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PARTITIONING STRATEGIES OF FOURTH AND FIFTH GRADE STUDENTS

ABSTRACT

Primary goal of this study is to analyze to what extent fourth and fifth graders can coordinate number of people sharing and number of things being shared to solve equal sharing problems, and which strategies in the taxonomy established by Charles and Nason they use to provide this coordination. To this aim, 3 partitioning problems were asked to a group of fourth and fifth grade students. They worked on each problem in pairs without any intervention about their solution processes. All papers on which students solved the problems were evaluated to determine partitioning strategies of students. Results showed that (i)regrouping, partition and quantify by part-whole notion, and whole to each person then half the remaining objects between half the people strategies were the most popular strategies, and (ii) only a small percent of fourth and fifth graders could use Class 1 strategies, and this means that they have low abstraction level of fractions.

Keywords: Fractions, Equal Sharing, Partitioning Strategies

DÖRT VE BEŞİNCİ SINIF ÖĞRENCİLERİNİN PAYLAŞTIRMA STRATEJİLERİ

ÖZET

Bu çalışmanın temel amacı, dördüncü ve beşinci sınıf öğrencilerinin eşit paylaştırma problemlerini çözmek için insan sayısı ile paylaşılan obje sayısını ne ölçüde koordine edebildiklerini ve bu koordinasyonu sağlamak için Charles and Nason tarafından belirlenen taksonomideki stratejilerden hangilerini kullandıklarını analiz etmektir. Bu amaçla, bir grup dört ve beşinci sınıf öğrencisine 3 paylaştırma problemi sorulmuştur. Öğrenciler her problem üzerinde, çözüm süreçleri ile ilgili herhangi bir müdahale düzeltme olmadan, ikili gruplar halinde çalışmışlardır. Öğrencilerin problemleri çözdükleri tüm kâğıtlar, onların paylaştırma stratejilerini belirlemek için değerlendirilmiştir. Sonuçlar (i) en popüler stratejilerin yeniden gruplama, parça-bütün fikri ile bölüştürme ve niceliğini belirtme ve herkese bir bütün sonra kalan nesnelerin yarısını kişilerin yarısına paylaştırma stratejileri olduğunu, (ii) öğrencilerin sadece küçük bir yüzdesinin Sınıf 1 stratejilerini kullanabildiklerini ve bunun da dört ve beşinci sınıf öğrencilerinin kesirleri soyutlama düzeyinin düşük olduğu anlamına geldiğini göstermiştir.

Anahtar Kelimeler: Kesirler, Matematik Öğretimi, Eş Paylaştırma, Paylaştırma Stratejileri

1. INTRODUCTION (GİRİŞ)

There is no domain of elementary school mathematics as mathematically rich, cognitively complicated, and difficult to teach as fractions [1]. The acknowledged difficulties in learning fractions are reflected and documented in a number of studies in which researchers have examined different aspects of this topic [eg. 2, 3, 4, 5). More recent studies by Mack [6], Tzur [7] and Anderson, Anderson and Wensell [8] revealed that understanding and using fractions are tasks that have traditionally been difficult for pupils. National and international assessment results showed that even older pupils had trouble in working with and understanding fractions [9, 10].

Many of aforementioned difficulties with learning fractions can be attributed to teaching efforts that have focused almost exclusively on the part-whole construct of fraction [11, 12], and many researches highlight the children's need for building a deep fractional understanding by using a variety of concrete and pictorial models to overcome these difficulties [12, 13, and 14]. In particular, partitioning activities are important mechanisms for building fractional understanding as Pothier and Sawada [15], Streefland [12], Lamon [14], Smith III [1], Pithkethly and Hunting [16], Charles and Nason [17], Empson [18,19] and Toluk [20] have pointed out in their studies. In a nutshell, studies that have investigated strategies for partitioning have found that (i) young children tend to use a variety of intuitive strategies for partitioning problems [12, 14, 15, 16, 17]; (ii) selection of partitioning strategies depends on student's prior knowledge and experiences, the context of the task, the type and number of objects being shared, and number of sharers (12, 14, 15, 17] (iii)young children's use of partitioning strategies is situationally specific, demonstrating a strong adherence to social practice [14].

As to classifying strategies used for partitioning problems, there are some determined and labeled strategies in the studies by Pothier and Sawada [15], Streefland [12], and Lamon [14]. But most recent and comprehensive study was done by Charles and Nason [17]. Since it has provided most important contribution to the theoretical framework of the present study, it is essential to give more detailed information about this study.

What Charles and Nason [17] did was not only identify new partitioning strategies, but also to develop taxonomy for categorizing young children's partitioning strategies in terms of their ability to facilitate the abstraction of fraction construction from the concrete activity of partitioning objects and/or sets of objects. They conducted clinical interviews with twelve purposely selected third grade students and presented each student a set of realistic partitioning tasks.

