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Optimal control processes associated with a class of stochastic sequential dynamical systems based on a parameter

Muhammet CANDAN[1]

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Abstract — This paper examines the optimal control processes represented by stochastic sequential dynamic systems involving a parameter obtained by unique solution conditions concerning constant input values. Then, the principle of optimality is proven for the considered process. Afterwards, the Bellman equation is constructed by applying the dynamic programming method. Moreover, a particular set defined as an accessible set is established to show the existence of an optimal control problem. Finally, it is discussed the need for further research.

Subject Classification (2020): 49L20, 93E20

1. Introduction

One of the most developing areas of current discrete systems theory is sequential dynamical systems theory. Sequential dynamic systems (SDS) are a mathematical modelling tool used to describe the behaviour of a system or to detect whether the desired state has been reached from step-by-step events. SDS provides the opportunity to define dynamic mathematical models for many problems in engineering, especially informatics.

In recent years, the study of optimal control processes in the general binary-dynamic system framework has gained interest [1-5]. Optimal control problems described by differential equations with continuous and discrete right-hand sides and correspond to differential inclusions are broadly used in modern engineering and technology [6-9].

An essential case of the aforesaid binary-dynamic system is presented with a stochastic sequential dynamic system based on a parameter λ (SSDSλ). So far, many significant real-world applications of the system theory have been conducted [3, 8,10-16]. SSDSλ is one of the rapidly growing fields of discrete system theory [3,7]. It plays a crucial role in many areas, such as synthesizing finite dynamic systems, imitation modelling problems, coding theory, identification problems, the base of computer science, and diagnosing problems in discrete events. Thus, it should be studied intensively.

[1]muhammetcandan@esenyurt.edu.tr (Corresponding Author)
[1]Department of Computer Engineering, Faculty of Engineering and Architecture, Istanbul Esenyurt University, Istanbul, Turkey
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The paper is devoted to introducing and investigating the fundamental properties of SSDSλ, obtaining the complete solution conditions, and determining the necessity and sufficiency conditions for discrete optimal processes given by SSDSλ.

In the present study, Section 2 provides some fundamental definitions and hypothesises concerning SSDSλ. Section 3 contains the main findings and investigates discrete optimal processes given by nonlinear stochastic binary dynamical systems. Moreover, it obtains Bellman's equation via dynamic programming. Section 4 establishes a particular set defined as an accessible set to show optimal control and proves the existence theorem. The last section discusses the need for further research.

2. Stochastic Sequential Dynamical System Based on Parameter

SSDSλ is a general class of finite sequential dynamic systems [3]. Despite the similarity of characteristics, SSDSλ contains a particular determined parameter. General notation of this system is defined by [4]:

$$K = \langle X, S, Y, s^0, p(\omega), F_v(.), G\rangle, v = 1,2,\dots k \tag{2.1}$$

where $X = [GF(2)]^r, S = [GF(2)]^m$, and $Y = [GF(2)]^q$ are input, state, and output index (alphabet) respectively; s^0 is the initial state vector, $p(\omega)$ is deterministic discrete probability distribution ($\Omega = \{\omega_1, \omega_2, \dots, \omega_p\}$ is a finite set, $p(\omega) = \{p(\omega_i): \omega_i \in \Omega, \sum_{\omega_i\in\Omega} p(\omega_i) = 1\}$), characteristic Boolean vector functions [16], known as transfer functions and denoted by $F_v(\cdot) = \{F_{v_1}(\cdot), \dots, F_{v_m}(\cdot)\}$, are nonlinear functions defined on the set $\mathbb{Z}^k \times [GF(2)]^m \times [GF(2)]^r$, G is an output characteristic function, defined on a Galois field $GF(2)$, and the symbol (.) denotes $(c, s(c), x(c))$ for simplicity.

