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NUMERICAL COMPUTATION OF MAIN WATER DEPTHS IN OGEE-CRESTED SPILLWAYS

ABSTRACT

As a component of dam structures in civil engineering, the spillway is one of the most important hydraulic structure with respect to dam safety. Hydraulic design of spillways is usually provided by using design curves and trial-and-error procedures. Therefore, one has to apply time-consuming trial-error method. Also, derived equations for determining of flow depths are nonlinear. However, water depths may be solved by utilizing an iterative technique commonly referred to as Newton-Raphson's iteration technique used to solve the system of nonlinear equations. In this paper, a numerical computation model based on utilizing modified Newton-Raphson technique with Chebyshev approximation to determine main water depths in ogee-crested spillway structure is introduced. An example for given the numerical computation model is presented.
Keywords: Spillway, Water Depth, Newton-Raphson Technique, Dam, Chebyshev Approximation

OGEE PROFİLLİ DOLUSAVAKDA TEMEL SU DERİNLİKLERİNİN SAYISAL HESAPLANMASI

ÖZET

Dolusavaklar, baraj emniyeti açısından baraj yapılarının en önemli öğelerinden biridir. Dolusavakların hidrolik tasarımı bazı grafikler ve deneme-yanılma yöntemleri yardımı ile yapılmaktadır. Bu nedenle, tasarım faaliyetleri uzun zaman alan ve deneme-yanılma gibi yorucu çalışmaları gerektirmektedir. Ayrıca, temel akım derinliklerini hesaplamak için çıkarılan denklemler lineer olmayan denklemlerdir. Newton-Raphson metodu bu denklemleri çözmek için kullanılabilecek etkin sayısal metotlardandır. Bu çalışmada, Chebyshev yaklaşımı ile modife edilen Newton-Raphson tekniği kullanılarak ogee profilli dolusavaklarda temel su derinliklerinin hesaplanmasını ön gören bir hesap modeli sunulmaktadır. Ayrıca, verilen sayısal hesap modeli bir örnek üzerinde tartışılmaktadır.
Anahtar Kelimeler: Dolusavak, Su Derinliği, Newton-Raphson Tekniği, Baraj, Chebyshev Yaklaşımı

1. INTRODUCTION (GİRİŞ)

In dam engineering, extreme water or flood discharge which is not wanted for storage in reservoir requires to convey from upstream to downstream. For this aim, a hydraulic structure called spillway is constructed. A spillway that is a component of a dam serves to evacuate the flood wave from the reservoir without damaging the structure and environment. In spillway, water flows over a spillway crest and falls freely to the downstream. Flow conditions are similar to the case over a sharp crested weir. The ideal spillway would take the form of the underside of the nappe (i.e., ogee profile) of a sharp crested weir when flow rate corresponds to the maximum design capacity of the spillway.

A spillway structure generally consists of four units. These are approach channel, spillway sill called ogee crest, channel (chute), and energy dissipation structure such as stilling basin or flip bucket. In literature, information related to spillway structure is available (e.g., USBR, 1987; USACE, 1990). Hydraulic design equations of spillways are derived from nonlinear equation form according to hydraulic analysis of spillway structures. The spillway design is exactly completed when hydraulic calculations of these units step by step are completed.

Therefore, hydraulic design of spillways is usually provided by using design curves, trial-and-error procedures (Chow, 1973; French, 1985; Chaudhry, 1993). But, hydraulic calculations are required tiring efforts and long time for “trial-error” method and using design curves (see USBR, 1987).

Computer applications related to channel, dam, bridge, and other hydraulic structures are increasing nowadays. This increased use of computer to solve engineering problems poses new challenges to the designer (e.g., engineer, student). Many of the existing design techniques may be conveniently computerized through use of numerical analysis techniques (e.g., Newton-Raphson iteration solution and its modified forms such as Chebyshev approximation). Conte and de Boor (1987) defined an algorithmic approach related to numerical analysis. Also, Griffiths and Smith (1991) summarized some numeric methods for engineers.

