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# Chaos Theory and Applications: The Physical Evidence, Mechanism are Important in Chaotic Systems

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**ABSTRACT** This editorial is presented for readers and researchers in the field of nonlinear dynamics, including dynamical control, synchronization stability and control, fractional order approach, boundary condition, memristive system, functional neural circuit, Hamilton energy and Lyapunov function. These short comments and clarifications are helpful to explain the motive of scientific research, physical principle and potential application of nonlinear circuits, statistical analysis and schemes, and thus the report and papers may become readable and instructive.

**KEYWORDS**
Time delay
Memristors
Hamilton energy
Fractional order

Most of the deterministic dynamical systems with nonlinear terms can be tamed in the intrinsic parameters or excited by using external stimulus for inducing chaotic states, and numerical solutions can be detected under reliable algorithm, as a result, Poincare section and Lyapunov exponent spectrum are calculated to predict the parameter regions for generating chaos. For experimental series of some variables, power spectrum analysis becomes available and dense orbits are confirmed in the phase space when chaos keeps survival. In experimental way, many nonlinear circuits are useful to reproduce the dynamical properties of the chaotic systems, some of the intrinsic parameters of electric components can be adjusted to change the energy exchange, output voltage from capacitor and channel current across each branch circuit. For most of the isolated nonlinear circuits, the standard electric components are ideal and linear relation for the input and output variables, while nonlinear components can be rebuilt by using equivalent branch circuits.

It is claimed that many chaotic and hyperchaotic systems can be used for secure communication and image encryption. On the other hand, some nonlinear circuits can be modified and improved as neural circuits, which similar firing patterns such as quiescent, spiking, bursting and chaotic modes can be reproduced as those series from biological neurons. An isolated nonlinear circuit has finite power release and it just describes the local kinetics of multi-agent and networks, therefore, the energy pumping along the coupling channel becomes a challenge by regulating the physical properties via external physical field. That is, it is important to clarify some of the physical principles and mechanism (Ma *et al.* 2019) before imposing any theoretical schemes for synchronization and control of chaos.

**Existence of Solutions means controllability in the dynamical systems.** In fact, before starting any control scheme, it is critical to confirm that reliable solutions can be obtained in theoretical or numerical way. For example, the numerical solution becomes divergent and overflow when the code for the dynamical system is run. For most of the chaotic system, infinite periodic orbits are combined and connected in enough transient period no matter whether equilibrium points exist or not.

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**Intermittent and discontinuous control benefit from intrinsic self-adaption of the chaotic systems.** For synchronization control of chaotic systems, the orbits become self-leading to reach the target orbits within certain transient period and thus external control and energy consumption become non-necessary. For chaotic systems, any external control from additive branch circuit will cost certain energy when the controller is activated. For synchronization stabilization, continuous coupling will consume energy in the coupling channel when resistor is used to connect the chaotic circuits because of cost in Joule heat in the coupling resistor. Therefore, discontinuous or intermittent coupling decreases certain energy costs in the coupling channels and controllers, and period for switch on-off for the controllers becomes worthy of investigation.

**Intrinsic response time delay and propagation time delay depend on the local property and coupling channels.** Some electric components can be activated only when the driving voltage or channel current are beyond the threshold, for example photocurrent can be generate in the phototube only when the frequency in the illumination is beyond the threshold even the illumination intensity is much large for the phototube. For an engine or motor, it needs certain transient period to reach a high speed and rotation rate. For some interneurons, autapse develops a close loop to connect the synapse and the body or muscle, and intrinsic time delay is considered and estimated by using the autaptic current.

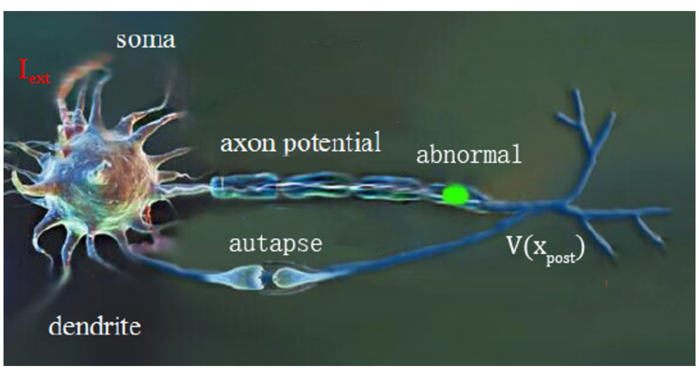


**Figure 1** Schematic diagram for autapse to neuron

In generic way, time delay $\tau$ and feedback gain $k$ are involved in the oscillator model to describe the response time delay as follows,

$$\frac{dx}{dt} = f\left(x, \mu\right) \pm kx\left(t - \tau\right) \tag{1}$$

The autaptic current or driving can be inhibitory or excitatory, and the firing modes of the nonlinear oscillator can be controlled effectively. For collective behaviors in spatiotemporal systems, distributed time delay can be introduced to estimate the effect of different links and coupling channels, it is obtained by,

$$\frac{dx_i}{dt} = f\left(x, \mu\right) + D\sum_{j=1, i\neq j}^{N} \varepsilon_{ij} x\left(t - \tau_j\right) \tag{2}$$

Where the connection matrix $\varepsilon_{ij} = 1$ indicates complete connection between the node $i$ and node $j$, otherwise, it is set as 0, the gain $D$ measure the connection intensity between two nodes, the time delay $\tau_j$ estimates the propagation period along the connection channel or link.