The paper by Charles and Nason [17] concluded with a taxonomy consisting eleven strategies sorted into four classes. They decided the classes based on three criteria: (i) fair sharing, (ii) accurate quantification of shares, and (iii) conceptual mapping. They explained the mean of "conceptual mapping" as follows:

…the process of abstraction from concrete experience begins with children initially acquiring a body of disconnected knowledge situated in a large number of everyday experiences such as sharing objects like pizza and cakes with their family or friends. They then note consistencies within different situations. Following the noting of these consistencies, children classify these situations into contexts in which the mathematical construct is situated. During this phase, children begin to construct the conceptual mappings which form the basis of abstraction. Finally they form abstract-general constructs by recognizing consistencies and/or similarities across several different contexts. ([17], p.193)

According to Charles and Nason [17], Class 1 strategies meet all three conditions. Class 2 strategies meet first two conditions, namely that

of generating equal and quantifiable shares. Class 3 strategies only meet the condition of generation equal shares. Class 4 strategies meet none of the conditions. They state that these classes can be viewed as an order of abstraction ability in which Class 1 demonstrates full abstraction, Class 2 and 3 demonstrate lesser degrees of abstraction, and Class 4 demonstrate nil abstraction (Table 2).

2. AIM AND SIGNIFICANCE OF RESEARCH (ÇALIŞMANIN AMACI VE ÖNEMİ)

Equal sharing problems are a good foundation and starting point to begin fraction instruction, because students are able to generate strategies for such problems using their informal knowledge. In other words, ability to use, internalize and reason about partitioning was present in children at an early age, and these abilities developed and became more sophisticated with instruction [16].

Despite the proliferation of studies on children's strategies for partitioning, none of these researches has examined these strategies based on any taxonomy at fourth and fifth grade level. Therefore, primary goal of this study is to analyze to what extent fourth and fifth graders can coordinate two quantities (number of people sharing and number of things being shared) in their solutions to equal sharing problems, and which strategies in the taxonomy established by Charles and Nason [17] they use to provide this coordination. Another goal is to examine whether there is any inconsistency between strategies in this taxonomy and strategies used by students in the present study.

3. ANALITYCAL STUDY (ANALİTİK ÇALIŞMA)

3.1 Participants (Katılımcılar)

The study was carried out in a sex-mixed elementary school in Bursa/Turkey. Determinative factor in selecting this school was positive attitudes and open mindedness of management board and teachers toward this kind of research. There were two 4th and two 5th classes in the school and one of each of them were randomly selected for application. Consequently, 30 fourth and 28 fifth grade students participated in the study.

3.2 Information about Problems Used in the Study (Çalışmada Kullanılan Problemlerle Ilgili Bilgi)

Problems used in the research were represented in a context including events that a famous cartoon family named Sizinkiler had experienced. Three problems were made use and the first problem was about sharing three objects among 4 people. Other problems included distribution of 6 objects among 9 people and 5 objects among 3 people, respectively (Table 1).

Table 1. Problems used in the study
(Tablo 1. Çalışmada kullanılan sorular)

Problem 1	*Sizinkiler* family goes to a *pide* (a Turkish food like pizza) restaurant for dinner. But 4 *pides* seem big for them and they order 3 *pides* instead of 4. In your opinion, how can the waiter serve 3 *pides* for 4 people? Can you draw the *pides* that each person gets?
Problem 2	At the table next to the *Sizinkiler*, there is another group that consists of 9 people. They order 6 *pides*. Now, again show each person's part with pictures.
Problem 3	One day Zeytin was constructing a picture by cutting and gluing colored papers. He needed a green piece of paper, but two of his friends also needed that at the same time. The teacher said: "I have 5 pieces of green paper. Share them equally among you." Can you help them with sharing?

These three partitioning tasks were based on prototypes created by Streefland [12] and children were asked to assume roles of the waiters, the

teacher and the kids mentioned in these problems. Difficulty levels of sharing procedure were increased in each problem. For example, second problem have more options (2/3, 4/6 and 6/9) than that of the first problem to express the result of sharing by fraction. Moreover, in the third problem, number of objects being shared is more than number of people so that students can be directed to use improper fractions.

3.3 Application (Uygulama)

Table 2. Classes of partitioning strategies([7], p.211)
Tablo 2. Paylaştırma stratejileri ile ilgili sınıflar([7], p.211)

Strategy Classes	Strategies in each class	Characteristics of each class
Class 1	1) Partitive quotient foundational strategy 2) Proceduralised partitive quotient strategy	• Generates fair shares • Accurate quantification of shares • Conceptual mapping
Class 2	3) Regrouping strategy 4) People by objects strategy 5) Half to each person then quarter to each person strategy	• Generates fair shares • Accurate quantification of shares • No conceptual mapping
Class 3	6) Partition and quantify by part-whole notion strategy 7) Halving the objects between half the people strategy 8) Whole to each person then half the remaining objects between half the people strategy	• Generates fair shares • Little or no accurate quantification of shares • No conceptual mapping
Class 4	9) Horizontal partitioning strategy 10) Repeated sizing strategy 11) Repeated halving/repeated sizing strategy	• Does not generate fair shares • Little or no accurate quantification of shares • No conceptual mapping

First a picture of Sizinkiler family consisting of a father (Babisko), mother (Cıtcıt) and two kids (Zeytin and Limon) was showed, and then questions like "Do you know this family?", "What are the names of them?" were asked to introduce the context.

