Hacettepe Journal of Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 51 (6) (2022), 1697 – 1709
DOI : 10.15672/hujms.957653

Research Article

The contagion dynamics of vaccine skepticism

Meltem Gölgeli

TOBB University of Economics and Technology, Department of Mathematics, Ankara, Turkey

Abstract

In this manuscript, we discuss the spread of vaccine refusal through a non-linear mathematical model involving the interaction of vaccine believers, vaccine deniers, and the media sources. Furthermore, we hypothesize that the media coverage of disease-related deaths has the potential to increase the number of people who believe in vaccines. We analyze the dynamics of the mathematical model, determine the equilibria and investigate their stability. Our theoretical approach is dedicated to emphasizing the importance of convincing people to believe in the vaccine without getting into any medical arguments. For this purpose, we present numerical simulations that support the obtained analytical results for different scenarios.

Mathematics Subject Classification (2020). 34D20, 92D30, 37N25

Keywords. vaccine refusal, mathematical model, stability

Vaccination is probably the most effective method of prevention against the contagious infectious diseases. However, people do not suddenly accept novel vaccines, and scientists are expected to persuade people to get vaccinated, even during a pandemic. The term vaccine hesitancy refers to people's attitudes and beliefs about vaccination, and it is often defined as a delay in accepting or refusing the vaccine [16, 22]. The World Health Organization (WHO) draws attention to the vaccine hesitancy in the list of ten threats to global health in 2019 [20, 43].

Historical evidence indicates that the success of immunization for a contagious infectious disease depends on high acceptance and coverage rates of a vaccination program [32]. For example, measles requires at least a 90% to 95% vaccine coverage for a sufficient immunization of the entire society [20]. Although it is a vaccine-preventable disease, WHO reported that more than $140,000$ people worldwide died from measles in 2018. The growing movement of vaccine hesitancy in recent years has resulted in lower vaccination rates in Europe and in the US. Scientists underline that the increase of measles outbreaks is linked to the decrease of the vaccination rate [13].

There are many factors that keep people from getting vaccinated. Religious objections, certain ideas about natural living, and exaggerated concerns about vaccine safety are driving people to reject the vaccine [21, 37]. The vaccine hesitancy is associated with media-based anti-vaccine messages and distrust in the healthcare system [5, 20, 30]. In addition, the lack of desired education of healthcare providers has also been reported as a reason for vaccine refusal [21].

Email address: mgolgeli@etu.edu.tr
Received: 25.06.2021; Accepted: 23.06.2022

During the pandemic, although people have a strong perception of the high risks that may occur after COVID-19 infection, there is still an increase in hesitancy to receive any vaccine [42]. Investigating the impact of vaccine rejection on potential epidemics and estimating the public health cost of small changes in vaccination rates are more interesting from a mathematical point of view than the social causes of vaccine hesitancy. Indeed, mathematical models have provided several methods for understanding the non-linear transmission dynamics of an infectious disease under the effect of vaccination. These models are widely used by decision makers to evaluate possible scenarios for local or global vaccination program [25].

Mathematical modelling and analysis of the transmission of infectious diseases have nearly a century of history in epidemiology [8, 18]. So far, compartment-based models known as SIR models have been widely studied to understand the transmission of various infectious diseases [14, 15, 19, 23, 40, 41]. The preventive effect of vaccination on disease transmission has been examined through an extension of the SIR modeling approach [2, 9, 29, 38, 45]. Further, several game-theoretical models in epidemiology have been proposed in order to understand the effect of vaccination on the disease dynamics[3, 4, 6, 35, 36]. In social and human sciences, mathematical models in epidemiology have been employed to determine the dynamics of changes in society's beliefs and voters' decision-making mechanisms in elections [27, 31]. In addition, studies focusing on the effects of awareness programs on the spread of infectious diseases are also encountered in the literature [12, 26, 33, 34, 44].

In this manuscript, we discuss the spread of vaccine refusal via a compartmental mathematical model. Our approach stands on the research of A. K. Misra [27] that described the spread of two political ideologies with an SIR-type model. We combine this idea with the researches on the effect of media in the spread of an infectious diseases [12, 26, 34, 44]. While the current study has similarities with the research of A.K. Misra [27] in that it uses the idea of modeling the spread of two opposing political views (i.e. vaccine believers and vaccine deniers instead of two political parties), but it has differences from the aforementioned study in that the model set-up includes non-linear media effect and news about disease related deaths associated with increased confidence in the vaccine. We prefer to use the terminology of vaccine skeptics and vaccine believers as in [35] and vaccine deniers as in [11]. Thus, we consider three discrete compartments in a homogeneously distributed population: vaccine skeptics S, vaccine believers B and vaccine deniers D. We assume that only the adult individuals (at least 18 years old) find a place in the compartment of S and people can enter into S when they become 18 years old. The positive role of media in the awareness of people and other environmental stimulus in the changing of refusal tendency of individuals are described by a separate compartment as an environmental effect E. The feature of compartment E is that, unlike other compartments, it includes media sources, not individuals. In this concept, we discuss how strong the media do influence the spread of vaccine refusal. We analyze the stability of the system around the equilibria and simulate our results for different scenarios.

1. Model

We consider a homogeneously distributed population of $N(t)$ which is the sum of three distinct sub-populations at time t: vaccine skeptics $S(t)$, vaccine believers $B(t)$ and vaccine deniers $D(t)$. We assume that only the adult individuals (at least 18 years old) find a place in the compartment S and individuals who have turned the age of 18 can enter the compartment with the rate Π. $S(t)$ denotes skeptic people who have not yet decided whether to get a vaccine. $B(t)$, $D(t)$ respectively represent the people who believe in the benefits of vaccine and who deny to take a vaccine because of a personal reason (distrust to health system, religion, beliefs etc.). We assume that people who ones believed in the benefit of vaccine cannot be directly a denier. Further, $E(t)$ refers to the total number of media sources which consist of advertisements, or news that can convince people to get a vaccine. Since messages about the personal health risks associated with the COVID-19 may encourage vaccine uptake [28], we consider $E(t)$ increases proportional to the number of the disease related deaths because of the vaccine refusals. $f(E)$ is the awareness function which we define by Holling Type II response $f(E(t)) = \frac{E(t)}{K+E(t)}$ where K is the capacity of the positive advertisements produced by social media. The Holling Type II functional response originally occurs in predator-prey systems and is characterized by a slowing rate of intake based on the assumption that the consumer's capacity to process food is limited. However, this idea is commonly applied to rumor-spreading models by treating the news as a food [17,33]. Moreover, we assume that the deniers may influence skeptics and infect them.