In addition to the definition, SSDSλ is formulated by:

$$\begin{gathered} \xi_v s(c) = F_v\big(c, s(c), x(c), \omega(c), \lambda(c)\big), v = 1,2, \dots k \\ y(c) = G(s(c)) \end{gathered} \tag{2.2}$$

where $s(c)$, $x(c)$, and $y(c)$ are m, r, and q dimensional state, input, and output vectors at the point c respectively, $\omega(c)$ is a random variable and $\lambda(c)$ is an arbitrary parameter. Here,

$$c = (c_1, c_2, \dots, c_k) \in G_d = \{c \mid c \in \mathbb{Z}^k, c_1^0 \le c_1 \le c_1^{L_1}, \dots, c_k^0 \le c_k \le c_k^{L_k}, c_i \in \mathbb{Z}\}$$

is point in $\mathbb{Z}^k$, determining position L_i, ($i = 1,2, \dots, k$ positive integer) is the duration of the stage i of this process [10]. $\mathbb{Z}$ is a set of integers and $\xi_v s(c)$ is a shift operator [2, 10] defined as follows:

$$\xi_v s(c) = s(c + e_v), e_v = (0, \dots, 0, \overset{v}{1}, 0, \dots, 0), v = 1,2, \dots, k$$

Since the considered dynamic system equation is over-determined, the condition of the unique solution is to be satisfied. Therefore, it is assumed that the parameter $\lambda(c)$, depending on initial and other necessary states in the proof of optimality condition, holds the condition of the unique solution for the fixed input value. We can construct the condition of the unique solution by the following theorem:

Theorem 2.1. There exists a unique solution for the given system in (2.2) if and only if,

$$\begin{gathered} F_v(c + e_\mu, F_\mu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\mu), \omega(c + e_\mu), \lambda(c + e_\mu)) \\ = F_\mu\Big(c + e_v, F_v\big(c, s(c), x(c), \omega(c), \lambda(c)\big), x(c + e_v), \omega(c + e_v), \lambda(c + e_v)\Big), v, \mu = 1,2, \dots, k \end{gathered} \tag{2.3}$$

for every fixed $x(c)$ and arbitrary parameter $\lambda(c)$.

Proof. (Necessity) By the definition of shift operator,

$$\xi_\nu s(c) = s(c + e_\nu)$$

If we apply the shift operator to both sides of the above equality,

$$\xi_\mu \xi_\nu s(c) = \xi_\mu s(c + e_\nu) = s(c + e_\nu + e_\mu) \tag{2.4}$$

Similarly,

$$\xi_\nu \xi_\mu s(c) = s(c + e_\mu + e_\nu) \tag{2.5}$$

Since the process of summation is commutative,

$$s(c + e_\nu + e_\mu) = s(c + e_\mu + e_\nu) \tag{2.6}$$

then, by considering (2.4), (2.5), and (2.6),

$$\xi_\mu \xi_\nu s(c) = \xi_\nu \xi_\mu s(c) \tag{2.7}$$

Moreover, if we consider (2.2), then

$$\begin{aligned} \xi_\mu \xi_\nu s(c) &= \xi_\nu s\big(c + e_\mu\big) \\ &= F_\nu(c + e_\mu, s(c + e_\mu), x(c + e_\mu), \omega(c + e_\mu), \lambda(c + e_\mu)) \\ &= F_\nu(c + e_\mu, F_\mu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\mu), \omega(c + e_\mu), \lambda(c + e_\mu)) \end{aligned} \tag{2.8}$$

In other words,

$$\xi_\mu \xi_\nu s(c) = F_\nu(c + e_\mu, F_\mu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\mu), \omega(c + e_\mu), \lambda(c + e_\mu)) \tag{2.9}$$

By similar calculating,

$$\xi_\nu \xi_\mu s(c) = F_\mu(c + e_\nu, F_\nu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\nu), \omega(c + e_\nu), \lambda(c + e_\nu)) \tag{2.10}$$

Therefore, by (2.7), (2.9), and (2.10),

$$\begin{gathered} F_\nu(c + e_\mu, F_\mu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\mu), \omega(c + e_\mu), \lambda(c + e_\mu)) \\ = F_\mu\Big(c + e_\nu, F_\nu\big(c, s(c), x(c), \omega(c), \lambda(c)\big), x(c + e_\nu), \omega(c + e_\nu), \lambda(c + e_\nu)\Big), \nu, \mu = 1,2, \dots, k \end{gathered} \tag{2.11}$$

This completes the proof of necessity.