After introduction, first problem about sharing 3 objects among 4 people were represented the children as written text. Children were asked to study in pairs. Each student worked on the problem with the other student who was sitting his/her next. Students had approximately 15 minutes to discuss the problem. In the meantime, researcher walked around the groups and tried to understand their reasoning by questions like "What does this drawing mean?", "Can you explain your sharing?", and "Is every person's share equal?" etc. Researcher just helped students to understand the question when needed, and she did not make any intervention or correction. In addition, particular notes were taken of children's partitioning strategies through their explanations. After that, all papers on which students solved the problem were collected and a general class discussion on different solutions of the problem was implemented. This process was performed for each problem and application took two lessons.

3.4 Analysis of data (Veri analizi)

Since students worked in pairs, 45 papers including students' solutions were gathered from 4th graders. This number was 42 for fifth graders. Thus, 87 papers were examined in total.

To classify strategies of students, the taxonomy developed by Charles and Nason [17] were made use of. This taxonomy is presented in Table 2. By the help of students' drawings, observations made during implementation, and notes about students' reasoning, frequencies for each strategy given in

abovementioned taxonomy were computed for each problem. In addition, qualitative evaluations were fulfilled to have more in-depth information about use of each strategy.

4. FINDINGS (BULGULAR)

In this chapter, all findings about each problem are represented on the basis of grade levels.

4.1. Findings about Fourth Grader's Partitioning Strategies (Dördüncü Sınıf Öğrencilerinin Paylaştırma Stratejileri ile İlgili Bulgular)

In Table 3, fourth graders' frequencies about use of each partitioning strategy in the taxonomy were presented.

Table 3. Frequencies about use of partitioning strategies at fourth grade level
(Tablo 3. Dördüncü sınıf düzeyinde paylaştırma stratejilerinin kullanımı ile ilgili frekanslar)

Strategy Classes	Strategy Number	Frequencies			
		Problem 1	Problem 2	Problem 3	Total
Wrong or unrelated answe	0	4	6	2	12
1	1	1	0	1	2
	2	1	1	0	2
2	3	5	4	4	13
	4	0	0	0	0
	5	1	0	0	1
3	6	2	2	4	8
	7	0	0	0	0
	8	0	0	4	4
4	9	1	2	0	3
	10	0	0	0	0
	11	0	0	0	0

When frequencies given in Table 3 are evaluated on the basis of Strategy Classes, percent of wrong or an unrelated answer is 27%. Percent of usage of Class 1 strategies is quite low (9%). Strategy Class that has the highest usage level is Class 2 (31%). Class 3 strategies are employed at 27 percent level. And finally Class 4 strategies are used at 6 percent level. Besides, the most preferred strategies by the 4 graders are *regrouping* and *partition and quantify by part-whole notion* strategies.

In addition, an in-depth and qualitative examination of student solutions for each problem can be summarized as follows.

- **First problem:** Almost 1/3 of students at 4th grade gave wrong or unrelated answer to this question. One of the interesting sharing in this category has been shown in Figure 1. Students in this group gave one whole *pide* for mother and father, and shared the remaining *pide* between two children.

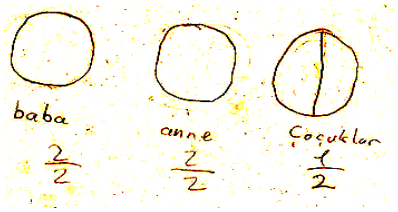


Figure 1. A sample of wrong answer by 4th graders to the first problem
Şekil 1. İlk probleme 4 sınıf öğrencileri tarafından verilen bir yanlış cevap örneği

Most popular strategy for this problem was *regrouping* and all groups that used this strategy wrote the correct fraction about consequence of partitioning (Figure 2).

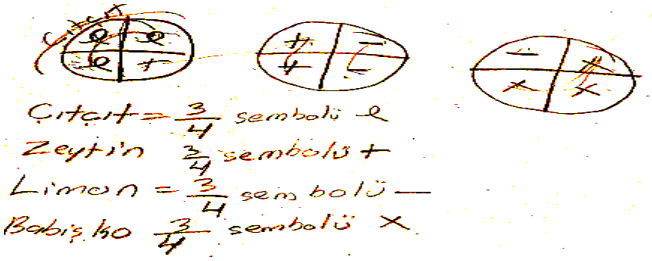


Figure 2. *Regrouping* strategy (4th grade-first problem)
Şekil 2. *Yeniden gruplama* stratejisi (4. sınıf-ilk problem)

The second notable strategy used in the solution of the first problem was *partition and quantify by part-whole notion.* In accordance with the characteristic of this strategy, students partitioned each whole into equal pieces, and shared one piece from each object to each person. But they were unable to accurately quantify each share as fraction due to their inability to successfully apply the part – whole system mapping even they could write it as decimal (Figure 3).

