psi$. Suppose that there exist $x_0, y_0, z_0 \in X$ such that

$$x_0 \preceq F(x_0, y_0, z_0),\ y_0 \succeq F(y_0, x_0, y_0) \text{ and } z_0 \preceq F(z_0, y_0, x_0).$$

Suppose either

(a) F is continuous, or

(b) X is regular.

Then there exist $x, y, z \in X$ such that

$$F(x, y, z) = x,\ \ F(y, x, y) = y \text{ and } F(z, y, x) = z.$$

Theorem 1.2. *Let $(X, d, \preceq)$ be a complete partially ordered metric space. Let $F : X^3 \to X$ be a mapping having the mixed monotone property on X. Assume that there exist constants $a, b, c \in [0, 1)$ such that $a + 2b + c < 1$ with*

$$(1.2)\qquad d(F(x,y,z),F(u,v,w)) \leq ad(x,u)+bd(y,v)+cd(z,w)$$

for all $x \succeq u$, $y \preceq v$, $z \succeq w$. Assume also that there exist $x_0, y_0, z_0 \in X$ such that

$$x_0 \preceq F(x_0, y_0, z_0), \quad y_0 \succeq F(y_0, x_0, y_0), \quad z_0 \preceq F(z_0, y_0, x_0).$$

Suppose either

(a) F is continuous, or

(b) X is regular.

Then there exist $x, y, z \in X$ such that

$$F(x, y, z) = x, \quad F(y, x, y) = y, \quad F(z, y, x) = z.$$

Remark 1.1. Since condition (1.2) follows from condition (1.1) of Theorem 1.1 (see further Remark 2.1), Theorem 1.1 follows from Theorem 1.2 (although it was stated the other way around in [14]). The proof of Theorem 1.2 can be found in [5], with the assumption $a + b + c < 1$.

Theorem 1.3. *In addition to hypotheses of Theorem 1.1, suppose that for all $(x, y, z), (u, v, w) \in X^3$ there exists $(a, b, c) \in X^3$ that is comparable with (x, y, z) and (u, v, w), w.r.t. $\sqsubseteq$. Then F has a unique triple fixed point.*

In [1], Abbas and Berzig proved the following results and formulated as Theorem 3 and Theorem 4, respectively.

Theorem 1.4. *Let $(X, d, \preceq)$ be a complete partially ordered metric space. Let $F : X^3 \to X$ be a mapping having the mixed monotone property. Suppose that the following conditions hold.*

(i) There exists $\lambda \in [0, 1)$ with

(1.3)

$$\begin{aligned}d(F(x,y,z),F(u,v,w)) \leq \lambda \max\{&d(x,u), d(y,v), d(z,w), d(F(x,y,x),x), \\ &d(F(x,y,x),u), d(F(y,x,y),y), d(F(y,x,y),v), d(F(z,y,z),z), d(F(z,y,z),w)\}\end{aligned}$$

for all $(x, y, z) \sqsubseteq (u, v, w)$.

(ii) There exist $x_0, y_0, z_0 \in X$ *such that*

$$x_0 \preceq F(x_0, y_0, x_0), \quad y_0 \succeq F(y_0, x_0, y_0) \text{ and } z_0 \preceq F(z_0, y_0, z_0). \tag{1.4}$$

(iii) $(X, d, \preceq)$ *is regular; further, if* $x_n \preceq y_n$ *for every* n *then* $\lim_{n\to\infty} x_n \preceq \lim_{n\to\infty} y_n$.

Then we have the following.