(**Sufficiency**) Suppose that the following equalities are satisfied:

$$\begin{gathered} (F_\nu^0(c,\cdot))(s) = s, (F_\nu^r(c,\cdot))(s) = F_\nu\underbrace{(c, F_\nu(c, \dots, F_\nu(c, s, x, \omega, \lambda)\dots))}_{rtimes} \\ (F_1(c^1,\cdot) \otimes F_2(c^2,\cdot) \otimes \dots \otimes F_m(c^m,\cdot))(s) = F_1(c^1, F_2(c^2, \dots, F_m(c^m, s, x, \omega, \lambda)\dots)) \end{gathered} \tag{2.12}$$

and let $K(c^1, c^2, \dots c^L)$ be the piecewise curve connecting the points $(c^1, c^2, \dots c^L)$. If we consider the over this curve, then

$$\pi(s) = \prod_{k(c^1,\dots,c^L)} \left(F_1{}^{\Delta l_1}\big(l, x(l), \omega(l), \lambda(l)\big) \otimes \dots \otimes F_k{}^{\Delta l_k}(l, x(l), \omega(l), \lambda(l))\right)(s)$$

$$= \prod_{i=1}^{L-1} \left(F_1^{c_1^{i+1}-c_1^i}\big(c^i, x(c^i), \omega(c^i), \lambda(c^i)\big) \otimes \dots \otimes F_k^{c_k^{i+1}-c_k^i}\big(c^i, x(c^i), \omega(c^i), \lambda(c^i)\big)\right)(s) \quad (2.13)$$

where the symbol $\otimes$ denotes modulo two multiplications on the Galois field. It is clear that the value of $\pi(s)$ is not only dependent on the initial and terminal point of $K(c^1, c^2, \dots c^L)$ but also on the points $c^i (1 < i < L)$.

To determine the value of $\pi(s)$ dependent only on the initial and terminal point of the curve $K(c^1, c^2, \dots c^L)$, it suffices to have (2.3). Thus,

$$\pi(s) = \prod_{(c^1,c^L)} \left(F_1{}^{\Delta l_1}\big(l, x(l), \omega(l), \lambda(l)\big) \otimes \dots \otimes F_k{}^{\Delta l_k}(l, x(l), \omega(l), \lambda(l))\right)(s) \quad (2.14)$$

If we consider the following function and equality:

$$s(c, c^0, s^0, x(c), \omega(c), \lambda(c)) = \prod_{(c^0,c)} \left(F_1{}^{\Delta l_1}(l, x(l), \omega(l), \lambda(l)) \otimes \dots \otimes F_k{}^{\Delta l_k}(l, x(l), \omega(l), \lambda(l))\right)(s^o) \quad (2.15)$$

If we apply shift operator to the considered function in (2.15)

$$\xi_v s(c, c^0, s^0, x(c), \omega(c), \lambda(c)) = (F_v(c, c^0, s^0, x(c), \omega(c), \lambda(c)) \otimes s(c, c^0, s^0, x(c), \omega(c), \lambda(c)))(s^0)$$

$$= F_v(c, s(c, c^0, s^0, x(c), \omega(c), \lambda(c))) \quad (2.16)$$

which means that s(c) is the unique solution for the given system in (2.2) ■

Each random variable ω poses a specific mission in SSDSλ. For instance, $\omega$is an input variable in the identification problem of the SSDS. However, ω is a set of all possible states to synthesise the optimal sequential dynamic system.

Furthermore, the system's state depends on the random variable ω which affects not only the parameters of the SSDSλ but also the input variable.

Finally, equation (2.2) is converted to:

$$\xi_v s(c) = F_v\left(c, s(c), x(c) \dot{\oplus} \omega(c), \lambda(c)\right), v = 1,2,\dots,k \quad (2.17)$$

where symbol $\dot{\oplus}$ means that $x \dot{\oplus} \omega$ is always in input alphabet X.

The discrete optimal processes governed by SSDSλ are characterized by functional:

$$\bar{J}(x) = M_\omega\{\varphi(s(c^L))\} \quad (2.18)$$

where $M_\omega\{.\}$ is a mathematical expected value of ω.