We draw a flow diagram figure 1 to summarize the above assumptions.

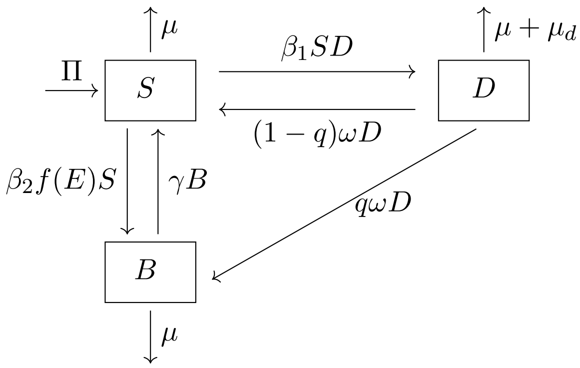

Figure 1. The flow diagram for the scenario of vaccine refusal

According to the flow diagram (see figure 1) we constitute a mathematical model via nonlinear ordinary differential equations (1.1) with initial conditions $S(0), B(0) > 0$, $D(0) \geq 0$ and $E(0) \geq E_0$.

$$\begin{aligned} \frac{dS(t)}{dt} &= \Pi - \beta_1 S(t)\,D(t) - \beta_2 S(t)\,\frac{E(t)}{K+E(t)} + \gamma B(t) + (1-q)\omega D(t) - \mu S(t) \\ \frac{dB(t)}{dt} &= \beta_2 S(t)\,\frac{E(t)}{K+E(t)} + q\omega D(t) - (\mu+\gamma)B(t) \\ \frac{dD(t)}{dt} &= \beta_1 S(t)\,D(t) - (\omega+\mu+\mu_d)D(t) \\ \frac{dE(t)}{dt} &= \alpha\mu_d D(t) - \delta(E(t)-E_0) \end{aligned} \tag{1.1}$$

Table 1. Parameters of the model (1.1)

Parameters	Description
Π	the rate of being adult
q	probability of gaining awareness
β_1	transmission rate for vaccine refusal
β_2	spread rate of awareness sources
ω	failure rate in refusal of deniers
γ	failure rate in awarenesses of believers
μ	natural death rate
μ_d	disease-related death rate of deniers
K	capacity of the positive advertisements produced by environmental sources
α	production rate of awareness sources
δ	decay rate of awareness sources
E_0	standard initial number of awareness sources

All parameters considered in the model (1.1) are positive and they are summarized in table 1. In order to achieve a significant model, we show the positivity of all solutions of the system (1.1) for all t.

Lemma 1.1. *The solutions $S(t)$, $B(t)$, $D(t)$, and $E(t)$ of the system* (1.1) *with initial conditions $S(0), B(0) > 0$, $D(0) \geq 0$, and $E(0) \geq E_0$ are positive for all $t > 0$.*

Proof. From the third equation of the system (1.1), we explicitly obtain

$$D(t) = D(0) exp\left(\int_0^t (\beta_1 S(s) - (\omega + \mu + \mu_d))ds\right)$$

which implies $D(t) \geq 0$ for all $t \geq 0$.

We have the solution of the last equation of the system (1.1)

$$E(t) = exp(-\delta t)\left(E(0) + \int_0^t exp(\delta s)\left(\alpha \mu_d D(s) + \gamma E_0\right) ds\right)$$

which is dependent on $D(t) \geq 0$ and $E_0 \geq 0$. Thus, we get the positivity of $E(t)$ for all $t \geq 0$. Let us check the positivity of the solutions $S(t)$ and $B(t)$ by using the positivity of $D(t)$ and $E(t)$. We have the solution of

$$B(t) = exp\left(-(\mu+\gamma)t\right)\left(B(0) + \int_0^t exp((\mu+\gamma)s)\left(\beta_2 S(s)\frac{E(s)}{K+E(s)} + q\omega D(s)\right) ds\right).$$

Then, we determine the integrating factor

$$\phi(t) = exp\left(\int_0^t \left(\beta_1 D(s) + \beta_2 \frac{E(s)}{K+E(s)} + \mu\right) ds\right)$$

to find the solution

$$S(t) = \frac{1}{\phi(t)}\left(S(0) + \int_0^t \left(\Pi + \gamma B(s) + (1-q)\omega D(s)\right)\phi(s)\, ds\right).$$

Suppose that $B(0) > 0$. By continuity of solutions, $B(t) > 0$ in some neighbourhood of $t = 0$. Then, we have $t_1 = sup\{s > 0 : B(t) > 0 \text{ on } [0, s)\}$. So, we need to show $t_1 = \infty$. For contradiction, we suppose that t_1 is finite. Then, $B(t) > 0$ on the interval $t \in [0, t_1)$ and $B(t_1) = 0$. In this case $S(t) > 0$ for $t \in [0, t_1]$ and $B(t_1) \geq exp\left(-(\mu+\gamma)t_1\right) B(0) > 0$. This contradiction proves that $t = \infty$. Hence, $B(t) > 0$ for all $t \geq 0$ whenever $B(0) > 0$. In similar way, we also have $S(t) > 0$ for all t. □

Lemma 1.2. *The positive solutions $S(t)$, $B(t)$, $D(t)$ and $E(t)$ of the system* (1.1) *with initial conditions $S(0), B(0) > 0$, $D(0) \geq 0$, and $E(0) \geq E_0$ are bounded for all $t > 0$.*