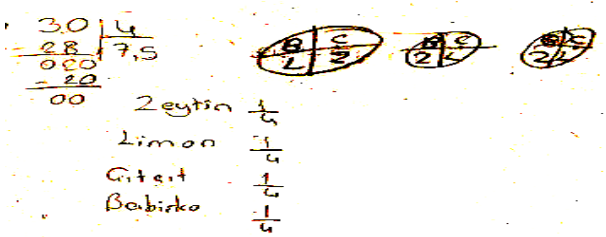
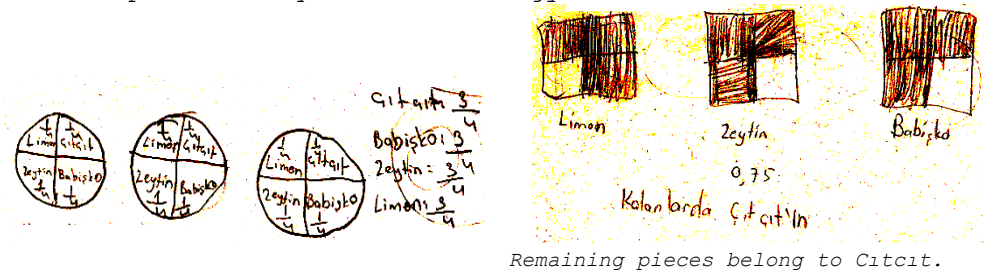


Figure 3. Partition and quantify by part-whole notion strategy (4th grade-first problem)
Şekil 3. Parça-bütün fikri ile bölüştürme *ve niceliğini belirtme* stratejisi (4. sınıf –ilk problem)

Only one group handled the problem by using *partitive quotient foundational* strategy (Figure 4a). Students in another group interestingly wrote the result of sharing as .75 and made the drawing shown in Figure 4b. When they asked to explain their reasoning, they said: "If we distribute 3 *pides* among 4 people, every person gets 3/4 *pide* and it equals to .75." They first found the decimal equivalent of 3/4 and then express the sharing by drawing. Their way of thinking was an explicit indicator of *proceduralised partitive quotient* strategy.



Remaining pieces belong to Cıtcıt.

(a) (b)

Figure 4. *Partitive quotient foundational* and *proceduralised partitive quotient* strategies (4th grade-first problem)
Şekil 4. *Parça belirleyici temel bölüm* ve *parça belirleyici işlemsel bölüm* stratejileri (4. sınıf-ilk problem)

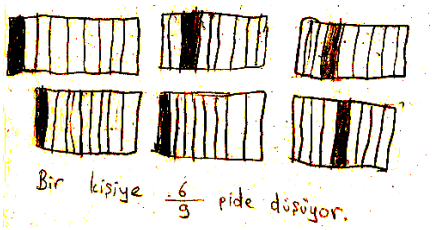
Another interesting solution was an obvious sample of *half to each person then quarter to each person* strategy. Students first gave a half *pide* to each person, then they divided the last *pide* into 4 equal parts, and lastly they allocated one quarter *pide* to every people (Figure 5a). Another group used *horizontal partitioning* strategy to tackle with the problem (Figure 5b).

pieces that every person gets

(a) (b)

Figure 5. Half to each person then quarter to each person and horizontal partitioning strategies (4th grade-first problem).
Şekil 5. Herkese bir bütün sonra kalanları çeyreğe bölerek dağıtma ve yatay bölüştürme stratejileri (4. sınıf-ilk problem)

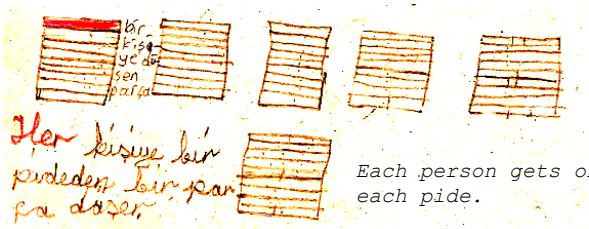
- **Second problem:** As to findings about this problem, number of wrong or unrelated solution exceeded 1/3 of students. Numbers of groups that applied *proceduralised partitive quotient* and *partition and quantify by part-whole notion* strategies were the same with that of the first problem. In a sample of *proceduralised partitive quotient* strategy (Figure 6), although students drew a rectangle for each *pide* and cut each of them into 9 equal parts, they did not need to show each persons share and they directly identified the right fraction.



Each person gets 6/9 of a pide.

Figure 6. *Proceduralised partitive quotient* strategy (4th grade-second problem)
Şekil 6. *Parça belirleyici işlemsel bölüm* stratejisi (4. sınıf-ikinci problem)

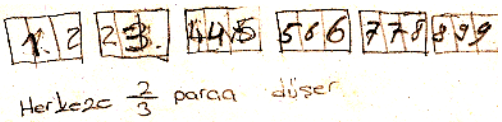
In Figure 7, sample of the latter strategy can be seen. These students cut the each *pide* into nine equal pieces, but they didn't mark each persons share. Instead of this, they just phrased that everybody would receive one piece from each *pide.* There was not any fraction on their paper.



Each person gets one piece from each pide.

Figure 7. Partition and quantify by part-whole notion strategy (4th grade-second problem)
Şekil 7. *Parça-bütün fikri ile bölüştürme ve niceliğini belirtme* stratejisi (4. sınıf –ikinci problem)

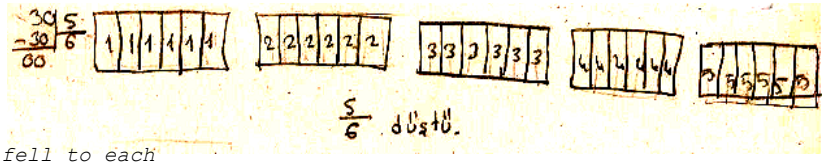
Most favorite strategy was *regrouping* again, but there were some changes in sharing procedure. Students in one pair separated each object into 3 parts instead of 6, and thought that there would be 18 parts in total. And they decided that every person could get two thirds (Figure 8).