3. Optimal Control Process and Principle of Optimality

This section interests in the optimal control problem provided in [7] for processes represented by SSDSλ. The main goal herein is to determine a control $x(c) \in X$ [15] such that the functional in (2.18) takes a minimum value when starting from the known initial "departure" state s^0 to any desired arrival state $s^*(c^L)$ by L steps. That is,

$$\xi_\nu s(c) = \hat{F}_\nu\big(c, s(c), x(c), \omega(c), \lambda(c)\big), c \in G_d, \nu = 1,2, \dots k \tag{3.1}$$

$$s(c^0) = s^0, x(c) \in X, c \in G_d \tag{3.2}$$

$$\begin{aligned} &\hat{F}_\nu(c + e_\mu, \hat{F}_\mu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\mu), \omega(c + e_\mu), \lambda(c + e_\mu)) \\ &= \hat{F}_\mu(c + e_\nu, \hat{F}_\nu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\nu), \omega(c + e_\nu), \lambda(c + e_\nu)) \end{aligned} \tag{3.3}$$

$$\bar{J}(x) = M_\omega\{\varphi(s(c^L)\} \to \min \tag{3.4}$$

where $\hat{F}_\nu(.)(\nu = 1, \dots, k)$ denotes the pseudo-Boolean expression of the Boolean vector function [16] $\hat{F}(.)(\nu = 1, \dots, k)$ and $L = L_1 + L_2 + \dots + L_k$ is the time duration of this process.

Dynamic programming is an efficacious method for the solution of optimal control processes. Therefore, (3.1)-(3.4) can be formulated as an optimal problem via this method:

$$\xi_\nu s(c) = \hat{F}_\nu(c, s(c), x(c), \omega(c), \lambda(c)), c \in G_d(\sigma) \tag{3.5}$$

$$s(\sigma) = \aleph \tag{3.6}$$

$$x(c) \in X, c \in G_d(\sigma) \tag{3.7}$$

$$\begin{aligned} &\hat{F}_\nu(c + e_\mu, \hat{F}_\mu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\mu), \omega(c + e_\mu), \lambda(c + e_\mu)) = \\ &\hat{F}_\mu(c + e_\nu, \hat{F}_\nu(c, s(c), x(c), \omega(c), \lambda(c)), x(c + e_\nu), \omega(c + e_\nu), \lambda(c + e_\nu)) \end{aligned} \tag{3.8}$$

$$\bar{J}(x) = M_\omega\{\varphi(s(c^L))\} \to \min \tag{3.9}$$

where $\aleph$ is an arbitrary element in S. Taking into consideration (3.5)-(3.9), if we substitute $\sigma = c^0$or $\aleph = s^0$ into (3.5)-(3.9), we reach the first problem stated above. If the conditions for the existence of a unique solution are satisfied, then for the given initial condition $s(\sigma) = \aleph$ and given $x(c)(c \in G_d(\sigma))$, we obtain a unique $s(c)$. That is, the functional (3.9) is the function of the parameters $\aleph$ and $x(c)(c \in G_d(\sigma))$:

$$\bar{J}(x) = \bar{J}(\aleph, x(G_d(\sigma)) \tag{3.10}$$

where $x(G_d(\sigma))$ denotes the range of the control $x(c)$ on the points $c \in G_d(\sigma)$:

$$x(G_d(\sigma)) = \{x(c), c \in G_d(\sigma)\} \tag{3.11}$$

From the unique solution condition of the system (19), the stochastic process can be investigated in the set $G_d(\sigma)$, and also $G_{d_1}(\sigma)$ is as follows:

$$G_{d_1}(\sigma) = \{c; c_1^0 \le c_1 < \sigma_1, \dots, c_k^0 \le c_k < \sigma_k\} \tag{3.12}$$

Definition 3.1. Control $x(c), (c \in G_d(\sigma))$ which minimizes the functional (2.18) in the problem (3.5)-(3.9) is defined as an optimal control concerning the initial pair $(\sigma, \aleph)$ on the region $G_d(\sigma)$.

Theorem 3.1. Let $x^0(c)$ be an optimal control concerning the initial pair (c^0, s^0) and $s^0(c)$ be possible optimal trajectory on the region G_d. Then, $x^0(c)$ is an optimal control corresponding to the initial pair $(\sigma, s^0(\sigma))$ on the region $G_d(\sigma)$ for every G_d.

