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Research Article

Reliability based modeling of the performance of solar plants with multistate PV modules

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Abstract

Solar energy is widely used as a renewable energy source in the world. Photovoltaic modules are the main components of a photovoltaic system to generate the solar power from the solar radiation. The photovoltaic modules may have multistate working conditions and different performance levels depending on the solar radiation. Each component can be in different states, namely, complete failure, partial working, and perfect functioning. In this study, we present a model for solar power systems with PV modules having various levels of operational performance. We develop a reliability model for the system's power regarding the m threshold value that is the minimum required total performance level for the system. This model reflects the performance levels of PV modules and working probabilities of modules. The problem is considered under different conditions regarding the dependency of two types of multistate PV modules. Two numerical examples are also conducted to evaluate the reliability and power generated by two solar plants located in two different regions. Beta and Weibull distributions are used for the numerical calculations to differ solar radiation regimes in the regions.

Mathematics Subject Classification (2020). 90B25, 62N05

Keywords. Photovoltaic (PV) system, solar power, multistate system, reliability

1. Introduction

Solar energy has recently become one of the remarkably rising sources of renewable energy. Photovoltaic (PV) system transforms the solar energy into electricity directly with no or lower air pollutants. The solar cells of a PV module contain materials that absorb light particles, emit electrons, and produce an electrical current. The performance of a PV system depends on the solar radiation that is a random variable. Due to its stochastic nature, researchers need to use the probabilistic and the statistical techniques to investigate some characteristics of the system. One of the characteristics of particular focus is the reliability of the system. The most of the studies in the literature are concerned with the reliability assessment methods for a PV module as an electronic component of

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a PV system such as, failure mode analysis, life testing, degradation analysis. For some results and references (see [2, 4, 13, 17, 25, 26]. Some other studies in the literature are focused on the evaluation methods of reliability (e.g. Markov process, reliability block diagram and fault tree analysis) and reliability indices for a PV system (e.g. loss of load expectation, loss of energy expectation, forced outage rate), see for example [1, 28]. The review studies [23] and [22] can be given for the further information on PV power and the performance analysis for long term reliability.

The PV system generates power if PV modules work properly and the solar radiation exists. Assuming that PV modules have different states (working, partially working and not working) depending on the solar radiation, the contribution of modules to the power generation of a system will also change. In this case, a PV module has multistate working conditions in terms of its different performance levels. Here, the term *performance* represents the power generated by PV modules. The reliability of a solar system consisting of multistate PV modules can be defined as the probability of the system performing satisfactorily a required level of power generation under specified conditions. Thus, the reliability for performance of the PV system should be evaluated by taking into account both states of PV modules and the solar radiation random variable. Considering each PV module has a weight according to its state and the system has a minimum required power level, the reliability for the performance of the PV system can be analyzed by weighted k-out-of-n system modeling. Wu and Chen [27] proposed an algorithm for weighted k-out-of-n system and defined the system for n components such that each component has its own weight. They assumed that the system works if and only if the total weight of all working components is at least a given threshold k. For more studies concerned with the weighted k-out-of-n model, one can see, [5, 7, 9, 15, 21, 29].

For the concept of power modeling, there are few studies related to the energy systems with components having various levels of performance. Eryilmaz an Kan [8] defined a reliability model for a particular kind of a multistate system and used the model to evaluate the power of wind energy system. Also, the number of wind turbines to be built in the wind plants is optimized by minimizing the total cost under the desired total power. The only study involving the reliability performance in the PV system context is by [6]. They consider a hybrid system that consists of a specified number of wind turbines and solar modules while assuming that the weights of wind turbines and solar modules corresponds to the mean power produced by the turbine and by the module depending on the energy source conditions, respectively. They evaluated the performance of a hybrid energy system using weighted k-out-of-n system reliability equations derived by [11]. They compute the amount of power generation of a solar module using a linear PV power output model. Eryilmaz and Kan [10] modeled a wind power system including of two wind farms using weighted k-out-of-n system when the wind speeds of two farms are statistically dependent. They used the two methods to model the dependency between wind speeds at two sites, namely, multivariate normal approximation and copulas based methods. Larsen et al. [14] defined a new multi-performance weighted multi-state K^{-}-out-of-n system and used this model for a power system that produces both electricity and heat. Eryilmaz and Ucum [12] studied the lost capacity by the weighted k-out-of-n system upon failure and applied the model to a power system that consists of a specified number of generating units.

