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ON BLOCKING SETS IN A SPECIAL PROJECTIVE PLANE OF ORDER 9

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Abstract. In this paper the numbers of $t-$fold blocking sets in the Baer subplane which is the projective subplane of order 3 of the projective plane of order 9 over the left semifield, using MATLAB are given.

1. Introduction

In the past, mathematicians went on to consider more general kinds of geometry, geometries of curved space, and more abstract even than curved space. There have been some attempts to find special subsets in projective geometry. Daskalov (Discrete Mathematics, 308 (2008) 1341-1345.) introduced *a geometric contruction of a* $(38, 2)-$*blocking set in* $PG(2, 13)$ *and the related* [145,3,133] [13] *code.* In [3], the numerical solutions of $t-$fold blocking sets in the Baer subplane which is the projective subplane of order 5 of the projective plane of order 25 over the Cartesian group, using MATLAB were given. In the present paper, we will $t-$fold blocking sets in a special the left semifield projective plane of order 9.

2. Preliminaries

One can consider an incidence structure $\circ$ while $\mathcal{N}$ and $\mathcal{D}$ are two distinct sets whose elements are called points and lines. An incidence structure satisfying the following three properties is called a projective plane and denoted by $\mathbb{P} = (\mathcal{N}, \mathcal{D}, \circ)$.

P1) Any distinct two points are incident with a unique line.

P2) Any two lines are incident with at least one point.

P3) There exist four points, no three incident with a common line.

There is an integer $n \geq 2$ with the properties: any line is incident with $n + 1$ points; any point is incident with $n + 1$ lines; The total number of its points and lines is equal and n^2+n+1. The integer n is called the order of the projective plane $\mathbb{P} = (\mathcal{N}, \mathcal{D}, \circ)$. The smallest projective plane has order 2 and this projective plane is called as Fano plane. Fano subplanes and blocking sets in some projective planes

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have been examined by many authors. For instance, Room-Kirpatrick [10], Çifçi-Kaya [4], Akça-Kaya [1], Akça-Günaltılı and Güney [2], Daskalov [5] and Walls [13] etc. A left semifield of order 9 is defined as fallows:

Definition 2.1. (see [7],[8] A left semifield is a system $(S, \oplus, \odot)$, where $\oplus$ and $\odot$ are binary operations on the set S and

(1) S is finite
(2) $(S, \oplus)$ is a group, with identity 0
(3) $(S \backslash \{0\}, \odot)$ is a semi-group, with identity 1
(4) $x \odot 0 = 0$ for all $x \in S$
(5) $\odot$ is left distributive over $\oplus$, that is $x \odot (y \oplus z) = (x \odot y) \oplus (x \odot z)$ for all $x, y, z \in S$
(6) Given $a, b, c \in S$ with $a \neq b$, there exists a unique $x \in S$ such that

$$-a \odot x \oplus b \odot x = c.$$

Let $(F_3, +, .)$ be the field of integers modulo 3. Let

$$S = \{a + \lambda b \ : \ a, b \in F_3, \ \lambda \notin F_3\}$$

and consider the addition and multiplication on S given by

$$(a + \lambda b) \oplus (c + \lambda d) = (a + c) + \lambda(b + d) \tag{1}$$

and

$$(a + \lambda b) \odot (c + \lambda d) = \begin{cases} ac + \lambda(ad), & \text{if} \quad b = 0 \\ ac - b^{-1}df(a) + \lambda(bc - (a-1)d), & \text{if} \quad b \neq 0 \end{cases} \tag{2}$$

where, $f(t) = t^2 - t - 1$ is a irreducible polynom on F_3.

For the sake of shortness if we use ab instead of $a + \lambda b$ in equation (1) and (2) then addition and multiplication tables as follows:

$\oplus$	00	01	02	10	11	12	20	21	22
00	00	01	02	10	11	12	20	21	22
01	01	02	00	11	12	10	21	22	20
02	02	00	01	12	10	11	22	20	21
10	10	11	12	20	21	22	00	01	02
11	11	12	10	21	22	20	01	02	00
12	12	10	11	22	20	21	02	00	01
20	20	21	22	00	01	02	10	11	12
21	21	22	20	01	02	00	11	12	10
22	22	20	21	02	00	01	12	10	11

Table 1.