(a) For every initial point $(x_0, y_0, z_0) \in X^3$ *such that condition* (1.4) *holds,*

$$x_n = F^n(x_0, y_0, z_0) \to x, y_n = F^n(y_0, x_0, y_0) \to y \text{ and } z_n = F^n(z_0, y_0, z_0) \to z$$

as $n \to \infty$, *where* x, y, z *satisfy*

$$x = F(x, y, x), \quad y = F(y, x, y), \quad z = F(z, y, z).$$

If $x_0 \preceq y_0$ *and* $y_0 \preceq z_0$ *in condition* (1.4), *then* $x = y = z$ *and* $\{x_n\}$, $\{y_n\}$ *and* $\{z_n\}$ *converge to the equilibrium of the equation*

$$x_{n+1} = F(x_n, y_n, x_n), \; y_{n+1} = F(y_n, x_n, y_n), \; z_{n+1} = F(z_n, y_n, z_n), \quad n = 0, 1, 2, \ldots \tag{1.5}$$

(b) In particular, every solution $\{u_n\}$ *of*

$$u_{n+1} = F(u_n, u_{n-1}, u_{n-2}), \quad n = 2, 3, \ldots$$

such that $u_0, u_1, u_2 \in [x_0, y_0]$ *(or* $[z_0, y_0]$*), converges to the equilibrium of* (1.5).

(c) The following estimates hold

$$\begin{aligned} d(x, F^n(x_0, y_0, x_0)) \le \frac{\lambda^n}{1-\lambda} &\max\{d(F(x_0, y_0, z_0), x_0), d(F(y_0, x_0, y_0), y_0), \\ &d(F(z_0, y_0, z_0), z_0)\}, \\ d(y, F^n(x_0, y_0, x_0)) \le \frac{\lambda^n}{1-\lambda} &\max\{d(F(x_0, y_0, z_0), x_0), d(F(y_0, x_0, y_0), y_0), \\ &d(F(z_0, y_0, z_0), z_0)\}, \\ d(z, F^n(x_0, y_0, x_0)) \le \frac{\lambda^n}{1-\lambda} &\max\{d(F(x_0, y_0, z_0), x_0), d(F(y_0, x_0, y_0), y_0), \\ &d(F(z_0, y_0, z_0), z_0)\}. \end{aligned}$$

Theorem 1.5. *In addition to the hypotheses of Theorem 1.4, suppose that for every* $(x, y, z), (x', y', z') \in X^3$, *there exists* $(u, v, w) \in X^3$ *such that* $(x, y, z) \sqsubseteq (u, v, w)$ *and* $(x', y', z') \sqsubseteq (u, v, w)$. *Then we obtain the uniqueness of the fixed point of 3-order.*

2. Main results

Our first result is the following theorem which generalizes and improves the above Theorems 1.1, 1.2, that is, Theorem 7 and Corollary 8 from [14].

Theorem 2.1. *Let* $(X, d, \preceq)$ *be a complete partially ordered metric space and let* $F : X^3 \to X$ *has the mixed monotone property. Suppose that the following conditions hold.*

(i) There exists $\lambda \in [0, 1)$ *with*

$$\begin{aligned} \max\{d(F(x, y, z), F(u, v, w)), &d(F(y, x, y), F(v, u, v)), d(F(z, y, x), F(w, v, u))\} \\ &\le \lambda \max\{d(x, u), d(y, v), d(z, w)\} \end{aligned} \tag{2.1}$$

for all $(x, y, z), (u, v, w) \in X^3$ *such that* $x \succeq u$, $y \preceq v$, $z \succeq w$.

(ii) There exist $x_0, y_0, z_0 \in X$ *such that*

$$x_0 \preceq F(x_0, y_0, z_0), \quad y_0 \succeq F(y_0, x_0, y_0) \text{ and } z_0 \preceq F(z_0, y_0, x_0).$$

Suppose that either

(1) F *is continuous, or*

(2) $(X, d, \preceq)$ *is regular.*

Then there exist $x, y, z \in X$ *such that*

$$F(x, y, z) = x, \quad F(y, x, y) = y \text{ and } F(z, y, x) = z.$$

Proof. According to Lemma 1.1, (1)–(2) the condition (2.1) implies the following contractive condition

$$D(T_F(Y), T_F(V)) \leq \lambda D(Y, V),$$

for all $Y, V \in X^3$ with $Y \sqsubseteq V$ or $Y \sqsupseteq V$. Further, the proof follows by ([18], Theorems 2.1 and 2.2) and from Lemma 1.1, (5). □