Proof. Assume the contrary. Then, there exists $x(c) \in X, c \in G_d(\sigma)$ such that following inequality holds,

$$\overline{J}(\aleph, x(G_d(\sigma)) < \overline{J}(\aleph, x^0(G_d(\sigma)) \tag{3.13}$$

Therefore, we can select a new control process $\tilde{x}(c), c \in G_d$ as follows:

$$\tilde{x}(c) = \begin{cases} x^0(c), c \in G_{d_1}(\sigma) \\ x(c), c \in G_d(\sigma) \end{cases} \tag{3.14}$$

It is concluded that (3.14) is an admissible control such that

$$\overline{J}(s^0, \tilde{x}(G_d)) = \overline{J}(s^0, \tilde{x}(G_{d_1}(\sigma) \cup G_d(\sigma)) \tag{3.15}$$

According to the condition, $s^0(\sigma) = \aleph$. Hence,

$$\begin{aligned} \overline{J}(s^0, \tilde{x}(G_{d_1}(\sigma) \cup G_d(\sigma)) &= \overline{J}(s^0(\sigma), \tilde{x}(G_d(\sigma)) \\ &= \overline{J}(\aleph, x(G_d(\sigma))) < \overline{J}(\aleph, x^0(G_d(\sigma))) \\ &= \overline{J}(s(\sigma), x^0(G_d(\sigma))) = \overline{J}(s^0, x^0(G_d)) \end{aligned} \tag{3.16}$$

and with the help of (3.15) and (3.16), we can obtain

$$\overline{J}(s^0, \tilde{x}(G_d)) < \overline{J}(s^0, x^0(G_d)) \tag{3.17}$$

Hence, (3.17) contradicts the hypothesis that the control $x^0(c), c \in G_d$ is optimal. This completes the proof.

Let a function (for every fixed σ and $\aleph$) be corresponding to the optimal value of pseudo-Boolean functional in the problem (3.5)-(3.9):

$$B(\sigma, \aleph, \lambda) = \min M_\omega\{\varphi(s(c^L))\} \tag{3.18}$$

where minimization on the set of possible control $x(c), c \in G_d(\sigma)$.

We try to apply the method of dynamical programming known as Bellman's equation [17] to $B(\sigma, \aleph, \lambda)$. Suppose that $x^0(c), c \in G_d$is the possible control concerning (3.5)-(3.9) with initial condition and $s^0(c), c \in G_d(\sigma)$ is also the optimal trajectory. Let the point $\xi_v s \in G_d(\sigma)(v = 1,2, \ldots, k)$ and any element $y(c) \in X$be specified. If $x(\sigma) = y(c)$, then the state of the system in the point $\xi_v \sigma$ is given by

$$s(\xi_v \sigma) = \hat{F}_v(\sigma, \aleph, y, \omega(\sigma), \lambda(\sigma)) \tag{3.19}$$

We mention the following problem:

$$\xi_v s(c) = \hat{F}_v(c, s(c), x(c), \omega(c), \lambda(c)), c \in G_d(\xi_v \sigma) \tag{3.20}$$

$$s(\xi_v \sigma) = \hat{F}_v(\sigma, \aleph, y, \omega(\sigma), \lambda(\sigma)) \tag{3.21}$$

$$x(c) \in X(c), c \in G_d(\xi_v \sigma) \tag{3.22}$$

$$\overline{J}(x) = M_\omega\{\varphi(s(c^L))\} \to \min \tag{3.23}$$

If $\hat{y}(c), c \in G_d(\xi_v \sigma)$ and $\hat{s}(c), c \in G_d(\xi_v \sigma)$ are optimal control and corresponding optimal trajectory respectively, then

$$M_\omega\{\varphi(\hat{s}(c^L))\} = B\big(\xi_v \sigma, \hat{F}_v(\sigma, \aleph, y, \lambda(\sigma), \omega(\sigma))\big) \tag{3.24}$$

can be found. For (3.5)-(3.9), let $\tilde{x}(c)$ be a possible control below.

$$\tilde{x}(c) = \begin{cases} y, c = \sigma \\ \hat{y}(c), c \in G_d(\xi_\nu \sigma) \end{cases} \tag{3.25}$$

Also, $\tilde{s}(c)$ can be obtained

$$\tilde{s}(c) = \begin{Bmatrix} \hat{F}_\nu(\sigma, \aleph, y, \omega(\sigma), \lambda(\sigma)), c = \sigma \\ \hat{s}(c), c \in G_d(\xi_\nu \sigma) \end{Bmatrix} \tag{3.26}$$