$\odot$	00	01	02	10	11	12	20	21	22
00	00	00	00	00	00	00	00	00	00
01	00	11	22	01	12	20	02	10	21
02	00	21	12	02	20	11	01	22	10
10	00	01	02	10	11	12	20	21	22
11	00	10	20	11	21	01	22	02	12
12	00	20	10	12	02	22	21	11	01
20	00	02	01	20	22	21	10	12	11
21	00	22	11	21	10	02	12	01	20
22	00	12	21	22	01	10	11	20	02

Table 2.

In [11], the system $(S, \oplus, \odot)$ is a left semifield of order 9.

In [2], the projective plane of order 9 coordinatized by elements of the above left semifield is considered and investigated.

Definition 2.2. The Plane $\mathbb{P}_2S$: The 91 points of $\mathbb{P}_2S$ are the elements of the set

$$\{(x, y)\ :\ x, y \in S\} \cup \{(m) : m \in S\} \cup \{(\infty)\}.$$

The points of the form (x, y) are called *proper points*, and the unique point (∞) and the points of the form (m) are called *ideal points*. The 91 lines of P_2S are defined to be set of points satisfying one of the three conditions:

$$[m, k] = \{(x, y) \in S^2 : y = m \odot x \oplus k\} \cup \{(m)\}$$
$$[\lambda] = \{(x, y) \in S^2 :\ x = \lambda\} \cup \{(\infty)\}$$
$$[\infty] = \{(m) \in S\} \cup \{(\infty)\}$$

The 81 lines having form $y = m \odot x \oplus k$ and 9 lines having equation of the form $x = \lambda$ are called the *proper lines* and the unique line $[\infty]$ is called *the ideal line.*

In [2],[6], the system of points, lines and incidence relation given above defines a projective plane of order 9, which is the left semifield plane .

Definition 2.3. (see [5],[13]) A $t-$blocking set B is a set of points in a projective plane $\mathbb{P}$ of order n such that every line in $\mathbb{P}$ intersects B in at least t points. A $t-$fold blocking set $k-$set is a $t-$fold blocking set with k points.

In this section $t-$fold blocking sets of the subplanes of order 2 and 3 of the left semifield plane of order 9 will be considered. That is the points have affine coordinates (x, y), where x, y are elements of $(S, \oplus, \odot)$. In this case, the set points, the set of lines and the incidence relation of subplanes of order 2 and 3 of P_2S are as follows respectively;

$$\mathcal{N} = \{(00, 00), (00, 10), (00, 20), (10, 00), (20, 20), (10, 10), (00)\}$$

$$\mathcal{D} = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$$

$\circ$:

$d_1 = [00]$	$\{(00, 00), (00, 10), (00, 20)\}$
$d_2 = [10, 00]$	$\{(00, 00), (10, 10), (20, 20)\}$
$d_3 = [00, 00]$	$\{(00, 00), (20, 00), (00)\}$
$d_4 = [00, 10]$	$\{(00, 10), (10, 10), (00)\}$
$d_5 = [00, 20]$	$\{(00, 20), (20, 20), (00)\}$
$d_6 = [20, 20]$	$\{(00, 20), (10, 10), (20, 00)\}$
$d_7 = [10, 10]$	$\{(00, 10), (20, 00), (20, 20)\}$

and

$$\mathcal{N} = \left\{ \begin{array}{l} \{N_1 = (00,00)\,, N_2 = (10,00)\,, N_3 = (20,00)\,, N_4 = (00)\,, N_5 = (00,10)\,, \\ \\ N_6 = (10,10)\,, N_7 = (20,10)\,, N_8 = (00,20)\,, N_9 = (10,20)\,, N_{10} =, (20,20)\,, \\ \\ N_{11} = (\infty)\,, N_{12} = (10)\,, N_{13} = (20)\,, \} \end{array} \right\}$$

$$\mathcal{D} = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}, d_{12}, d_{13}\}$$

$\circ$:

$d_1 = [00]$	$\{(00,00)\,,(00,10)\,,(00,20)\,,(\infty)\}$
$d_2 = [10]$	$\{(10,00)\,,(10,10)\,,(10,20)\,,(\infty)\}$
$d_3 = [20]$	$\{(20,00)\,,(20,10)\,,(20,20)\,,(\infty)\}$
$d_4 = [10,20]$	$\{(00,20)\,,(10,00)\,,(20,10)\,,(10)\}$
$d_5 = [10,00]$	$\{(00,00)\,,(10,10)\,,(20,20)\,,(10)\}$
$d_6 = [10,10]$	$\{(00,10)\,,(10,20)\,,(20,00)\,,(10)\}$
$d_7 = [00,20]$	$\{(00,20)\,,(10,20)\,,(20,20)\,,(00)\}$
$d_8 = [00,10]$	$\{(00,10)\,,(10,10)\,,(20,10)\,,(00)\}$
$d_9 = [00,00]$	$\{(00,00)\,,(10,00)\,,(20,00)\,,(00)\}$
$d_{10} = [\infty]$	$\{(00)\,,(10)\,,(20)\,,(\infty)\}$
$d_{11} = [20,00]$	$\{(00,00)\,,(10,20)\,,(20,10)\,,(20)\}$
$d_{12} = [20,20]$	$\{(00,20)\,,(10,10)\,,(20,00)\,,(20)\}$
$d_{13} = [20,10]$	$\{(00,10)\,,(10,00)\,,(20,20)\,,(20)\}$

3. Blocking Sets in the Projective Plane $\mathbb{P}_2S$

In this section, our program was written in MATLAB for the classification of the t-fold blocking sets in the $\mathbb{P}_2S$ which is the projective subplane of order 3 of the projective plane of order 9, using MATLAB in [9] and [12] was presented and were obtained some tests of the t-fold blocking sets in this work has been obtained using a computer-based exhaustive search. Now, we will give some examples of $t-$fold blocking sets of master thesis of second author of this paper in [12] and numbers , respectively.

Theorem 3.1. *There are* $1-$*fold,* $2-$*fold and* $3-$*fold blocking sets of Fano subplane of* $\mathbb{P}_2S$ *.*

Proof. Firstly, in the Fano plane every collinear 3 points form a $1-$fold bolcking set. Because every line in the Fano plane intersects every other line exactly once. Besides ,a set of three points, four points and five points such that three are collinear also form $1-$ fold blocking sets.

Secondly, in the Fano plane every six points form a $2-$fold blocking set. Since, every line intersects six points exactly twice.

Finally, all seven points in the Fano plane form $3-$fold blocking set in as much as; every line intersects all seven points exactly three time. □

Remark 3.1. The number of $1-$fold blocking sets with four points of subplane of order 3 of $\mathbb{P}_2S$ is 13.

Example 3.1. we give the first example of such a set $B = \{N_1, N_2, N_3, N_4\}$. Consider the following table.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
$s(B \cap d_n)$	1	1	1	1	1	1	1	1	4	1	1	1	1

Remark 3.2. The number of $1-$fold blocking sets with five points of subplane of order 3 of $\mathbb{P}_2 S$ is 117.

Example 3.2. If $B = \{N_1, N_2, N_3, N_4, N_5\}$, then the following example of $1-$fold blocking set is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
$s(B \cap d_n)$	2	1	1	2	1	1	1	2	4	1	1	1	2

Remark 3.3. The number of $1-$fold blocking sets with six points of subplane of order 3 of $\mathbb{P}_2 S$ is 702.

Example 3.3. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6\}$, then the following example of $1-$fold blocking set is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
$s(B \cap d_n)$	2	2	1	2	2	1	1	3	4	1	1	2	2

Remark 3.4. The number of $1-$fold blocking sets with seven points of subplane of order 3 of $\mathbb{P}_2 S$ is 1248.

Example 3.4. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7\}$, then the following example of $1-$fold blocking set is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
$s(B \cap d_n)$	2	2	2	2	2	2	1	4	4	1	2	2	2

Remark 3.5. The number of 1−fold blocking sets with eight points of subplane of order 3 of $\mathbb{P}_2 S$ is 1170.

Example 3.5. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8\}$, then the following example of 1−fold blocking set is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_8	1	0	0	0	0	1	1	0	0	0	0	1	0
$s(B \cap d_n)$	3	2	2	2	2	3	2	4	4	1	2	3	2

Remark 3.6. The number of 1−fold and 2−fold blocking sets with nine points of subplane of order 3 of $\mathbb{P}_2 S$ are 468 and 234, respectively.