2/3 of one pide fall to each persons share.

Figure 8. *Regrouping* strategy (4th grade-second problem)
Şekil 8. *Yeniden gruplama* stratejisi (4. sınıf-ikinci problem)

- **Third problem:** Only two pairs answered wrongly this problem. Moreover, beginning point of one of them was correct. They partitioned each object into six pieces so that they could mete out them equally. But they divided total number of pieces (30) by number of object (5) instead of number of person (3). As a consequence, they were not able to reach right answer (Figure 9).

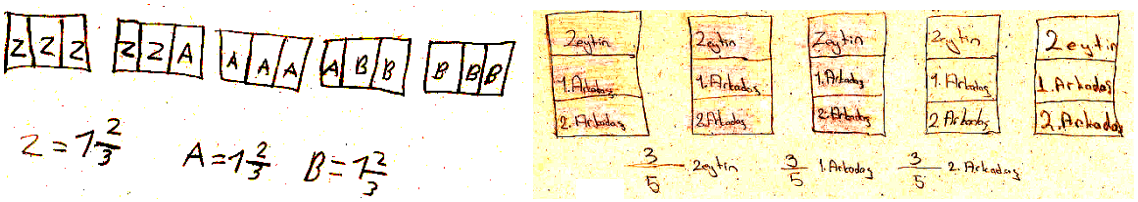


5/6 of one pide fell to each persons share.

Figure 9. A sample of wrong answer by 4th graders to the third problem
Şekil 9. Üçüncü probleme 4 sınıf öğrencileri tarafından verilen bir yanlış cevap örneği

Interestingly, there were 3 popular strategies in student solutions about this problem.

One of them was *regrouping* as it happened for the first two problems. Differently, students used improper fractions to express the result of sharing (Figure 10a). There was an increasing in the usage of *partition and quantify by part-whole notion* strategy, and all pairs did not manage to write each person's share as fractions as it is expected since nature of the strategy (Figure 10b).

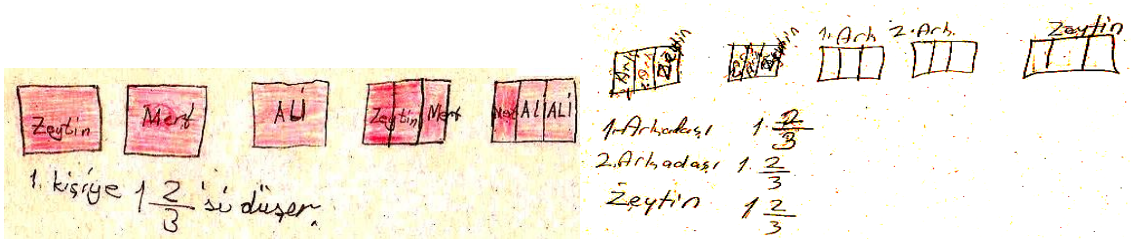


(a) (b)

Figure 10. *Regrouping* and *partition and quantify by part-whole notion* strategies (4th grade-third problem)
Şekil 10. *Yeniden gruplama* ve *parça-bütün fikri ile bölüştürme ve niceliğini belirtme* stratejileri (4. sınıf-üçüncü problem)

A breakthrough of usage of *whole to each person then half the remaining objects between half the people* strategy was another point which is worth to mention here. In pursuant of this strategy, 4 pairs first gave a whole *pide* to each person. Then they applied *partitive quotient foundational* or *regrouping* strategy for the remaining two *pides* (Figure 11a and 11b). All of these pairs found out the correct fraction.

One person takes 1 2/3 of a pide.



(a) (b)

Figure 11. Whole to each person then half the remaining objects between half the people strategy (4th grade-third problem)
Şekil 11. Herkese bir bütün sonra kalan nesnelerin yarısını kişilerin yarısına paylaştırma stratejisi(4. sınıf-üçüncü problem)

4.2 Findings about Fifth Grader's Partitioning Strategies (Beşinci Sınıf Öğrencilerinin Paylaştırma Stratejileri ile İlgili Bulgular)

Frequencies about fifth graders' use of partitioning strategies in Charles and Nason [17]'s taxonomy were shown in Table 4.

Table 4. Frequencies about use of partitioning strategies at fifth grade level
(Tablo 4. Beşinci sınıf düzeyinde paylaştırma stratejilerinin kullanımı ile ilgili frekanslar)

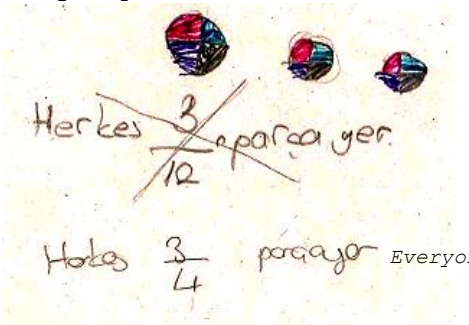
Strategy Classes	Strategy Number	Frequencies			
		Problem 1	Problem 2	Problem 3	Total
Wrong or unrelated answer	0	4	3	3	10
1	1	1	0	1	2
	2	0	1	0	1
2	3	8	8	6	22
	4	0	0	0	0
	5	1	0	0	1
3	6	0	1	1	2
	7	0	0	0	0
	8	0	0	3	3
4	9	0	1	0	1
	10	0	0	0	0
	11	0	0	0	0