It is clear that the value of $\overline{J}(x) = M_\omega\{\varphi(s(c^L))\}$ to control $\tilde{x}(c)$ is determined by

$$M_\omega\{\varphi(\tilde{s}(c^L))\} = M_\omega\{\varphi(\hat{s}(c^L))\} = B(\xi_\nu \sigma, \hat{F}_\nu(\sigma, \aleph, y, \lambda(\sigma), \omega(\sigma))) \tag{3.27}$$

Since $\tilde{x}(c), c \in G_d(\sigma)$ is not optimal control, we can show

$$M_\omega\{\varphi(\tilde{s}(c^L))\} \geq M_\omega\{\varphi(s^0(c^L))\} = B(\sigma, \aleph, \lambda) \tag{3.28}$$

Thus,

$$B(\sigma, \aleph, \lambda) \leq B(\xi_\nu \sigma, \hat{F}_\nu(\sigma, \aleph, y, \lambda(\sigma), \omega(\sigma))) \tag{3.29}$$

Furthermore, if $y(c) = x^0(\sigma)$, then by the principle of optimality [6],

$$\hat{y}(c) = x^0(c)(c \in G_d(\xi_\nu \sigma)) \tag{3.30}$$

Therefore,

$$B(\sigma, \aleph, \lambda) = B(\xi_\nu \sigma, \hat{F}_\nu(\sigma, \aleph, x^0(\sigma), \omega(\sigma), \lambda^0(\sigma))) \tag{3.31}$$

By (3.29) and (3.30), Bellman's equation can be determined by

$$B(\sigma, \aleph, \lambda) = \min B\left(\xi_\nu \sigma, \hat{F}_\nu\big(\sigma, \aleph, y, \lambda(\sigma), \omega(\sigma)\big)\right), \aleph \in S \tag{3.32}$$

4. Determination of accessible set

We consider the optimal control problem (3.5)-(3.9); for each fixed $x(c)$, the system satisfies unique solution conditions. Since the considered problem satisfies unique solution conditions, the solution of the system is independent of the discrete curve's form. Therefore, the solution of nonlinear multi-parameter discrete equations system can be investigated along the discrete curve as follows:

$$\hat{L} = \hat{L}(c^0, c^1, \ldots, c^{L_1}, c^{L_1+1}, \ldots, c^{L_1+L_2}, \ldots, c^L) \tag{4.1}$$

where $c^i = (c_1^0 + i, c_2^0, \ldots, c_k^0)$, $i = \overline{0, L_1}$; $c^{L_1+i} = (c_1^0 + L_1, c_2^0 + i, c_3^0, \ldots, c_k^0)$, $i = \overline{0, L_2}$; $c^{L_1+L_2+i} = (c_1^0 + L_1, c_2^0 + L_2, c_3^0 + i, c_4^0, \ldots, c_k^0)$, $i = \overline{0, L_3}$; ...; $c^{L_1+L_2+\ldots+L_\nu+i} = (c_1^0 + L_1, \ldots, c_\nu^0 + L_\nu, c_{\nu+1}^0 + i, c_{\nu+2}^0, \ldots, c_k^0)$, $i = \overline{0, L_{\nu+1}}$; $\nu = \overline{1, k-1}$, and $L = L_1 + L_2 + \ldots + L_k$.

From the initial condition, $s(c^0) = s^0$, where s,

$$S = \{s(c) = (s_1(c), s_2(c), \ldots, s_m(c)), c \in Z^k\} = [GF(2)]^m \tag{4.2}$$

We can set a sequence of sets to indicate the existence of optimal control for the considered problem. After analysing this system, all possible state sets which can be reached from the initial state s^0 along the discrete curve $\hat{L} = \hat{L}(c^0, c^1, \ldots, c^{L_1}, c^{L_1+1}, \ldots, c^{L_1+L_2}, \ldots, c^L)$ by one step, can be determined by

$$R_1(s^0) = \{s: s = F_1(c^0, s(c^0), x(c^0), \omega(c^0), \lambda(c^0)), s(c^0) \in S\} \tag{4.3}$$