The Newton-Raphson method is suitable for solving of nonlinear flow equations in open channel. A number of authors have found this method to be suitable for solving the gradually varied flow equation (GVF), e.g., Henderson (1966), Fread and Harbaugh (1971), French (1985), Paine (1992) and Rhodes (1995, 1998). Recently, Dey (2000) used the Chebyshev approximation as aid in computing GVF. Moreover, Bagatur (2005) applied this method for analysis of flow profile in a stilling basin.

In this paper, a numerical computation model based on utilizing the Newton-Raphson technique to determine main water depths in ogee-crested spillway structure is developed. This computational model is applied by computer programs. A computer programs which have Visual Basic (VBA) code were prepared under the Microsoft Excel Macro. The main flow depths both in the approach channel and downstream channel were estimated by modified Newton-Raphson technique with Chebyshev approximation implemented in computer program prepared with Visual Basic computer language (Appendix 1). Also, numerical example is presented.

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

The spillway is most important hydraulic structure with respect to dam safety. Hydraulic design of spillways is usually provided by using design curves and trial-and-error procedures. Therefore, one has to apply time-consuming trial-error method. Also, derived equations for determining of flow depths are nonlinear. However, water depths may be solved by utilizing an iterative technique commonly referred to as Newton-Raphson's iteration technique used to solve the system of nonlinear equations. In this paper, a numerical computation model based on utilizing modified Newton-Raphson technique with Chebyshev approximation to determine main water depths in ogee-crested spillway structure is introduced. This study has practical importance for engineers and students in spillway design.

3. DETERMINATION OF DISCHARGE COEFFICIENT FOR SPILLWAY CREST (DOLUSAVAK KRETİ İÇİN DEBİ KATSAYISININ BELİRLENMESİ)

Design discharge of a spillway can be determined by integrating the velocity distribution over the cross-sectional flow area on the spillway from the crest to the free surface.

Kindsvater and Carter (1959) presented a form of the discharge equation for either a fully or partially contracted sharp-crested vertical weir. Figure 1 shows a longitudinal cross-section of a spillway. Design discharge of a spillway can be determined by integrating the velocity distribution over the cross-sectional flow area on the spillway from the crest to the free surface. The empirical relationship between the flow rate and upstream head for a sharp-crested weir/spillway is given by:

$$Q = C_o \ L \ H_o^{3/2} \quad (1)$$

where Q is the design discharge of the spillway which can be determined from the reservoir routing performed for a design inflow hydrographic having a certain recurrence interval, C_o is the spillway discharge coefficient selected from graphs given by US Bureau of Reclamation (USBR, 1987), L is the effective length of spillway crest and H_o is the total head above the crest excluding the velocity head.

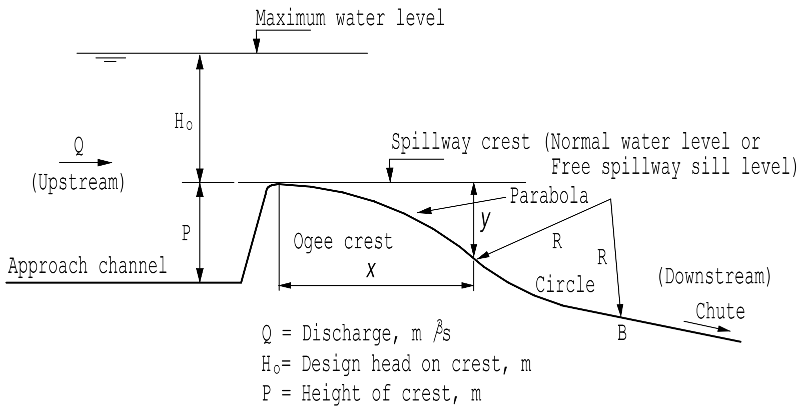

Figure 1. Longitudinal cross-section of uncontrolled spillway crest
(Şekil 1. Kontrolsüz dolusavak kretinin boyuna kesiti)

The design discharge coefficient for ogee-crested spillways can be selected from design curves or charts given by USBR (1987). But, the figures may give reading difficulties. Therefore, if they is defined in equation form, using curve equations of graphs will present practical skills. For this aim, regression analysis of curves in graphs is carried out in polynomial form.

In order to computerize solutions, regression equations for spillway discharge coefficients are presented in Tables 1 and 2. These equations are similar to those of given by Yanmaz (1997).