Remark 2.1. It is clear that condition (1.1) implies (2.1). Firstly, (1.1) implies the following

$$\begin{aligned} d(F(x, y, z), F(u, v, w)) &\leq ad(x, u) + bd(y, v) + cd(z, w) \\ &\leq \lambda \max\{d(x, u), d(y, v), d(z, w)\}, \end{aligned}$$

since $\phi \in \Phi$ is non-negative and $\psi \in \Psi$ is non-decreasing, where $\lambda = a + b + c \leq a + 2b + c < 1$. Now, since $y \preceq v$, $x \succeq u$ and $y \preceq v$ it follows from (1.1) that

$$\begin{aligned} d(F(y, x, y), F(v, u, v)) &\leq \lambda \max\{d(y, v), d(x, u), d(y, v)\} \\ &= \lambda \max\{d(x, u), d(y, v)\} \\ &\leq \lambda \max\{d(x, u), d(y, v), d(z, w)\}. \end{aligned}$$

Further, by a similar argument we get that

$$\begin{aligned} d(F(z, y, x), F(w, v, u)) &\leq \lambda \max\{d(z, w), d(y, v), d(x, u)\} \\ &= \lambda \max\{d(x, u), d(y, v), d(z, w)\}. \end{aligned}$$

Hence, taking the maximum of the left-hand sides, we get (2.1) from (1.1) (or (1.2)). Thus, Theorems 1.1 and 1.2, i.e., Theorem 7 and Corollary 8 from [14] are immediate consequences of Theorem 2.1 if $a, b, c \in [0, 1)$ and $a + b + c < 1$.

Remark 2.2. We note now that Theorem 1.3, that is, Theorem 9 of [14] is an immediate consequence of our Theorem 2.1 and ([18], Theorem 2.3).

Theorem 2.1 is more general than Theorems 1.1 and 1.2. The following example illustrates our claim.

Example 2.1. Let $X = \mathbb{R}$ be equipped with the standard metric and order. Consider the mapping $F : X^3 \to X$ given by

$$F(x, y, z) = \frac{1}{2}x - \frac{1}{3}y,$$

and let $a = \frac{k}{2}$, $b = \frac{k}{4}$, $c = 0$, $k \in [0, 1)$ (then $a + 2b + c = k < 1$ holds). Obviously, F has the mixed monotone property. We will check that condition (2.1) of Theorem 2.1 is fulfilled, while condition (1.2) of Theorem 1.2 is not.

Indeed, putting $x = 1 \succeq 0 = y = u = v$, $z \succeq w$ arbitrary, we get that $d(F(1, 0, z), F(0, 0, w)) = |\frac{1}{2} - 0| = \frac{1}{2} > \frac{k}{2} = \frac{k}{2} \cdot d(1, 0) + \frac{k}{4} \cdot d(0, 0) + 0 \cdot d(z, w)$.

On the other hand, F satisfies (2.1) since

$$\begin{aligned}
d(F(x,y,z),F(u,v,w)) &= |\tfrac{1}{2}x - \tfrac{1}{3}y - (\tfrac{1}{2}u - \tfrac{1}{3}v)| = |\tfrac{1}{2}(x-u) + \tfrac{1}{3}(v-y)| \\
&\le \tfrac{5}{6}\max\{d(x,u),d(y,v),d(z,w)\}, \\
d(F(y,x,y),F(v,u,v)) &= |\tfrac{1}{2}y - \tfrac{1}{3}x - (\tfrac{1}{2}v - \tfrac{1}{3}u)| = |\tfrac{1}{2}(y-v) + \tfrac{1}{3}(u-x)| \\
&\le \tfrac{5}{6}\max\{d(x,u),d(y,v),d(z,w)\}, \\
d(F(z,y,x),F(w,v,u)) &= |\tfrac{1}{2}z - \tfrac{1}{3}y - (\tfrac{1}{2}w - \tfrac{1}{3}v)| = |\tfrac{1}{2}(z-w) + \tfrac{1}{3}(v-y)| \\
&\le \tfrac{5}{6}\max\{d(x,u),d(y,v),d(z,w)\},
\end{aligned}$$