Most of the studies in the literature are related to the modeling of the power generated by the PV system and how to evaluate the reliability indices for the system. To the best of our knowledge, there is no paper that considers the solar energy system with dependent components in a multi-state model. In this study, multistate PV modules in a solar system and power output model consisting of both linear and quadratic relationship between the solar radiation and the power are combined to evaluate the reliability for the system performance. In such a system, performance levels of PV modules at their states occur in a stochastic setting while the components are assumed different dependency conditions.

It consists of two types of PV modules which are categorized with respect to their state probabilities and weights for performances. We obtain the reliability model for the system which are defined as the probability that the total power of the system is at least a given level, say m, under three cases related to the dependency of the components. We use the PV power output model given by [19] for the definition of the states of PV modules in our model. We give examples under two different distribution functions for solar radiation. In the next section, we give the definitions and preliminaries for the power output model and the proposed reliability models. In Section 3, we obtain reliability models for the system considering the state probabilities of components under three different cases. In Section 4, we give two numerical examples of the theoretical findings assuming that the components are independent. We used beta and Weibull distributions for solar radiation to evaluate the reliability and power generated by two solar plants located in two regions. In Section 5, we give the conclusions.

Notation

The following notations will be used throughout the paper:
G_{bi}: The global solar radiation random variable (W/m^2).
G_{std}: The solar radiation in a standard environment (W/m^2).
R_c: The certain irradition point (W/m^2).
P_{sn}: The rated capacity of a PV module (W).
P_{PV}: The power generated by a PV module.
$F_{G_{bi}}$: The cumulative distribution function (cdf) of G_{bi}.
$f_{G_{bi}}$: The probability density function (pdf) of G_{bi}.
n_1: The number of components of Type I.
n_2: The number of components of Type II.
t_j: The performance of components of Type I when they are in state j, $j = 1, 2, 3, 4$.
t_j^*: The performance of components of Type II when they are in state j, $j = 1, 2, 3, 4$.
p_j: The probability that a component of Type I is in state j, $j = 1, 2, 3, 4$.
p_j^*: The probability that a component of Type II is in state j, $j = 1, 2, 3, 4$.
m: The minimum required total performance level for the system.
T_1: The total performance of components of Type I.
T_2: The total performance of components of Type II.

2. Definitions and preliminaries

Marwali et al. [16] developed a model representing a relationship between the power output and the solar radiation. Park et al. [19] used this model for the reliability evaluation indices of solar cell generators and Equation (2.1) represents the relationship between the power output from a PV module and solar radiation. This model will be used throughout the paper.

$$P_{PV} = h(G_{bi}) = \begin{cases} P_{sn}\dfrac{G_{bi}^2}{G_{std}\times R_c}, & 0 \le G_{bi} < R_c, \\[2ex] P_{sn}\dfrac{G_{bi}}{G_{std}}, & R_c \le G_{bi} < G_{std}, \\[2ex] P_{sn}, & G_{bi} \ge G_{std}, \end{cases} \tag{2.1}$$

where P_{PV} is the power generated by a PV module, G_{bi} is the global solar radiation random variable, P_{sn} is the rated capacity of a PV module, G_{std} is the solar radiation in a standard environment and R_c is the certain irradition point.

In this model, the PV module generates power when the solar radiation, G_{bi}, is between zero and certain irradiation point, R_c, that is, a quadratic relationship between the solar radiation and the power is observed. Also, the module generates power when the solar radiation is between certain irradiation point and the solar radiation in a standard environment, G_{std}, then a linear relationship between the solar radiation and the power is observed.