Similarly, we can establish the set which can be achieved in two steps:

$$R_{\eta_1}^1(s^0) = \begin{Bmatrix} s: s = F_1(c^{\eta_1 - 1}, s(c^{\eta_1 - 1}), x(c^{\eta_1 - 1}), \omega(c^{\eta_1 - 1}), \lambda(c^{\eta_1 - 1})), s(c^{\eta_1 - 1}) \\ \in R_{\eta_1 - 1}(s^0), x(c^{\eta_1 - 1}) \in \hat{X} \end{Bmatrix} \tag{4.4}$$

By applying the same method,

$$R_{\eta_\nu}^\nu(s^0) = \begin{Bmatrix} s: s = F_\nu(c^{\eta_\nu - 1}, s(c^{\eta_\nu - 1}), x(c^{\eta_\nu - 1}), x(c^{\eta_\nu - 1}), \omega(c^{\eta_\nu - 1}), \lambda(c^{\eta_\nu - 1})), s(c^{\eta_\nu - 1}) \\ \in R_{\eta_\nu - 1}^\nu(s^0), x(c^{\eta_\nu - 1}) \in \hat{X} \end{Bmatrix}, \tag{4.5}$$

$$\eta_\nu = \overline{1, L_\nu} \nu = \overline{1, k}$$

Thus, $R_{\eta_\nu}^\nu(s^0)$ is the state obtained from the initial state s^0 by discrete curve $\hat{L} = \hat{L}(c^0, c^1, \ldots, c^L)$ in $L_1 + L_2 + \ldots + L_\nu$ steps. The last one is called an accessible set.

We concluded that every possible optimal control is only dependent on state $s(c^L)$. So, we can convert the optimal control problem to find the minimal value of $J(x) = M_\omega\{\varphi(s(c^L))\}$ in accessible set $R_{\eta_\nu}^\nu(s^0) = R_L(s^0)$. It is seen that the accessible set is not given, but it is obtained depending on the initial state with the help of a nonlinear stochastic dynamic system.

Theorem 4.1. Assume that nonlinear binary stochastic dynamic system has a unique solution for every fixed control $x(c)$, and this system is strictly dependent on state sets. Then, $x^* = \{x^*(c^0), x^*(c^1), \ldots, x^*(c^{L-1})\}$ is optimal control for desired initial state $s(c^0) = s^0$.

Proof. Since possible optimal control depends only on the final state of the system, by utilizing accessible sets, the considered optimal control problem can be converted to finding the minimal value of pseudo-Boolean functional $\varphi(s(c^L))$ in the accessible set $R_L(s^0)$.

Since the nonlinear stochastic binary dynamic system has a unique solution, the value of discussed functional $J(x) = M_\omega\{\varphi(s(c^L))\}$ is independent of discrete curve connecting point c^0 and point c^L at point $s(c^L)$.

Moreover, by the condition of strict dependence,

$$R_{\hat{L}}(s(c^0)) = S = [GF(2)]^m \tag{4.6}$$

can be obtained. So,

$$M_\omega\{\varphi(s(c^L))\} = J(s(c^0), x(c^1), \ldots, x(c^{L-1})) \tag{4.7}$$

it is clear that functional pseudo-Boolean is defined as uniquely on binary sets with dimensional $m + rL$ and takes a finite value for desired initial condition $s(c^0)$. Since $R_{\hat{L}}(s^0)$ is a bounded set, the minimum value of $M_\omega\{\varphi(s(c^L))\}$ is determined concerning the possible $x(c^k)$; $k = \overline{0, L-1}$, $x(c^k) \in [GF(2)]^r$. This completes the proof.

5. Conclusions

It is shown that the parameter λ in accordance with the fixed input variable is obtained from the statement of the unique solution condition. Therefore, it is observed that the determined parameter λ plays a vital role in the case of statements that are used in the proof of optimality principle and optimal control problem. Moreover, the Bellman equation is obtained by applying the dynamic programming method, which is the solution method of optimal control problem for multi-step stochastic discrete processes. The results obtained in the paper can be applied in the research and calculation of optimal nonlinear impulsive systems, in error-correcting coding theory, in optimal control problems of multidimensional systems, in integer programming.

In future studies, methods that provide the exact solution of the multi-parameter linear stochastic binary equations system, which are necessary for the solution of the optimal controlling problem, can also be investigated. Approximations can be made to obtain the numerical solutions of the optimal control problems gained for the processes represented by SSDS.

Author Contributions

The author read and approved the last version of the manuscript.

Conflicts of Interest

The author declares no conflict of interest.

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