Frequencies given in Table 4 show that percent of wrong or an unrelated answer is 24%. Usage of Class 1 strategies at fifth grade level is low again (7%). Class 2 strategies have the highest usage (55%). Class 3 strategies are employed at 12 percent level. And usage level of Class 4 strategies is 2%. The most preferred strategies by the 5 graders are *regrouping, whole to each person then half the remaining objects between half the people* and *partition and quantify by part-whole notion* strategies.

To support the quantitative findings, more detailed information obtained through observations, notes and student writings were represented in the following.

- **First problem:** Answers given by fifth grades to the first problem focused on only three strategies (except the wrong or unrelated ones). One of them was *partitive quotient foundational* strategy, and only one group made use of it (Figure 12). Students in this group first divided each pide into 4 equal parts. Then they gave one piece of each pide to each person by using different colored pencil for

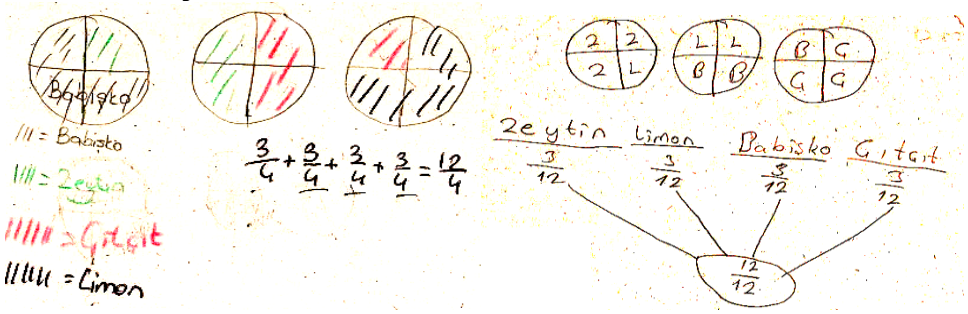
everybody. First they determined the fraction as 3/12, but after a short discussion, they replaced their answer with 3/4.



Everyone eats 3/4 of a pide.

Figure 12. Partitive quotient foundational strategy (5th grade-first problem)
Şekil 12. Parça belirleyici temel bölüm stratejisi (5. sınıf-ilk problem)

The other strategy used for this problem was *regrouping*. Most widespread usage of this strategy was observed here (more than half of students). But only one group could accurately quantify the share (Figure 13a). All of the other groups seemed to confuse the number of pieces in each whole (4) with the number of pieces in total (12), so they represented the share as 3/12 (figure 13b)



(a) (b)

Figure 13. *Regrouping* strategy (5th grade-first problem)
Şekil 13. *Yeniden gruplama* stratejisi (5. sınıf-ilk problem)

The last strategy encountered among student solutions was *half to each person then quarter to each person* strategy. There was only one answer including this strategy, but the answer was quite conspicuous, because shares had been generated by *horizontal partitioning* (Figure 14). So, two strategies were combined here.

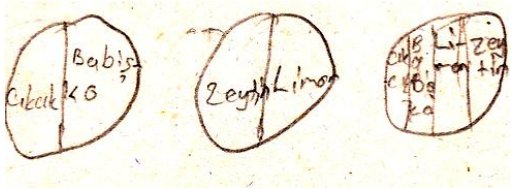


Figure 14. Half to each person then quarter to each person strategy (5th grade-first problem)
Şekil 14. Herkese bir bütün sonra kalanları çeyreğe bölerek dağıtma stratejisi (5. sınıf-ilk problem)

- **Second problem:** Like 4th grade students, only one group employed the *proceduralised partitive quotient* strategy for this problem (Figure 15) and students in this group exhibited the same way of thinking shown in Figure 6.

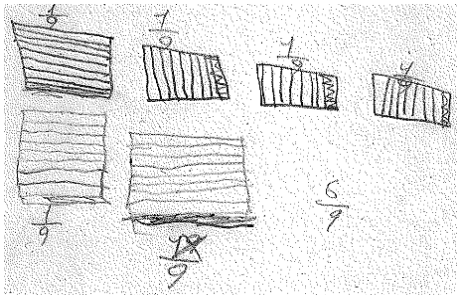


Figure 15. Proceduralised partitive quotient strategy (5th grade-second problem)
Şekil 15. Parça belirleyici işlemsel bölüm stratejisi (5. sınıf-ikinci problem)

There were 8 groups that overcame this problem by using *regrouping* strategy. The variety of sharing methods reached the highest point here. Four groups broke each shape into 3 equal pieces, and got 18 pieces in total. Therefore every person got 2 pieces, meaning that share of every person was 2/3 (Figure 16a). One group cut each shape into 6 equal pieces, obtained 36 pieces totally, delivered 4 pieces to everybody, and came up to 4/6 (Figure 16b). Partitioning each shape into 9 pieces, 3 groups gave 6 pieces to each person from overall 54 pieces (Figure 16c).