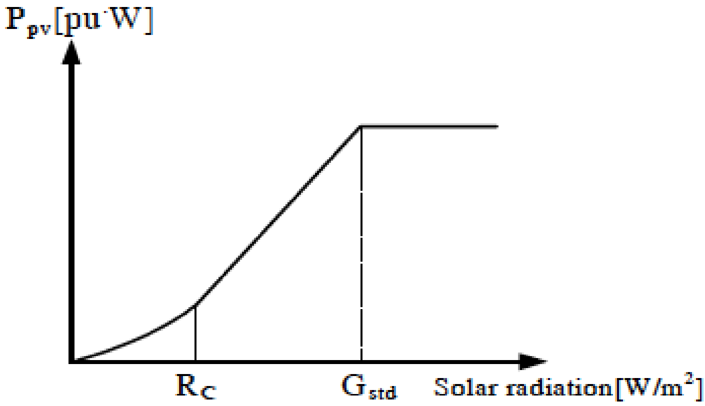

Figure 1. Power output of PV module by [19],

As it is seen in Figure 1, if the solar radiation is greater than the solar radiation in a standard environment, the module generates power at a constant rate P_{sn}.

Consider a PV system which consists of $n = n_1 + n_2$ multistate components (PV modules) classified into two categories on the basis of their performance levels and state probabilities, where n_1 and n_2 represent the number of components of Type I and Type II, respectively. According to the model in Equation (2.1), we can define the states for each PV module as the following: complete failure (state 1), partial working (state 2, state 3), and perfect functioning (state 4) which are given in the followings:
State 1 : The PV module generates no power, i.e. there is no solar radiation or the PV module is broken down.
State 2 : The PV module generates power at a rate depending on the solar radiation in the interval $[0, R_c)$.
State 3: The PV module generates power at a rate depending on the solar radiation in the interval $[R_c, G_{std})$.
State 4: The PV module generates power at the rated capacity.

Note that, the model assumes that all PV modules in a system are subject of the same solar radiation and have the same power curve. However, PV modules consisting of Type I and Type II have different performances as weights t_j and t_j^* according to its state, respectively. In our PV system, the total performance as a weight of components of Type I can be defined by the random variable

$$T_1 = \sum_{i=1}^{n_1} \sum_{j=1}^{4} t_j I(X_i = j), \tag{2.2}$$

where X_i represents the state of the ith component and $I(E) = 1$, if E occurs, and $I(E) = 0$, if else. In a similar way, the total performance of the components for Type II is written by

$$T_2 = \sum_{i=1}^{n_2} \sum_{j=1}^{4} t_j^* I(X_i^* = j), \tag{2.3}$$

where X_i^* represents the state of the ith component for Type II. Note that, the power is not generated when the components are in complete failure state, $t_1 = t_1^* = 0$.

In this study, our objective is to obtain the reliability model as the probability that the PV system's performance is at least m threshold value (the minimum required performance level for the system) i.e. $P\{T_1 + T_2 \geq m\}$, using weighted k-out-of-n system. In real life problems, when calculating system reliability, it is necessary to consider different situations depending on how components affect each other. When the two groups of components are planted in two regions with various random factors, dependency may exist. In some situations, in the same plant, the performance of a module can be affected by another module due to the random environmental effects. Hence, the reliability models are obtained under the following three cases that could occur statistically for the two groups of components:

- Case A: All components are considered as dependent.
- Case B: There are dependent components of the same type, but different types are independent.
- Case C: All components are considered as independent.

3. Reliability models

In this section, we present the reliability models for multistate PV system under the considerations of the defined cases above. As mentioned in previous section, when PV modules work properly and the solar radiation exists, PV system produces power. Also, in our model, PV modules work in different states depending on solar radiation and this property affects the performance of the system. Therefore, we must pay attention to the probabilities of being in different states, when we evaluate the reliability of the system.