Proof. By taking into account the equality $N(t) = S(t) + B(t) + D(t)$, we reduce the (1.1) and obtain the simplified model

$$\begin{aligned}
\frac{dN(t)}{dt} &= \Pi - \mu N(t) - \mu_d D(t) \\
\frac{dB(t)}{dt} &= \beta_2 (N(t) - B(t) - D(t))\frac{E(t)}{K+E(t)} + q\omega D(t) - (\mu+\gamma)B(t) \\
\frac{dD(t)}{dt} &= \beta_1 (N(t) - B(t) - D(t))\, D(t) - (\omega + \mu + \mu_d)D(t) \\
\frac{dE(t)}{dt} &= \alpha \mu_d D(t) - \delta(E(t) - E_0).
\end{aligned} \tag{1.2}$$

Using the first equation of the model (1.2), we get $\frac{dN(t)}{dt} \leq \Pi - \mu N(t)$ and after integrating the both sides of the inequality in the interval $[0, t]$ we obtain $N(t) \leq N(0)\exp(-\mu t) + \frac{\Pi}{\mu}$. Thus we have $\lim_{t\to\infty} \sup N(t) \leq \frac{\Pi}{\mu}$, which means $N(t) \leq \frac{\Pi}{\mu}$ for a large $t > 0$. Since we know that $B(t) > 0$, we rewrite $S(t) = N(t) - D(t) \geq 0$. It implies the inequality $0 \leq D(t) \leq N(t) \leq \frac{\Pi}{\mu}$. Similarly, we have $0 \leq B(t) \leq N(t) \leq \frac{\Pi}{\mu}$. Using last equation of the system (1.2), we achieve $\lim_{t\to\infty} \sup E(t) \leq \frac{\Pi\alpha\, \mu_d}{\delta\, \mu} + E_0$.

□

Therefore, we define the positively invariant set of the system (1.2)

$$\Omega = \left\{ (N, B, D, E) \in \mathbb{R}_+^4 : 0 \leq B, D \leq N \leq \frac{\Pi}{\mu}, 0 \leq E \leq \frac{\Pi\alpha\, \mu_d}{\delta\, \mu} + E_0 \right\}$$

and investigate the dynamical processes on Ω below.

2. Equilibria and stability analysis

2.1. Denier-free and denier-existing equilibria

We first seek a denier-free equilibrium (DFE) of the model by setting the right hand sides of the differential equations (1.2) to zero and substitute $D = 0$ for the denier-free case. Thus, we define a denier-free equilibrium in the form $E_{df} = (\frac{\Pi}{\mu}, \frac{\beta_2 \Pi E_0}{\mu(\beta_2 E_0 + (K+E_0)(\mu+\gamma))}, 0, E_0)$.

Let us represent the denier-existing equilibrium by $E_{de} = (N^*, B^*, D^*, E^*)$. We will solve the following system (2.1) of algebraic equations to determine E_{de}:

$$\begin{aligned}
0 &= \Pi - \mu N^*(t) - \mu_d D^*(t) \\
0 &= \beta_2 (N^*(t) - B^*(t) - D^*(t)) \frac{E^*(t)}{K + E^*(t)} + q\omega D^*(t) - (\mu + \gamma) B^*(t) \\
0 &= \beta_1 (N^*(t) - B^*(t) - D^*(t)) D^*(t) - (\omega + \mu + \mu_d) D^*(t) \\
0 &= \alpha \mu_d D^*(t) - \delta (E^*(t) - E_0).
\end{aligned} \tag{2.1}$$

As a result of algebraic computations we get the following expressions $N^* = \frac{\Pi - \mu_d D^*}{\mu}$, $B^* = \frac{\beta_1(\Pi - (\mu_d + \mu)D^*) - \mu(\omega + \mu + \mu_d)}{\mu\beta_1}$, $E^* = \frac{\alpha\mu_d D^* + \delta E_0}{\delta}$ and the implicit function

$$\begin{aligned}
H(D^*) &= \beta_2 \left(\frac{\Pi - (\mu_d + \mu)D^*}{\mu} - \frac{\beta_1(\Pi - (\mu_d + \mu)D^*) - \mu(\omega + \mu + \mu_d)}{\mu\, \beta_1} \right) \frac{\alpha\mu_d D^* + \delta E_0}{\delta K + \alpha\mu_d D^* + \delta E_0} \\
&\quad -(\mu + \gamma) \frac{\beta_1(\Pi - (\mu_d + \mu)D^*) - \mu(\omega + \mu + \mu_d)}{\mu\, \beta_1} + q\omega D^*.
\end{aligned}$$

Thus, we show that N^* and E^* are positive in Ω, while B^* is positive under the condition $\frac{\beta_1(\Pi - (\mu_d + \mu)D^*)}{\mu(\omega + \mu + \mu_d)} > 1$ which implies $D^* < \frac{\beta_1 \Pi - \mu(\omega + \mu + \mu_d)}{\beta_1(\mu_d + \mu)}$. Further, we have the quadratic equation of D^* which is written in the form $a_1(D^*)^2 + a_2 D^* + a_3 = 0$ where

$a_1 = \beta_1 \alpha \mu_d \left((\mu + \gamma)(\mu_d + \mu) + q\omega\mu \right)$,

$a_2 = \beta_2 \alpha\mu\mu_d(\omega + \mu + \mu_d) + \beta_1 \delta(\mu + \gamma)(K + E_0)(\mu_d + \mu) + \beta_1 q\omega\delta(K + E_0) + \alpha\mu_d\mu(\mu + \gamma)(\omega + \mu + \mu_d) - \Pi\beta_1\alpha\mu_d(\mu + \gamma)$,

$a_3 = \beta_2 \mu\delta E_0(\omega + \mu + \mu_d) + \mu(\mu + \gamma)\delta(\omega + \mu + \mu_d)(K + E_0) - (\mu + \gamma)\beta_1 \Pi\delta(K + E_0)$.

We conclude $a_1 > 0$ and $a_3 < 0$ when the condition $1 < \frac{\beta_1 \Pi(\gamma+\mu)(K+E_0)}{\mu(\omega+\mu+\mu_d)(\beta_2 E_0 + (\gamma+\mu)(K+E_0))}$ holds. Thus, we obtain a unique positive solution D^* that also ensures the existence of positive solutions E^*, N^*, and B^*.