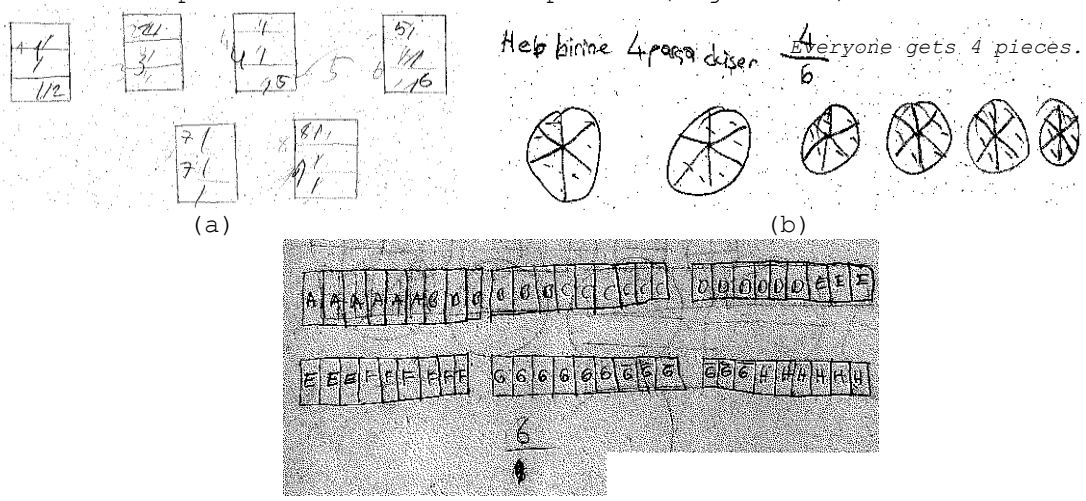


Figure 16. Regrouping strategy (5th grade-second problem)
Şekil 16. Yeniden gruplama stratejisi (5. sınıf-ikinci problem)

One group's answer to this problem was quite complex and hard to put into any category. Students in this group unequally divided each shape into 6 parts and thought that everybody has 4 parts if all parts are distributed fairly. They used *regrouping* and *horizontal partitioning* strategies together, and they did not succeed in writing fraction name of each share (Figure 17).

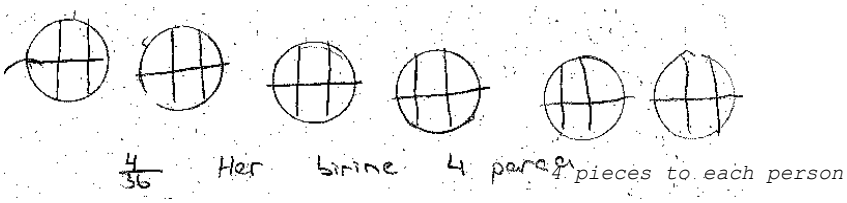
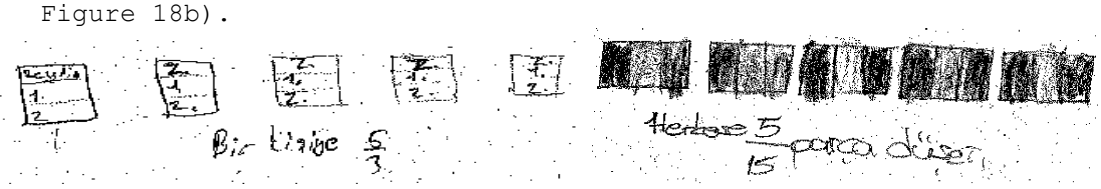


Figure 17. Common use of *regrouping* and *horizontal partitioning* strategies (5th grade-second problem)
Şekil 17. *Yeniden gruplama* ve *yatay bölüştürme* stratejilerinin beraber kullanımı (5. sınıf-ikinci problem)

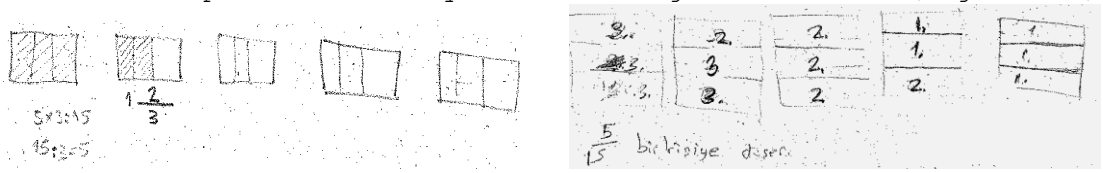
- **Third problem:** Regardless of wrong or unrelated answers, all student solutions for this problem indicated usage of 4 strategies: Two of them (*partitive quotient foundational* and *partition and quantify by part-whole notion*) were preferred by only one group. (Figure 18a and Figure 18b).