Now, assume a PV module is working correctly with probability p and it is down for a fixed period of time with probability $1 - p$. Then the probabilities of being in states 1, 2, 3, and 4 for PV modules depending on the solar radiation can be computed by the followings, respectively.

$$\begin{aligned} p_1 &= 1 - p \\ p_2 &= pP\{0 \leq G_{bi} < R_c\} = p[F_{G_{bi}}(R_c)] \\ p_3 &= pP\{R_c \leq G_{bi} < G_{std}\} = p[F_{G_{bi}}(G_{std}) - F_{G_{bi}}(R_c)] \\ p_4 &= pP\{G_{bi} \geq G_{std}\} = p[1 - F_{G_{bi}}(G_{std})] \end{aligned} \tag{3.1}$$

Then, for the case A, the total performances of Type I and Type II components, T_1 and T_2 can be written as

$$T_1 = \sum_{j=1}^{4} t_j S_j^{(1)} \text{ and } T_2 = \sum_{j=1}^{4} t_j^* S_j^{(2)}, \tag{3.2}$$

where $S_j^{(1)} = \sum_{i=1}^{n_1} I(X_i = j)$ and $S_j^{(2)} = \sum_{i=1}^{n_2} I(X_i^* = j)$, $j = 1, 2, 3, 4$. The joint probability mass function $P^A_{n_1,n_2}(c_2, c_3, c_4, d_2, d_3, d_4)$ of the random variables $S_j^{(1)}$ and $S_j^{(2)}$ can be written as follows:

$$\begin{aligned} &P\{S_2^{(1)} = c_2, S_3^{(1)} = c_3, S_4^{(1)} = c_4, S_2^{(2)} = d_2, S_3^{(2)} = d_3, S_4^{(2)} = d_4\} \\ &\quad = \binom{n_1}{c_2}\binom{n_1 - c_2}{c_3}\binom{n_1 - c_2 - c_3}{c_4}\binom{n_2}{d_2}\binom{n_2 - d_2}{d_3}\binom{n_2 - d_2 - d_3}{d_4} \\ &\qquad P\{X_1 = 2, \ldots, X_{c_2} = 2, X_{c_2+1} = 3, \ldots, X_{c_2+c_3} = 3, \\ &\qquad X_{c_2+c_3+1} = 4, \ldots, X_{c_2+c_3+c_4} = 4, X_{c_2+c_3+c_4+1} = 1, \ldots, X_{n_1} = 1, \\ &\qquad X_1^* = 2, \ldots, X_{d_2}^* = 2, X_{d_2+1}^* = 3, \ldots, X_{d_2+d_3}^* = 3, X_{d_2+d_3+1}^* = 4, \ldots, \\ &\qquad X_{d_2+d_3+d_4}^* = 4, X_{d_2+d_3+d_4+1}^* = 1, \ldots, X_{n_2}^* = 1\} \end{aligned} \tag{3.3}$$

for $0 \le c_2 + c_3 + c_4 \le n_1$ and $0 \le d_2 + d_3 + d_4 \le n_2$. Note that, since $S_2^{(1)} = c_2, S_3^{(1)} = c_3, S_4^{(1)} = c_4, S_2^{(2)} = d_2, S_3^{(2)} = d_3, S_4^{(2)} = d_4$ implies $S_1^{(1)} = n_1 - c_2 - c_3 - c_4$, $S_1^{(2)} = n_2 - d_2 - d_3 - d_4$ with probability 1, we do not need to write $S_1^{(1)}$ and $S_1^{(2)}$ in the left hand side of the Equation (3.3). However, $S_1^{(1)}$ and $S_1^{(2)}$ are considered in the right hand side of the equation. Thus, for the case of all components are dependent, the reliability of the PV system for a given level m is

$$\begin{aligned} R^A_{n_1,n_2}(m) &= P\{T_1 + T_2 \ge m\} \\ &= \sum_{c_2=0}^{n_1} \sum_{c_3=0}^{n_1-c_2} \sum_{c_4=0}^{n_1-c_2-c_3} \sum_{d_2=0}^{n_2} \sum_{d_3=0}^{n_2-d_2} \sum_{\substack{d_4=0 \\ t_2c_2+t_3c_3+t_4c_4+t_2^*d_2+t_3^*d_3+t_4^*d_4 \ge m}}^{n_2-d_2-d_3} P^A_{n_1,n_2}(c_2,c_3,c_4,d_2,d_3,d_4). \end{aligned} \tag{3.4}$$