In the current terminology of mathematical epidemiology, the average number of secondary infections generated by an infectious individual in a completely susceptible population is called the basic reproduction number $\mathcal{R}_0$. Since we adapt this approach to the spread of vaccine-refusal, $\mathcal{R}_0$ refers a threshold of a vaccine-refusal epidemics,and indicates whether the vaccine refusal will die out or become an endemic. Since we have only one 'infected' compartment, we obtain a threshold value $\mathcal{R}_0 = \frac{\beta_1 \Pi(\gamma+\mu)(K+E_0)}{\mu(\omega+\mu+\mu_d)(\beta_2 E_0+(\gamma+\mu)(K+E_0))}$ for a vaccine-refusal endemic in terms the method of next generation matrix [7, 41].

The denier-free equilibrium is locally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$ according to the theorem given in [41]. Consequently, if $\mathcal{R}_0 > 1$ then $a_1 > 0$ and $a_3 < 0$. Thus, the quadratic equation $a_1(D^*)^2 + a_2 D^* + a_3 = 0$ has a unique positive solution D^* which enables to find the positive solutions N^*, E^*, B^*. The system has a unique vaccine refusal-existing equilibrium if $\mathcal{R}_0 > 1$.

2.2. Local stability of equilibria

Let us consider the Jacobian matrix of the system (1.2). Thus, linearization of the system (1.2) around the denier-free equilibrium E_{df} yields the matrix $J_{E_{df}}$:

$$J_{E_{df}} = \begin{bmatrix} -\mu & 0 & -\mu_d & 0 \\ \beta_2 \frac{E_0}{K+E_0} & -\beta_2 \frac{E_0}{K+E_0} - (\mu+\gamma) & -\beta_2 \frac{E_0}{K+E_0} + q\omega & \beta_2 \left(\frac{\Pi}{\mu} - \frac{\beta_2 \Pi E_0}{\mu(\beta_2 E_0 + (K+E_0)(\mu+\gamma))} \right) \frac{K+2E_0}{(K+E_0)^2} \\ 0 & 0 & J_{33} & 0 \\ 0 & 0 & \mu_d \alpha & -\delta \end{bmatrix}$$

where $J_{33} = \frac{\beta_1 \Pi}{\mu} \left(\frac{(\gamma+\mu)(K+E_0)}{(\beta_2 E_0 + (\gamma+\mu)(K+E_0))} \right) - (\omega+\mu+\mu_d)$. All eigenvalues of the matrix $J_{E_{df}}$ are negative while $\mathcal{R}_0 < 1$. However, $J_{33} > 0$ if $\mathcal{R}_0 > 1$. In other words, the denier-free equilibrium $J_{E_{df}}$ is locally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.

The matrix $J_{E_{de}}$ gives the linearization of the system (1.2) around the denier-existing equilibrium E_{de}:

$$J_{E_{de}} = \begin{bmatrix} -\mu & 0 & -\mu_d & 0 \\ \beta_2 \frac{E^*}{K+E^*} & -\beta_2 \frac{E^*}{K+E^*} - (\mu+\gamma) & q\omega - \beta_2 \frac{E^*}{K+E^*} & \frac{\beta_2 (K+2E^*)(N^*-B^*-D^*)}{(K+E^*)^2} \\ \beta_1 D^* & -\beta_1 D^* & \beta_1 (N^* - B^* - 2D^*) - (\omega+\mu+\mu_d) & 0 \\ 0 & 0 & \alpha\mu_d & -\delta \end{bmatrix}$$

The corresponding characteristic equation has the form $\lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0$, where all coefficients can be calculated algebraically:

$$\begin{aligned} b_1 &= \beta_2 \frac{E^*}{K+E^*} + \gamma + 2\mu + \delta + \beta_1 D^* \\ b_2 &= \delta \left(\beta_2 \frac{E^*}{K+E^*} \gamma + \mu \right) + \frac{\beta_2 K (N^* - B^* - D^*)}{(K+E^*)^2} + \beta_1 D^* (\gamma + \delta + \mu) \\ &\quad + \mu_d \beta_1 D^* + \mu \left(\frac{\beta_2 E^*}{K+E^*} + \gamma + \delta + \mu \right) + \mu \beta_1 D^* \\ b_3 &= \beta_1 \delta (\gamma + \mu) D^* + \beta_1 \beta_2 \alpha \mu_d \frac{K(N^* - B^* - D^*)}{(K+E^*)^2} D^* + \beta_1 \mu_d (\beta_2 + \delta + \mu) D^* \\ &\quad + \beta_1 \mu (\beta_2 + \delta + \mu) D^* + \mu\delta \left(\frac{\beta_2 E^*}{K+E^*} + \gamma + \mu \right) \\ b_4 &= \beta_1 \mu \delta (\gamma + \mu) + \beta_1 \beta_2 \alpha \mu \mu_d \frac{K(N^* - B^* - D^*)}{(K+E^*)^2} D^* + \beta_1 \delta \mu_d (\gamma + \mu) D^* \end{aligned}$$

According to Routh-Hurwitz criterion all roots of the characteristic equation have negative real parts if and only if $b_1 b_2 - b_3 > 0$ and $b_1 b_2 b_3 - b_3^2 - b_1^2 b_4 > 0$. Hence, the denier-existing equilibrium $J_{E_{de}}$ exists when $\mathcal{R}_0 > 1$ and these inequalities constitute the condition for the local asymptotic stability of the denier-existing equilibrium E_{de}.