which implies that

$$\begin{aligned}
&\max\{d(F(x,y,z),F(u,v,w)),d(F(y,x,y),F(v,u,v)),d(F(y,x,y),F(v,u,v))\} \\
&\qquad\qquad\le \tfrac{5}{6}\max\{d(x,u),d(y,v),d(z,w)\}.
\end{aligned}$$

We conclude that Theorem 2.1 is a proper generalization of Theorem 1.2 (and so of Theorem 1.1).

Our second result is the following theorem which generalizes and improves the above Theorem 1.4(a), that is, Theorem 3(a) from [1].

Theorem 2.2. *Let $(X,d,\preceq)$ be a complete partially ordered metric space and let $F : X^3 \to X$ be a mapping having the mixed monotone property. Suppose that the following conditions hold.*

(i) There exists $\lambda \in [0,1)$ with

(2.2)

$$\begin{aligned}
&\max\{d(F(x,y,z),F(u,v,w)),d(F(x,y,x),F(u,v,u)),d(F(z,y,z),F(w,v,w))\} \\
&\quad\le \lambda\max\{d(x,u),d(y,v),d(z,w),d(F(x,y,x),x),d(F(x,y,x),u),d(F(y,x,y),y), \\
&\qquad d(F(y,x,y),v),d(F(z,y,z),z),d(F(z,y,z),w),d(F(u,v,u),u),d(F(u,v,u),x), \\
&\qquad d(F(v,u,v),v),d(F(v,u,v),y),d(F(w,v,w),w),d(F(w,v,w),z)\}
\end{aligned}$$

for all $(x,y,z) \sqsubseteq (u,v,w)$.

(ii) There exist $x_0, y_0, z_0 \in X$ such that

$$x_0 \preceq F(x_0,y_0,x_0), \quad y_0 \succeq F(y_0,x_0,y_0) \text{ and } z_0 \preceq F(z_0,y_0,z_0). \tag{2.3}$$

Suppose either
(1) F is continuous, or
(2) $(X,d,\preceq)$ is regular.
Then F has a fixed point of 3-order.

Proof. Firstly, by Lemma 1.1, (1) and (3), the condition (2.2) becomes

(2.4)

$$\begin{aligned}
&D(S_F(Y),S_F(V)) \\
&\quad\le \lambda\max\{D(Y,V),D(S_F(Y),Y),D(S_F(V),V),D(Y,S_F(V)),D(S_F(Y),V)\}
\end{aligned}$$

for all $Y,V \in X^3$ with $Y \sqsubseteq V$. It follows from (2.3) that there exists $Y_0 \in X^3$ such that $Y_0 \sqsubseteq S_F(Y_0)$. Again, by Lemma 1.1, (1) and (4), all conditions of ([15], Corollary 2) are satisfied. It follows that S_F has a fixed point in X^3. According to Lemma 1.1, (5), F has a fixed point of 3-order. □

Remark 2.3. Note that condition (1.3) of Theorem 1.3 implies condition (2.2) of the previous theorem. Hence, Theorem 2.2 is more general than Theorem 1.3(a), i.e., Theorem 3(a) of [1].

Indeed, the right-hand side of (2.2) is greater than the right-hand side of (1.3). When x, y, z (resp. u, v, w) in (1.3) are replaced by x, y, x (resp. u, v, u) and then by z, y, z (resp. w, v, w), and taking the maximum of both sides, the condition (2.2) is obtained. This is possible since $(x, y, x) \sqsubseteq (u, v, u)$ and $(z, y, z) \sqsubseteq (w, v, w)$ if $(x, y, z) \sqsubseteq (u, v, w)$).

By a similar example as Example 2.1, it can be shown that this generalization is proper.

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