For the case B, when different types of components are independent, the joint probability mass function can be defined as

$$\begin{aligned} P^B_{n_1,n_2}(c_2,c_3,c_4,d_2,d_3,d_4) = &\binom{n_1}{c_2}\binom{n_1-c_2}{c_3}\binom{n_1-c_2-c_3}{c_4}\binom{n_2}{d_2}\binom{n_2-d_2}{d_3}\binom{n_2-d_2-d_3}{d_4} \\ &P\{X_1 = 2,\dots,X_{c_2} = 2, X_{c_2+1} = 3,\dots,X_{c_2+c_3} = 3, \\ &X_{c_2+c_3+1} = 4,\dots,X_{c_2+c_3+c_4} = 4, X_{c_2+c_3+c_4+1} = 1,\dots,X_{n_1} = 1\} \\ &\times P\{X_1^* = 2,\dots,X_{d_2}^* = 2, X_{d_2+1}^* = 3,\dots,X_{d_2+d_3}^* = 3, \\ &X_{d_2+d_3+1}^* = 4,\dots,X_{d_2+d_3+d_4}^* = 4, X_{d_2+d_3+d_4+1}^* = 1,\dots,X_{n_2}^* = 1\}. \end{aligned} \tag{3.5}$$

The probability that the performance of the system is at least a given level m is

$$\begin{aligned} R^B_{n_1,n_2}(m) &= P\{T_1 + T_2 \ge m\} \\ &= \sum_{c_2=0}^{n_1} \sum_{c_3=0}^{n_1-c_2} \sum_{c_4=0}^{n_1-c_2-c_3} \sum_{d_2=0}^{n_2} \sum_{d_3=0}^{n_2-d_2} \sum_{\substack{d_4=0 \\ t_2c_2+t_3c_3+t_4c_4+t_2^*d_2+t_3^*d_3+t_4^*d_4 \ge m}}^{n_2-d_2-d_3} P^B_{n_1,n_2}(c_2,c_3,c_4,d_2,d_3,d_4). \end{aligned} \tag{3.6}$$

For the case C that is all components are independent, the joint probability mass function becomes

$$\begin{aligned} P^C_{n_1,n_2}(c_2,c_3,c_4,d_2,d_3,d_4) = &\binom{n_1}{c_2}\binom{n_1-c_2}{c_3}\binom{n_1-c_2-c_3}{c_4}\binom{n_2}{d_2}\binom{n_2-d_2}{d_3}\binom{n_2-d_2-d_3}{d_4} \\ &\times p_2^{c_2} p_3^{c_3} p_4^{c_4} (1 - p_2 - p_3 - p_4)^{n_1-c_2-c_3-c_4} \\ &\times (p_2^*)^{d_2} (p_3^*)^{d_3} (p_4^*)^{d_4} (1 - p_2^* - p_3^* - p_4^*)^{n_2-d_2-d_3-d_4}, \end{aligned} \tag{3.7}$$

where p_j and p_j^* represent the probability that a component of Type I and Type II is in state j, $j = 1, 2, 3, 4$, respectively. Thus, the reliability

$$\begin{aligned} R^C_{n_1,n_2}(m) &= P\{T_1 + T_2 \ge m\} \\ &= \sum_{c_2=0}^{n_1} \sum_{c_3=0}^{n_1-c_2} \sum_{c_4=0}^{n_1-c_2-c_3} \sum_{d_2=0}^{n_2} \sum_{d_3=0}^{n_2-d_2} \sum_{\substack{d_4=0 \\ t_2c_2+t_3c_3+t_4c_4+t_2^*d_2+t_3^*d_3+t_4^*d_4 \ge m}}^{n_2-d_2-d_3} P^C_{n_1,n_2}(c_2,c_3,c_4,d_2,d_3,d_4). \end{aligned} \tag{3.8}$$