2.3. Global stability of E_{de}

We consider the following positive definite function

$$V(t) = D(t) - D^* - D^* \ln(\frac{D(t)}{D^*}) + \frac{1}{2}a_1(B(t) - B^*)^2 + \frac{1}{2}a_2(N(t) - N^*)^2 + \frac{1}{2}a_3(E(t) - E^*)^2$$

where a_1, a_2, and a_3 are positive constants in order to investigate the global stability of E_{de} by using the Lyapunov direct method. Let us take $a_2 = \frac{\beta_1}{\mu_d}$ and compute the derivative of $V(t)$ with respect to time t:

$$\begin{aligned}\frac{dV}{dt} = & -\beta_1(D - D^*)^2 - a_1\left(\frac{\beta_2 E^*}{K + E^*} + (\gamma + \mu)\right)(B - B^*)^2 - \frac{\beta_1 \mu}{\mu_d}(N - N^*)^2 \\ & -a_3\delta(E - E^*)^2 - \beta_1(B - B^*)(D - D^*) - a_1\frac{\beta_2 E^*}{K + E^*}(B - B^*)(D - D^*) \\ & +a_1\frac{\beta_2 K(N - D - B)}{(K + E)(K + E^*)}(E - E^*)(B - B^*) + a_1\frac{\beta_2 E^*}{K + E^*}(N - N^*)(B - B^*) \\ & +a_3\alpha\mu_d(D - D^*)(E - E^*).\end{aligned}$$

$\frac{dV}{dt}$ is negative in the region of Ω if

$$\begin{aligned}T(N, B, D, E) = & -\beta_1(B - B^*)(D - D^*) - a_1\frac{\beta_2 E^*}{K + E^*}(B - B^*)(D - D^*) \\ & +a_1\frac{\beta_2 K(N - D - B)}{(K + E)(K + E^*)}(E - E^*)(B - B^*) \\ & +a_1\frac{\beta_2 E^*}{K + E^*}(N - N^*)(B - B^*) + a_3\alpha\mu_d(D - D^*)(E - E^*) < 0.\end{aligned} \tag{2.2}$$

We reorganize the terms of (2.3) to find the sufficient conditions for $T(N, B, D, E) < 0$, and have the following inequalities:

$$\begin{aligned} a_1 &> \frac{15\beta_1}{4}\frac{K + E^*}{\beta_2 E^* + (K + E^*)(\gamma + \mu)} \\ a_3 &< \frac{4\beta_1\delta}{9\alpha^2\mu_d^2} \\ a_1\left(\frac{\beta_2\Pi}{\mu(K + E^*)}\right)^2 &< a_3\frac{4\delta}{15}\left(\frac{\beta_2 E^* + (K + E^*)(\gamma + \mu)}{K + E^*}\right).\end{aligned}$$

Solving these inequalities for a_1 we obtain the global stability condition of E_{de}

$$\frac{15\beta_1}{4}\frac{K + E^*}{\beta_2 E^* + (K + E^*)(\gamma + \mu)} < \frac{4\beta_1\delta}{9\alpha^2\mu_d^2}\frac{4\delta}{15}\left(\frac{\beta_2 E^* + (K + E^*)(\gamma + \mu)}{K + E^*}\right)\left(\frac{\mu(K + E^*)}{\beta_2\Pi}\right)^2$$

or in simpler form

$$\frac{225\alpha^2\mu_d^2}{64\delta^2} < (\beta_2 E^* + (K + E^*)(\gamma + \mu))^2\left(\frac{\mu}{\beta_2\Pi}\right)^2.$$

Hence, we conclude that α has an effect on the stability whereas $\mathcal{R}_0$ does not dependent on α.

2.4. Sensitivity analysis

We focus on a sensitivity analysis to determine the robustness of the model to the parameter values that are correlated with the basic reproduction number $\mathcal{R}_0$. We calculate the sensitivity indices to determine the parameters that are most efficient by the transmission of vaccine refusal epidemic. We follow the sensitivity approach given in [23] and define the normalized sensitivity index of a variable $\mathcal{R}_0$ with respect to the parameter p as

$$\epsilon^{p}_{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial p} \times \frac{p}{\mathcal{R}_0}. \tag{2.3}$$

The sensitivity indices for the parameters of the model (1.2) are $\epsilon^{\Pi}_{\mathcal{R}_0} = \epsilon^{\beta_1}_{\mathcal{R}_0} = 1$, $\epsilon^{\omega}_{\mathcal{R}_0} = -\frac{\omega}{\omega+\mu+\mu_d}$, $\epsilon^{\mu}_{\mathcal{R}_0} = -\frac{\omega+2\mu+\mu_d}{\omega+\mu+\mu_d} = -(1+\frac{\mu}{\omega+\mu+\mu_d})$, and $\epsilon^{\mu_d}_{\mathcal{R}_0} = -\frac{\mu_d}{\omega+\mu+\mu_d}$ which show the significance of Π and β_1 for the model (1.2). However, we consider that the number of new adults Π come in to the vaccine skeptic compartment S and the natural death rate μ do not change radically. We conclude that, an increase of the value of β_1 promote the basic reproduction number whereas an increase of the value of ω diminishes $\mathcal{R}_0$.

3. Simulations

We provide numerical solutions of the model (1.2) with estimated parameters for supporting the analytical results given in section 2. Since the equilibrium of denier-free case is not interesting, we focus on the case where the basic reproduction rate remains $\mathcal{R}_0 > 1$ which ensures the existence of a denier-existing equilibrium. With this intention, we observe the different scenarios for the set of parameters given in table 2.

Table 2. Parameters values for the simulations

Parameters	Values
Π	$5\,\text{day}^{-1}$
q	0.1
β_1	$0.000003\,\text{day}^{-1}$
β_2	varies in $[0, 0.026]$ day^{-1}
ω	$0.2\,\text{day}^{-1}$
γ	$0.007\,\text{day}^{-1}$
μ	$0.00036\,\text{day}^{-1}$
μ_d	$0.00001\,\text{day}^{-1}$
K	1000
α	$60, 100\,\text{day}^{-1}$
δ	$0.05\,\text{day}^{-1}$
E_0	500

Even though the set of parameters $P = \{\Pi, \beta_1, \beta_2, K, \gamma, q, \omega, \mu, \mu_d, \alpha, \delta, E_0\}$ is not directly obtained from an experimental research, we choose the parameters according to a plausible scenario. The Supreme Election Council (YSK) of Turkey announced the number of voters as 57058636 for the 31 March 2019 Local Administrations General Elections. Further, according to the statistical data of the General Directorate of Population and Citizenship Affairs (NVI), 1380026 people at the age 17 were living in Turkey in 2018. In other words, we roughly assume 3780 people turn 18 per day. Thus, for a initial value of 60000 people we assume to have 5 people daily enter to the system. Turkish Statistical Institute (TUIK) reported the average life expectancy as 78 in Turkey, therefore natural death rate per day is taken as 0.000035 [39]. Due to the current pandemic, the disease related death rate has been chosen as $\mu_d = 0.00001$ per day associated with COVID-19 [1]. In the hypothetical scenario we choose a quite small rate for the transmission of vaccine refusal, $\beta_1 = 0.000003$ and manipulate the parameters β_2 and α for observing possible results. All other parameters are determined for a vaccine-refusal endemic state by choosing the basic reproduction rate, $\mathcal{R}_0 > 1$. For numerical solution of (1.2), we employed the ODE solver 'ode45' of `Matlab` [24].