Table 1. Regression equations for spillway discharge coefficient of vertical faced ogee crest
(Tablo 1. Menba yüzü düşey olan ogee kretinin dolusavak debi katsayısı için regresyon denklemleri)

Number	Case	Equation
1	$\frac{P}{H_o} < 1.2$	$C_o = 1.74 + 1.84\ (\frac{P}{H_o}) - 4.03\ (\frac{P}{H_o})^2 + 4.91\ (\frac{P}{H_o})^3 - 3.12\ (\frac{P}{H_o})^4 + 0.8\ (\frac{P}{H_o})^5$
2	$1.2 \leq \frac{P}{H_o} < 2.8$	$C_o = 2.34 - 0.49\ (\frac{P}{H_o}) + 0.44\ (\frac{P}{H_o})^2 - 1.62\ (\frac{P}{H_o})^3 + 0.02\ (\frac{P}{H_o})^4$
3	$\frac{P}{H_o} \geq 2.8$	$C_o = 2.18$

Table 2. Regression equations for spillway discharge coefficient of inclined upstream faced ogee crest
(Tablo 2. Menba yüzü meyilli ogee kretinin dolusavak debi katsayısı için regresyon denklemleri)

Number	Discharge coefficient ratio	Slope of spillway upstream face	Equation
1	$\frac{C_{inclined}}{C_{vertical}} = \frac{C_m}{C_o}$	1:3	$\frac{C_m}{C_o} = 1.013 - 0.022\ (\frac{P}{H_o}) + 0.017\ (\frac{P}{H_o})^2 - 0.005\ (\frac{P}{H_o})^3$
2		2:3	$\frac{C_m}{C_o} = 1.034 - 0.052\ (\frac{P}{H_o}) + 0.032\ (\frac{P}{H_o})^2 - 0.008\ (\frac{P}{H_o})^3$
3		3:3	$\frac{C_m}{C_o} = 1.058 - 0.137\ (\frac{P}{H_o}) + 0.108\ (\frac{P}{H_o})^2 - 0.030\ (\frac{P}{H_o})^3$

In the spillway design, firstly head over the spillway and height of crest are determined. Then, slope of spillway upstream face (vertical/horizontal) is selected. The design discharge coefficient for vertical faced ogee-crested spillway can be determined with regression equations (see Table 1) given as a function of the spillway height divided by the total head, (P/H_o).

4. OGEE CREST PROFILE (OGEE KRET PROFİLİ)

The ideal shape of ogee-crested spillway profile which has parabolic geometry form is defined by:

$$\frac{y}{H_o} = -K\ (\frac{x}{H_o})^n \qquad (2)$$

where H_o is the total head above the crest, x is total horizontal length of parabolic geometry, y is total vertical height of parabolic geometry and, K and n are equation coefficients. These values can be determined from design charts given by USBR (1987) or regression equations presented in Tables 3 and 4 depending on h_a, flow velocity head in upstream (i.e., $V^2/2g$).

Table 3. Regression equations for values of K to be used in ogee profile
(Tablo 3. Ogee profilinin çıkarılmasında kullanılan K değerleri için regresyon denklemleri)

Number	Slope of spillway upstream face	Equation coefficient	Equation
1	Vertical and 1:3	K	$K = 0.499 + 0.305 \left(\frac{h_a}{H_o}\right) - 1.567 \left(\frac{h_a}{H_o}\right)^2 - 3.977 \left(\frac{h_a}{H_o}\right)^3$
2	2:3		$K = 0.526 + 0.159 \left(\frac{h_a}{H_o}\right) - 1.299 \left(\frac{h_a}{H_o}\right)^2 - 2.636 \left(\frac{h_a}{H_o}\right)^3$
3	3:3		$K = 0.539 + 0.0108 \left(\frac{h_a}{H_o}\right) - 0.214 \left(\frac{h_a}{H_o}\right)^2 - 5.498 \left(\frac{h_a}{H_o}\right)^3$

Table 4. Regression equations for values of n to be used in ogee profile
(Tablo 4. Ogee profilinin çıkarılmasında kullanılan n değerleri için regresyon denklemleri)