In order to make the probabilities more clear, additional explanations are required for all cases given above. Recall that if a PV module operates right depend on solar radiation, then it generates power. When t_j denotes the PV module power produced in the states $j = 1, 2, 3, 4$, then it is obvious that $t_1 = 0$ and $t_4 = P_{sn}$. Indeed, t_2 and t_3 indicate the mean power generated by a PV module when the solar radiation is in the interval $[0, R_c)$ and $[R_c, G_{std})$, respectively. For the state 2, the power can be written as

$$t_2 = \int_0^{R_c} P_{sn} \frac{g_{bi}^2}{G_{std} \times R_c} k_1(g_{bi}) dg_{bi}, \tag{3.9}$$

where k_1 is the truncated pdf of the solar radiation on $(0, R_c)$ that is $k_1(g_{bi}) = \dfrac{f(g_{bi})}{F(R_c)}$ and $k_1(g_{bi}) = 0$, otherwise. And for the state 3, it can be written as

$$t_3 = \int_{R_c}^{G_{std}} P_{sn} \frac{g_{bi}}{G_{std}} k_2(g_{bi}) dg_{bi}, \tag{3.10}$$

where k_2 is the truncated pdf of the solar radiation on (R_c, G_{std}) that is $k_2(g_{bi}) = \dfrac{f(g_{bi})}{F(G_{std}) - F(R_c)}$ and $k_2(g_{bi}) = 0$, otherwise.

4. Numerical examples

In this section, we give two numerical examples for illustrating the theoretical results in the case C, i.e., all components are statistically independent, under two different probability distributions of solar radiation. The probability that the power generated by two solar plants located in two different locations exceeds a given level m can be evaluated under the PV power model given in Equation (2.1). Note that, all computations have been made using Mathematica v.11.3.

4.1. Example 1

Beta distribution is widely used for the statistical modeling of the solar radiation data (see, [3, 18, 20, 24]). The pdf and the cdf for the beta distribution are given respectively by

$$f(s; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} s^{\alpha-1}(1 - s)^{\beta-1}$$

and

$$F(s; \alpha, \beta) = \frac{B(s; \alpha, \beta)}{B(\alpha, \beta)} = I_s(\alpha, \beta),$$

where $0 \leq s \leq 1$ is the random solar radiation (kW/m^2) and $\alpha, \beta \geq 0$ are the shape parameters.

In this numerical example, first we calculated the states probabilities are defined in Equation (3.1), then we computed the performances of PV modules for each state using Equations (3.9) and (3.10). Thus, we obtained the reliability of the system defined as the probability that the PV system provides power for at least the desired level of production. Our assumptions are as follows: the rated capacities of two solar plants located in two different locations consisting of Type I and Type II PV modules are $P_{sn} = 220, 270\ W$, respectively. Also, the solar radiation in a standard environment is $G_{std} = 900\ W/m^2$, and the certain irradiation point is $R_c = 150\ W/m^2$ for both types of PV modules. Suppose that all PV modules work properly with probability $p = 0.95$ for a set amount of time. Assume that the solar radiation is distributed as $Beta(1.3, 2.5)$ and $Beta(1.5, 2.3)$ for one year period in the regions of plants 1 and 2, respectively. Since PV modules in the plants were assumed to be independent, we have used Equation (3.8) for all computations. Under these assumptions, the corresponding state probabilities for two types of PV modules using Equations (3.1) are given in Table 1. Clearly, $p_1 = p_1^* = 0.05$ for both types of PV module, as the working probability of all PV modules are 0.95 and $p_1 = 1 - p$ represents the probability of the complete failure state. Also, $p_3 = 0.7315$ and $p_3^* = 0.7931$ represent that the probability of the PV module generates power at a rate depending on the solar radiation in the interval $[R_c, G_{std})$ for the Type I and II of PV modules in plants 1 and 2, respectively. From Table 1, it is said that the probability of being in state 2 is lower than the probability of being in state 3 for both types of PV modules depending on the solar radiation in two regions.

Table 1. The corresponding probabilities of state j for PV modules, $j = 1, 2, 3, 4$ when $p = 0.95$ under beta distribution.