Except the electromagnetic field, signal propagation along the coupling channels and links in the network often require finite period, and thus time delay becomes important. In a non-uniform network, distributed time delay should be applied for different links and coupling channels (Wang and Ma 2018).

**Controllability and standard criterion for controllers are useful in practice.** From mathematical viewpoint, a variety of controllers and control schemes can be applied for all the dynamical systems and networks. In fact, the controller can be in simple form, lower energy consumption, finite and shorter transient period before reaching target orbits. In particular, fewer controllers are appreciated, for example, local pinning control is more effective than global control because control all the nodes becomes much difficult.

**Dimensional homogeneity and scale transformation are critical for dynamical analysis and control.** For some realistic systems, dynamical equations can be obtained to estimate the correlation between different physical variables. For nonlinear circuit, Kirchhoff's law is often used to obtain the circuit equations composed of physical variables (voltage, current, magnetic flux, charges) with different physical units. Therefore, standard scale transformation (Wang *et al.* 2019) should be applied for the physical parameters, variables, field energy, and thus dimensionless dynamical system and Hamilton energy can be obtained for finding numerical solutions via reliable algorithm. For example, the sampled time units from nonlinear oscillator or dimensionless dynamical systems should be discerned by using time units than seconds or milliseconds. For example, the common known physical variables (voltage $V$, current $i$, magnetic flux $\varphi$, charge $q$, time $t$) can be mapped into dimensionless variables (Wang *et al.* 2019) as follows,

$$x = \frac{V}{V_0};\ y = \frac{iR}{V_0}[or, y = \frac{i}{I_0}]; \tag{3}$$
$$w = \frac{\varphi}{LI_0} = \frac{\varphi R}{LV_0}, z = \frac{q}{CV_0};\ \tau = \frac{t}{\sqrt{LC}} = \frac{t}{RC}$$

Where $L$, $C$, $R$ represents the inductance, capacitance, resistance of electric components of the nonlinear circuit, and $V_0$, $I_0$ denote the scale value, for example, $V_0$, $I_0$ can be selected from the amplitudes from external realistic signal or intrinsic value of nonlinear electric components.

**Hamilton energy function meets the most suitable Lyapunov function.** For any dynamical systems, continuous energy pumping and exchange are critical to keep and change the dynamics and firing modes. The Helmholtz theorem provides helpful guidance to estimate the sole Hamilton energy function and then guides how the dynamical system can be controlled in reliable scheme. Lyapunon function scheme seems to confirm the controllability of any chaotic systems and networks, however, the most suitable Lyapunov function (Zhou *et al.* 2021) must be the Hamilton energy function and any arbitrary setting for gains in the Lyapunov function and the controllers just regulate a damaged system than the original dynamical system. For generic dynamical system, the Hamilton energy function H can be obtained from the following criterion,

$$\frac{dX}{dt} = F_c(X) + F_d(X);\ \nabla H^T F_c(X) = 0; \tag{4}$$
$$\frac{dH}{dt} = \nabla H^T F_d(X)$$

Where the vector $X$ describes the variable of the system, $F_c(X)$ represents the conservative field containing the full rotation and $F_d(X)$ is the dissipative field containing the divergence. Indeed, the Hamilton energy function is composed of all system variables and some intrinsic parameters, and effective control of the energy flow will control the stability and firing modes completely (Ma *et al.* 2017).

**Memristive and boundary effect, fractional order calculation are relative to the intrinsic property of the dynamical systems.** The nonlinear circuits involved with any memristor can be mapped into memristive system and becomes dependent on the initial value for memristive variable such as magnetic flux, and thus the dynamics is switched between different modes when any slight changes occur for initial values. For spatiotemporal systems with finite size, no-flux boundary condition is applied while periodic boundary condition is more suitable for networks with infinite size or system with globular, ring types. Local memory, boundary value and non-uniform diffusive effect require fractional calculation and approach (Zhou *et al.* 2020). Indeed, more electric components such as phototube, Josephson junction, piezoelectric ceramic can be incorporated into the nonlinear circuits for enhancing more specific biophysical functions in neural circuits.

## LITERATURE CITED


Ma, J., F. Wu, W. Jin, P. Zhou, and T. Hayat, 2017 Calculation of hamilton energy and control of dynamical systems with different types of attractors. Chaos: An Interdisciplinary Journal of Nonlinear Science **27**: 053108.

Ma, J., Z.-q. Yang, L.-j. Yang, and J. Tang, 2019 A physical view of computational neurodynamics. Journal of Zhejiang University-Science A **20**: 639–659.

Wang, C. and J. Ma, 2018 A review and guidance for pattern selection in spatiotemporal system. International Journal of Modern Physics B **32**: 1830003.

Wang, C., J. Tang, and J. Ma, 2019 Minireview on signal exchange between nonlinear circuits and neurons via field coupling. The European Physical Journal Special Topics **228**: 1907–1924.

Zhou, P., X. Hu, Z. Zhu, and J. Ma, 2021 What is the most suitable lyapunov function? Chaos, Solitons & Fractals **150**: 111154.

Zhou, P., J. Ma, and J. Tang, 2020 Clarify the physical process for fractional dynamical systems. Nonlinear Dynamics **100**: 2353–2364.



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