In figure 2, the effect of positive broadcasting on the vaccine can be observed. The parameter values $\beta_2 = 0$, $\beta_2 = 0.013$, and $\beta_2 = 0.026$ represent no-media effect, a medium effect of media and a high effect of media, respectively. As expected, the media are able to persuade skeptic people to take a vaccine, i.e. the number of vaccine believers increases whereas the number of vaccine deniers decreases.

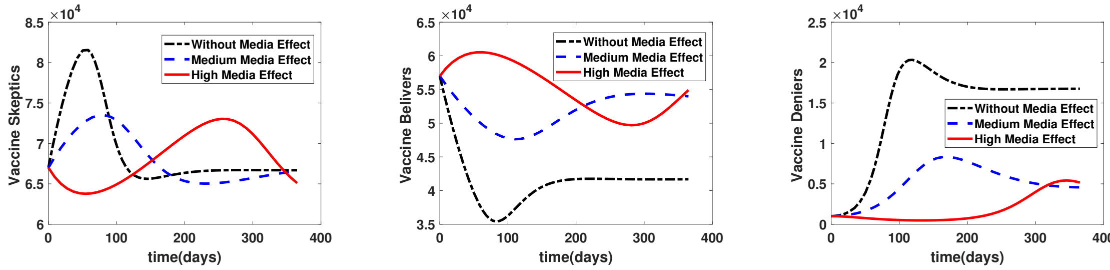

Figure 2. The effect of media in the spread of vaccine-refusal ($\alpha = 60$, $\mathcal{R}_0 = 2.0829$ for $\beta_2 = 0$, $\mathcal{R}_0 = 1.3532$ for $\beta_2 = 0.013$, $\mathcal{R}_0 = 1.0021$ for $\beta_2 = 0.026$).

Further, we observe the stability of E_{de} in figure 3, where the solution trajectory approaches to the denier existing equilibrium.

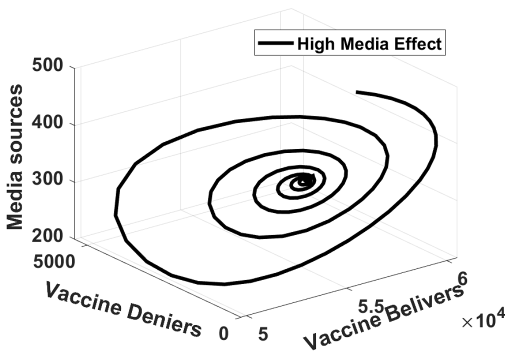

Figure 3. Long term behavior of system with high media effect.

The analytical results of the subsection 2.3 show that α has an effect on stability even if $\mathcal{R}_0$ does not consist of this term. Moreover, α is one of the possible candidates that may convert a stable state to an unstable state. Therefore, we choose a larger value of $\alpha = 100$ and perform simulations to see the change of stability state.

In figure 4, we observe the behavior of vaccine skeptics, vaccine believers, and vaccine deniers for a rate $\alpha = 100$ in a year. This figure represents the positive effect of media against vaccine deniers in a year as in figure 2 and the dynamic behavior of the model is just slightly different from the figure 2 for $\alpha = 60$. See figure 5 to find out the difference of these two cases, numerically.

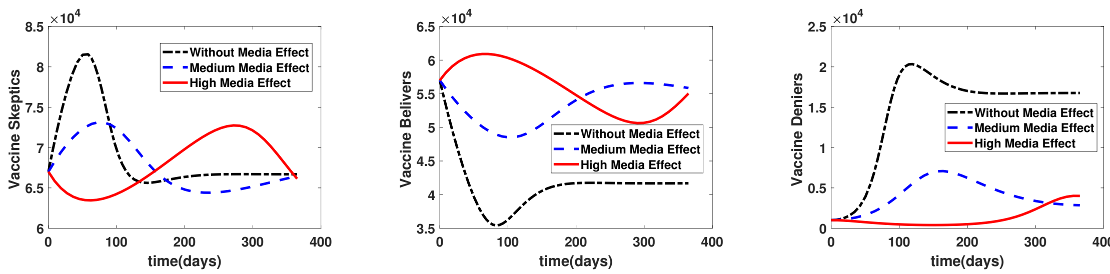

Figure 4. The effect of media in the spread of vaccine-refusal ($\alpha = 100$, $\mathcal{R}_0 = 2.0829$ for $\beta_2 = 0$, $\mathcal{R}_0 = 1.3532$ for $\beta_2 = 0.013$, $\mathcal{R}_0 = 1.0021$ for $\beta_2 = 0.026$).

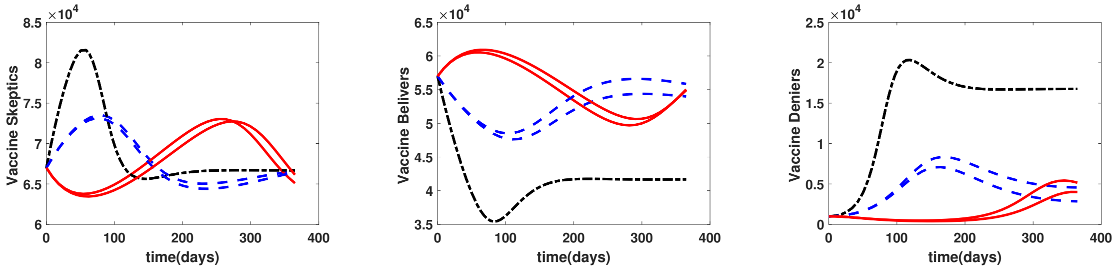

Figure 5. Slight different behavior of the model according to the increase of α from 60 to 100.