		$j = 1$	$j = 2$	$j = 3$	$j = 4$
Type I	p_j	0.0500	0.2139	0.7315	0.0046
Type II	p_j^*	0.0500	0.1478	0.7931	0.0091

Table 2 presents the obtained performances of PV modules using Equations (3.9) and (3.10) for two types of PV modules. For example, $t_3^* = 0.13375$ (kW) indicates the mean power generated by a Type II of PV module in the plant 2 in any day of the year when the module is in state 3. It is obvious that, $t_1 = t_1^* = 0$, because of no solar radiation or the failure of the PV module.

Table 2. Estimated performances of PV modules (kW) for each state $j = 1, 2, 3, 4$ when $p = 0.95$ under beta distribution.

		$j = 1$	$j = 2$	$j = 3$	$j = 4$
Type I	t_j	0	0.01373	0.10131	0.220
Type II	t_j^*	0	0.01855	0.13375	0.270

Finally, we give the reliability for corresponding m (kW) values, i.e. $R_{n_1,n_2}(m) = P\{T_1 + T_2 \geq m\}$, which are calculated using Equation (3.8) with different working probabilities of PV modules, p, in Figure 2. We assume that plant 1 consists of $n_1 = 5$ Type I and plant 2 consists of $n_2 = 5$ Type II of PV modules. As expected, the reliability values are decreasing while the minimum desired total performance level for the system, m, is increasing. Also, we can say that if a PV module has higher working probability value, then we can provide more power for the same level of reliability. If much more level of power is required, the number of PV modules can be increased or different types of modules with higher rated capacity can be installed.

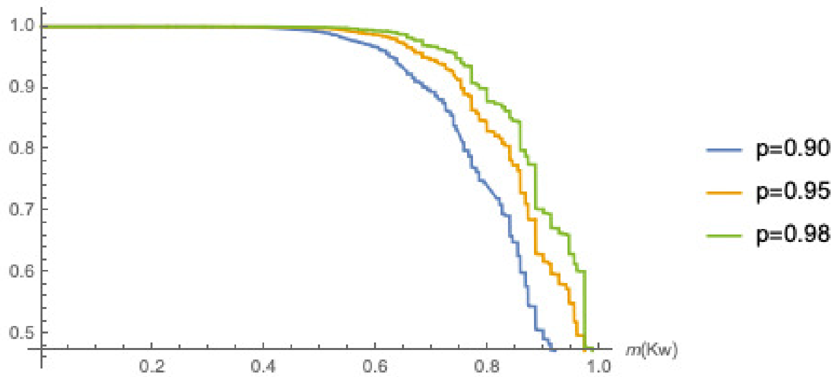

Figure 2. Reliability for the performance of PV system under beta distribution.

4.2. Example 2

Weibull distribution is considered by [20] for solar radiation data. Thus, the results of this paper are also relevant to a region where probability distribution of solar radiation is different from beta. The pdf and the cdf for Weibull distribution are given respectively by

$$f(s; \alpha, \beta) = \frac{\alpha}{\beta}\left(\frac{s}{\beta}\right)^{\alpha-1} \mathrm{e}^{-(s/\beta)^{\alpha}}$$

and

$$F(s;\alpha,\beta) = 1 - \mathrm{e}^{-(s/\beta)^{\alpha}},$$

where $s \geq 0$ is the random solar radiation (kW/m^2) and $\alpha, \beta \geq 0$ are the shape and scale parameters, respectively.

Our assumptions for this example are as follows: the rated capacities of the Type I and Type II PV modules are $P_{sn} = 25, 50\ W$, respectively. Also, the solar radiation in a standard environment is $G_{std} = 1000\ W/m^2$, and the certain irradiation point is $R_c = 150\ W/m^2$ for both types of PV modules. Suppose that all PV modules work properly with probability $p = 0.95$ for a set amount of time and the plant 1 consists of $n_1 = 3$ PV modules and plant 2 consists of $n_2 = 3$ PV modules. In addition, we assume that the solar radiation distributions in the regions of plants 1 and 2 are $WE(4.2, 1.3)$ and $WE(4.5, 0.8)$ for one year period, respectively. Under these assumptions, the corresponding state probabilities for two types of PV modules using Equations (3.1) are given in Table 3. Similar as the previous example, since the working probability of all PV modules are 0.95, we have $p_1 = p_1^* = 0.05$ for both types of PV modules. It is seen from Table 3 that Type I of PV modules have high probability of staying in state 4, while Type II of PV modules have high probability of staying in state 3. It means that Type I of PV modules in plant 1 generates much more power at the rated capacity, while Type II of PV modules in plant 2 generates much more power at a rate depending on the solar radiation in the interval $[R_c, G_{std})$.