(a) (b)

Figure 18. *Partitive quotient foundational* and *partition and quantify by part-whole notion* strategies (5th grade-third problem)
Şekil 18. *Parça belirleyici temel bölüm* ve *parça-bütün fikri ile bölüştürme ve niceliğini belirtme* stratejileri (5. sınıf-üçüncü problem)

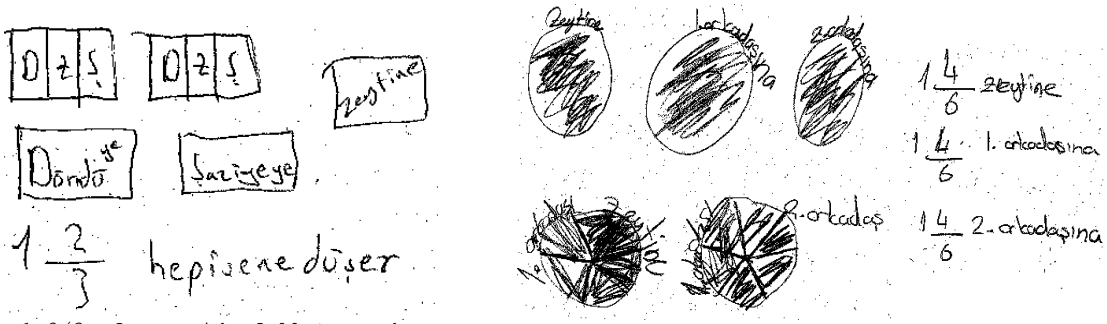
Most widely adopted strategy was *regrouping* again and 3 out of 6 groups that utilized this strategy did not need to complete all sharing procedure to determine quantity of each share (Figure 19a). The other 3 groups went on sharing until last piece, but they were distracted by the number of total piece while they were deciding the fraction (Figure 19b).



(a) (b)

Figure 19. Regrouping strategy (5th grade-third problem)
Şekil 19. Yeniden gruplama stratejisi (5. sınıf-üçüncü problem)

Lastly, 3 groups put forward a solution for this problem correspondent with *whole to each person then half the remaining objects between half the people* strategy. As fourth graders did, first one whole were delivered to each person, and *partitive quotient foundational* or *regrouping* strategy was implemented for the last two whole (Figure 20a and Figure 20b)



1 2/3 of one pide fall to each persons share.

(a) (b)

Figure 20. Whole to each person then half the remaining objects between half the people strategy (5th grade-third problem)

Şekil 20. Herkese bir bütün sonra kalan nesnelerin yarısını kişilerin yarısına paylaştırma stratejisi (5. sınıf-üçüncü problem)

5. DISCUSSION AND CONCLUSIONS (TARTIŞMA VE SONUÇLAR)

The overall goal of the present study was to increase our knowledge about fourth and fifth grade students' partitioning strategies based a previously developed taxonomy. When low percents of wrong or unrelated answers (approximately 26% and 24% for fourth and fifth grade, respectively) are taken into consideration, first thing that can be said about the results is that most of the fourth and fifth graders can put forward strategies for partitioning problems by using their insights. This result of the present study is parallel with the findings of the previous studies carried out with younger children [12, 14, 15, 17, 18], but it was justified for older children in this study. Charles and Nason [17] established taxonomy for partitioning strategies, but they did not investigate what the most widely used strategies are and what the levels of students in the sense of abstraction of fraction construction are. It has done in this study, and findings showed that only 9 percent of students at fourth grade and 7 percent of students at fifth grade used Class 1 strategies which are most proper for conceptual mapping and abstraction. In addition, it was found that most popular strategies (*regrouping*, *partition and quantify by part-whole notion*, and *whole to each person then half the remaining objects between half the people* strategies for each grade) were under categories of Class 2 and Class 3. Students rarely referred to Class 4 strategies. Another point which should be mentioned here is accordance between strategies determined by Charles and Nason [17] and strategies used by participants of the current study. However, students brought up different ideas in the present study as summarized in the following:

- Some students benefited from decimals to quantify each share directly. It has been thought within *proceduralized partitive quotient* strategy, but maybe it could have been labeled as a separate strategy.
- Beside of horizontal lines, students in this study often used vertical lines on circular or elliptic shapes while they were sharing objects. These solutions were also considered in the scope of *horizontal partitioning* strategy.
- Students' solutions for second problem showed that if number of objects being shared and number of sharers have common factors more than one, *regrouping* strategy can be used in different ways. Especially fifth graders displayed this variety in their solutions.
- Some answers were an indicator of simultaneous use of two different strategies, which made solution difficult to put under any title.
- Solutions for third problem revealed a different strategy which was not mentioned in other studies. Students used a version of *whole to each person then half the remaining objects between half the people* strategy. They first distributed a whole to each person in accordance with strategy, but the procedure of sharing remaining objects was different. They did not use halves after this point; instead, they preferred to use one of *partitive quotient foundational* or *regrouping* strategies. Therefore, it can be said that two different strategies was used successively here, or a new title can be written for this thinking style.

As a result of all different points listed above, some changes on the taxonomy such as adding new strategies, or enlarging scope of extant strategies may be suggested. It seems that one source of these differences are grade levels which were dealt with here, since fourth and fifth grade students are supposed to be more sophisticated, experienced about

partitioning when compared to younger pupils. But, when implications for teaching are considered, there is a deficiency that this research pointed out: Since lack of emphasizing on partitioning problems, fourth and fifth graders have low abstraction level of fractions, which make generating mathematical constructs difficult for them. To remove this deficiency, it should be allocated more time and effort for partitioning problems in curriculum from the beginning of the first grade.

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