Example 3.6. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9\}$, then the following example of 1−fold blocking sets is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_8	1	0	0	0	0	1	1	0	0	0	0	1	0
N_9	0	1	0	1	0	0	1	0	0	0	1	0	0
$s(B \cap d_n)$	3	3	2	3	2	3	3	4	4	1	3	3	2

If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_{10}, N_{12}\}$, then the following example of 2−fold blocking sets is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_{10}	0	0	1	0	1	0	1	0	0	0	0	0	1
N_{12}	0	0	0	1	1	1	0	0	0	1	0	0	0
$s(B \cap d_n)$	2	2	3	3	4	3	2	4	4	2	2	2	3

Remark 3.7. The number of 1−fold and 2−fold blocking sets with ten points of subplane of order 3 of $\mathbb{P}_2 S$ are 52 and 234, respectively.

Example 3.7. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}\}$, then the following example of 1−fold blocking sets is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_8	1	0	0	0	0	1	1	0	0	0	0	1	0
N_9	0	1	0	1	0	0	1	0	0	0	1	0	0
N_{10}	0	0	1	0	1	0	1	0	0	0	0	0	1
$s(B \cap d_n)$	3	3	3	3	3	3	4	4	4	1	3	3	3

If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_9, N_{10}, N_{12}\}$, then the following example of 2−fold blocking sets is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_9	0	1	0	1	0	0	1	0	0	0	1	0	0
N_{10}	0	0	1	0	1	0	1	0	0	0	0	0	1
N_{12}	0	0	0	1	1	1	0	0	0	1	0	0	0
$s(B \cap d_n)$	2	3	3	4	4	3	3	4	4	2	3	2	3

Remark 3.8. The number of 2−fold blocking sets with eleven points of subplane of order 3 of $\mathbb{P}_2 S$ is 78.

Example 3.8. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_{8,} N_9, N_{10}, N_{11}\}$, then the following example of 2−fold blocking sets is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_8	1	0	0	0	0	1	1	0	0	0	0	1	0
N_9	0	1	0	1	0	0	1	0	0	0	1	0	0
N_{10}	0	0	1	0	1	0	1	0	0	0	0	0	1
N_{11}	1	1	1	0	0	0	0	0	0	1	0	0	0
$s(B \cap d_n)$	3	4	4	3	3	2	3	4	4	2	3	2	3

Remark 3.9. The number of $3-$fold blocking sets with twelve points of subplane of order 3 of $\mathbb{P}_2 S$ is 13.

Example 3.9. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}\}$, then the following example of $3-$fold blocking sets is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_8	1	0	0	0	0	1	1	0	0	0	0	1	0
N_9	0	1	0	1	0	0	1	0	0	0	1	0	0
N_{10}	0	0	1	0	1	0	1	0	0	0	0	0	1
N_{11}	1	1	1	0	0	0	0	0	0	1	0	0	0
N_{12}	0	0	0	1	1	1	0	0	0	1	0	0	0
$s\,(B \cap d_n)$	4	4	4	4	4	4	4	4	4	3	3	3	3

Remark 3.10. The number of $4-$fold blocking sets with thirteen points of subplane of order 3 of $\mathbb{P}_2 S$ is 1.

Example 3.10. If $B = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}\}$, then the following example of $4-$fold blocking sets is given.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}
N_1	1	0	0	0	1	0	0	0	1	0	1	0	0
N_2	0	1	0	0	0	1	0	0	1	0	0	0	1
N_3	0	0	1	1	0	0	0	0	1	0	0	1	0
N_4	0	0	0	0	0	0	1	1	1	1	0	0	0
N_5	1	0	0	1	0	0	0	1	0	0	0	0	1
N_6	0	1	0	0	1	0	0	1	0	0	0	1	0
N_7	0	0	1	0	0	1	0	1	0	0	1	0	0
N_8	1	0	0	0	0	1	1	0	0	0	0	1	0
N_9	0	1	0	1	0	0	1	0	0	0	1	0	0
N_{10}	0	0	1	0	1	0	1	0	0	0	0	0	1
N_{11}	1	1	1	0	0	0	0	0	0	1	0	0	0
N_{12}	0	0	0	1	1	1	0	0	0	1	0	0	0
N_{13}	0	0	0	0	0	0	0	0	0	1	1	1	1
$s\,(B \cap d_n)$	4	4	4	4	4	4	4	4	4	4	4	4	4

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