Table 3. The corresponding probabilities of state j for PV modules, $j = 1, 2, 3, 4$, when $p = 0.95$ under Weibull distribution.

		$j=1$	$j=2$	$j=3$	$j=4$
Type I	p_j	0.0500	0.0001	0.2684	0.6815
Type II	p_j^*	0.0500	0.0005	0.8875	0.0620

In Table 4, the performances of each type of PV module are given using Equations (3.9) and (3.10). For example, $t_2^* = 0.0052$ (kW) indicates the mean power generated by the Type II of PV module in the plant 2 in any day of the year when the module is in state 2.

Table 4. Estimated performances of PV modules (kW) for each state $j = 1, 2, 3, 4$ when p=0.95 under Weibull distribution.

		$j=1$	$j=2$	$j=3$	$j=4$
Type I	t_j	0	0.0025	0.0198	0.025
Type II	t_j^*	0	0.0052	0.0353	0.050

Figure 3 represents $R_{n_1,n_2}(m)$ values calculated by Equation (3.8) for corresponding m (kW) with different p. We assume that plant 1 consists of $n_1 = 5$ Type I and plant 2 consists of $n_2 = 5$ Type II of PV modules. It can be understand from the figure that the reliability of a system is around 0.84 where the minimum required level of performance, m, is 0.28 (kW) when the working probability of a PV module, p, is 0.98. Hence, the reliability value is lower for the same performance level when the p is 0.90. As expected, the reliability values are decreasing while the minimum desired total performance level for the system, m, is increasing.

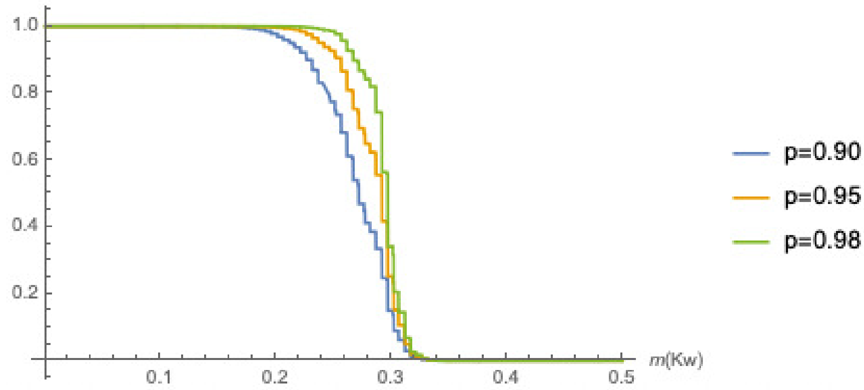

Figure 3. Reliability for the performance of PV system under Weibull distribution.

5. Conclusions

By the results of this study, important decisions can be made in terms of both the number of solar panels and the provision of energy. Considering the statistical distribution of solar radiation, evaluating the performance of the solar plant supports sustainable and energy-efficient timing strategies. Our study offers a different perspective on the models including the multistate PV modules and statistical dependency concept to obtain the reliability of the system. In this study, we investigate the reliability models constructed for the defined three cases using the state probabilities of two types of multistate PV modules. Numerical examples for illustrating the theoretical results are presented assuming that the solar radiation distributions are beta and Weibull. It has been seen from the numerical studies that the reliability value of the system varies depending on the solar radiation distribution, the working probability and the type and number of PV modules. The defined reliability models can be reconstructed for more than two types of PV modules. As a future work, the modeling of the power for PV systems in the dependency concept can be considered in more detail.

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