However, we see that the long term behavior of the model under high media effect for $\alpha = 60$ is totally different from the long term behavior of the system for $\alpha = 100$, see figure 6. The increase of the production rate of awareness sources, i.e. the parameter α allows to change the stability state of E_{de} from stable to unstable where we show with the simulation of long term behavior of the system in figure 6.

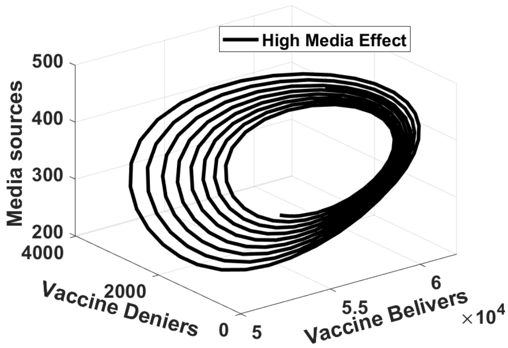

Figure 6. Long term behavior of system with high media effect.

4. Conclusions

According to the World Health Organization (WHO), vaccine hesitancy is a severe but not novel threat to global health. Rumours about the side effects of vaccination started just after Dr. Edward Jenner invented the smallpox vaccine in 1796. From the bodies of people who received the vaccine, cow heads would erupt. In 1892, Edward Joshua Edwardes mentioned in the report "Vaccination and Small-Pox in England and Other Countries Showing That Compulsory Re-Vaccination Is Necessary" these rumours and emphasized the need for compulsory vaccination [10]. In 2021, mankind continue to find novel irrational stories to believe in. Thus, we present a mathematical model that underlines the possible positive effect of media on the confidence in the vaccine. Mathematical analysis of the model results a vaccine-denier free equilibrium and a vaccine-denier existing equilibrium. The denier-free equilibrium is locally asymptotically stable if $\mathcal{R}_0 < 1$ and the denier-existing equilibrium exists for $\mathcal{R}_0 > 1$ and is globally asymptotically stable under the conditions given in the subsection 2.3, i.e. denier existing equilibrium will persist. Stability analysis around the equilibria shows that denier existing equilibrium is highly dependent on the awareness parameter of media and it may turn from stable to unstable as awareness parameter increases. Further, this research concludes that, according to the scenario created in this model, transmission of the opinion of vaccine refusal is more important than the production rate of awareness sources in the short-term future. However, the effect of the production rate of awareness sources can be seen in the long-term future. Finally, we present a view of how strong the indirect influence of the media can be, while developing vaccination policies, in controlling the epidemics that our world is likely to encounter.

Acknowledgment. The author would like to thank the editor and two anonymous reviewers for their valuable comments and suggestions.

References

[1] A. Antos, M-L. Kwong, T. Balmorez, A. Villanueva, and S. Murakami, *Unusually high risks of COVID-19 mortality with age-related comorbidities: An adjusted meta-analysis method to improve the risk assessment of mortality using the comorbid mortality data*, Infect. Dis. Rep. **13** (3), 700–711, 2021.

[2] J. Arino, C.C. McCluskey, and P. van den Driessche, *Global results for an epidemic model with vaccination that exhibits backward bifurcation*, J. Appl. Math. **64** (1), 2002.

[3] C.T. Bauch and D.J.D. Earn, *Vaccination and the theory of games*, Proc. Natl. Acad. Sci. **101** (36), 13391–13394, 2004.

[4] C.T. Bauch, A.P. Galvani, and D.J.D. Earn, *Group interest versus self-interest in smallpox vaccination policy*, Proc. Natl. Acad. Sci. **100** (18), 10564–10567, 2003.

[5] C. Betsch, F. Renkewitz, T. Betsch, and C. Ulshöfer, *The influence of vaccine-critical websites on perceiving vaccination risks*, J. Health Psychol. **15** (3), 446–455, 2010.

[6] M-G. Cojocaru, C.T. Bauch, and M.D. Johnston, *Dynamics of vaccination strategies via projected dynamical systems*, Bull. Math. Biol. **69** (5), 1453–1476, 2007.

[7] O. Diekmann, J.A.P. Heesterbeek, and J.A.J. Metz, *On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations*, J. Math. Biol. **28** (4), 1990.

[8] K. Dietz and J.A.P. Heesterbeek, *Daniel Bernoullis epidemiological model revisited*, Math. Biosci. **180** (1), 1–21, 2002.

[9] X. Duan, S. Yuan, and X. Li, *Global stability of an SVIR model with age of vaccination*, Appl. Math. Comput. **226**, 528-540, 2014.

[10] J.E. Edwardes, *Vaccination and small-pox in England and other countries showing that compulsory re-vaccination is necessary*, RCP Library, J. A. Churchill, 1892. Wiley Digital Archives: The Royal College of Physicians. Accessed 2021.

[11] D. Esernio-Jenssen and P. Offit, *Don't hesitate – vaccinate!*, J. Child Adolesc. Trauma **13** (3), 337–341, 2020.

[12] S. Funk, E. Gilad, and V.A.A. Jansen, *Endemic disease, awareness, and local behavioural response*, J. Theor. Biol. **264** (2), 501–509, 2010.

[13] L. Gardner, E. Dong, K. Khan, and S. Sarkar, *Persistence of US measles risk due to vaccine hesitancy and outbreaks abroad*, Lancet Infect. Dis. **20** (10), 1114–1115, 2020.

[14] M. Gölgeli, *A mathematical model of hepatitis B transmission in Turkey*, Commun. Fac. Sci. **68** (2), 1586–1595, 2019.

[15] M. Gölgeli and F.M. Atay, *Analysis of an epidemic model for transmitted diseases in a group of adults and an extension to two age classes*, Hacet. J. Math. Stat. **49**, 921–934, 2020.

[16] M.J. Hornsey, J. Lobera, and C. Díaz-Catalán, *Vaccine hesitancy is strongly associated with distrust of conventional medicine, and only weakly associated with trust in alternative medicine*, Soc. Sci. Med. **255**, 113019, 2020.

[17] L. Huo, J. Jiang, S. Gong, and B. He, *Dynamical behavior of a rumor transmission model with Holling-Type II functional response in emergency event*, Phys. A: Stat. Mech. Appl. **450**, 228–240, 2016.

[18] W.O. Kermack and A.G. McKendrick, *A contribution to the mathematical theory of epidemics*, Proc. R. Soc. Lond. Series A **115** (772), 700–721, 1927.

[19] M. Kröger, M. Turkyilmazoglu, and R. Schlickeiser, *Explicit formulae for the peak time of an epidemic from the SIR model: Which approximant to use?*, Phys. D: Nonlinear Phenom. **425**, 132981, 2021.

[20] M. LaCour and T. Davis, *Vaccine skepticism reflects basic cognitive differences in mortality-related event frequency estimation*, Vaccine **38** (21), 3790–3799, 2020.

[21] K.M. Lisenby, K.N. Patel, and M.T. Uichanco, *The role of pharmacists in addressing vaccine hesitancy and the measles outbreak*, J. Pharm. Pract. **34** (1), 127–132, 2021.

[22] N.E. MacDonald, *Vaccine hesitancy: Definition, scope and determinants*, Vaccine **33** (34), 4161–4164, 2015.

[23] M. Martcheva, *An Introduction to Mathematical Epidemiology*, Springer-Verlag GmbH, 2015.

[24] MATLAB-Mathworks, *MATLAB version 9.4.0.813654 (R2018a)*, The Mathworks, Inc., Natick, Massachusetts, 2018.

[25] C.J.E. Metcalf, V. Andreasen, O.N. Bjørnstad, K. Eames, W.J. Edmunds, S. Funk, T.D. Hollingsworth, J. Lessler, C. Viboud, and B.T. Grenfell, *Seven challenges in modeling vaccine preventable diseases*, Epidemics **10**, 11–15, 2015.

[26] A.K. Misra, A. Sharma, and J.B. Shukla, *Modeling and analysis of effects of awareness programs by media on the spread of infectious diseases*, Math. Comput. Model. **53** (5-6), 1221–1228, 2011.

[27] A.K. Misra, *A simple mathematical model for the spread of two political parties* Nonlinear Anal.: Model Control **17** (3), 343–354, 2012.

[28] M. Motta, S. Sylvester, T. Callaghan, and K. Lunz-Trujillo, *Encouraging COVID-19 vaccine uptake through effective health communication*, Front. Polit. Sci. **3**, 2021.

[29] A. Mukhopadhyay, A. De Gaetano, O. Arino, *Modeling the intra-venous glucose tolerance test: A global study for a single-distributed-delay model*, Discrete Contin. Dyn. Syst. Ser B. **4** (2), 407–417, 2004.

[30] X. Nan and K. Madden, *HPV vaccine information in the blogosphere: How positive and negative blogs influence vaccine-related risk perceptions, attitudes, and behavioral intentions*, Health Commun. **27** (8), 829–836, 2012.

[31] F. Nyabadza, T.Y. Alassey, and G. Muchatibaya, *Modelling the dynamics of two political parties in the presence of switching*, SpringerPlus **5** (1), Art. No. 1018, 2016.

[32] S.B. Omer, D.A. Salmon, W.A. Orenstein, M.P. deHart, and N. Halsey, *Vaccine refusal, mandatory immunization, and the risks of vaccine-preventable diseases*, N. Engl. J. Med. **360** (19), 1981–1988, 2009.

[33] N.N. Pelen and M. Gölgeli, *Vector-borne disinformation during disasters and emergencies*, Phys. A: Stat. Mech. Appl. **596**, 127157, 2022.

[34] S. Samanta, S. Rana, A. Sharma, A.K. Misra, and J. Chattopadhyay, *Effect of awareness programs by media on the epidemic outbreaks: A mathematical model*, Comput. Appl. Math. **219** (12), 6965-6977, 2013.

[35] E. Shim, J.J. Grefenstette, S.M. Albert, B.E. Cakouros, and D.S. Burke, *A game dynamic model for vaccine skeptics and vaccine believers: Measles as an example*, J. Theor. Biol. **295**, 194–203, 2012.

[36] E. Shim, B. Kochin, and A. Galvani, *Insights from epidemiological game theory into gender-specific vaccination against rubella*, Math. Biosci. Eng. **6** (4), 839–854, 2009.

[37] T.C Smith, *Vaccine rejection and hesitancy: A review and call to action*, Open Forum Infect. Dis. **4** (3), 2017.

[38] E. Tornatore, P. Vetro, and S.M. Buccellato, *SIVR epidemic model with stochastic perturbation*, Neural. Comput. Appl.**24** (2), 309–315, 2012.

[39] TUIK, *Adrese dayalı nüfus kayt sistemi sonuçları, 2019*, Tech. report, 2020.

[40] M. Turkyilmazoglu, *Explicit formulae for the peak time of an epidemic from the SIR model*, Phys. D: Nonlinear Phenom. **422**, 132902, 2021.

[41] P. van den Driessche and J. Watmough, *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission*, Math. Biosci. **180** (1-2), 29–48, 2002.

[42] P. Verger and E. Dubé, *Restoring confidence in vaccines in the COVID-19 era*, Expert Rev. Vaccines **19** (11), 991–993, 2020.

[43] WHO, *Ten threats to global health in 2019*, World Health Organization, 2019.

[44] Y. Xiao, T. Zhao, and S. Tang, *Dynamics of an infectious diseases with media/psychology induced non-smooth incidence*, Math. Biosci. Eng. **10** (2), 445–461, 2013.

[45] D. Zhao and S. Yuan, *Persistence and stability of the disease-free equilibrium in a stochastic epidemic model with imperfect vaccine*, Adv. Differ. Equ. **2016** (280), 2016.