Number	Slope of spillway upstream face	Equation coefficient	Equation
1	Vertical	n	$n = 1.871 - 0.560 \left(\frac{h_a}{H_o}\right) + 1.789 \left(\frac{h_a}{H_o}\right)^2 + 0.756 \left(\frac{h_a}{H_o}\right)^3$
2	1:3		$n = 1.851 - 0.589 \left(\frac{h_a}{H_o}\right) + 2.317 \left(\frac{h_a}{H_o}\right)^2 - 1.083 \left(\frac{h_a}{H_o}\right)^3$
3	2:3		$n = 1.800 - 0.723 \left(\frac{h_a}{H_o}\right) + 3.976 \left(\frac{h_a}{H_o}\right)^2 - 5.195 \left(\frac{h_a}{H_o}\right)^3$
4	3:3		$n = 1.781 - 0.639 \left(\frac{h_a}{H_o}\right) + 3.377 \left(\frac{h_a}{H_o}\right)^2 - 3.300 \left(\frac{h_a}{H_o}\right)^3$

5. THE UPSTREAM FLOW DEPTH BEHIND SPILLWAY SILL STRUCTURE (DOLUSAVAK EŞİK YAPISININ ARKASINDAKİ MEMBA AKIM DERİNLİĞİ)

In spillway hydraulic calculations, the upstream flow depth (d) behind spillway sill structure is firstly determined. Figure 2 shows longitudinal cross-section of spillway structure system.

According to hydraulic assumptions and analysis made, the upstream flow depth (d) is computed by Equation (3) for rectangular cross-sections. In any case, the energy applied to sections 1 and 2 (see Figure 2) gives:

$$f(d) = (d + \frac{Q^2}{d^2\ L^2\ 2g}) - (H_o + P) \qquad (3)$$

where Q is design discharge, L is the effective length of spillway crest, P is height of crest, g is gravitational acceleration and H_o is the total head above the crest excluding the velocity head.

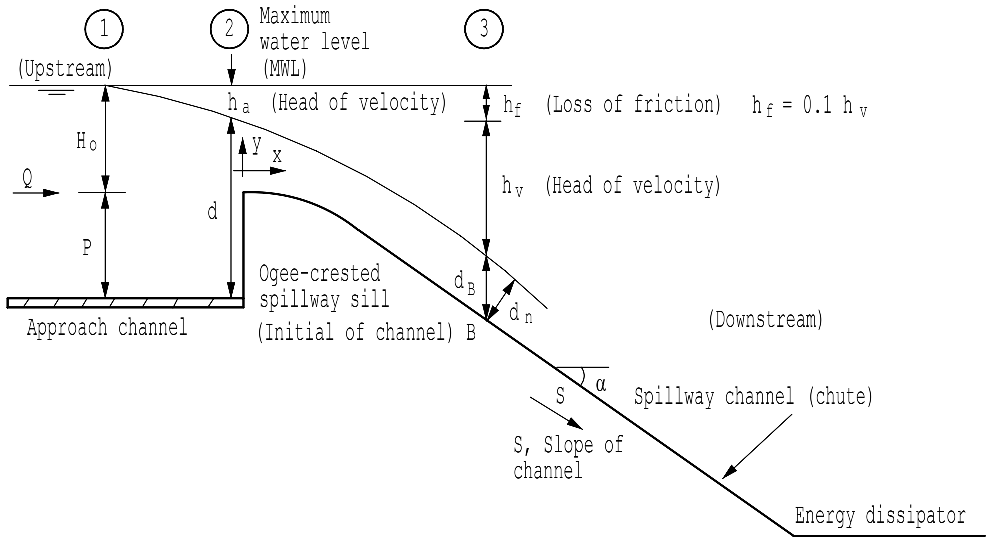

Figure 2. Longitudinal cross-section of spillway structure system
(Şekil 2. Dolusavak yapısının boyuna kesiti)

Equation (3) is nonlinear with respect to the unknown variable d. Since a direct solution of the this equation cannot be obtained, a numerical treatment must be applied. To determine the water depth, one has to apply time-consuming trial-error methods. But, Equation (3) can be solved for d by utilizing an iterative technique commonly referred to as Newton-Raphson's iteration technique which is used to solve the system of nonlinear equations.

6. THE INITIAL FLOW DEPTH FOR SPILLWAY CHANNEL (DOLUSAVAK KANALI İÇİN BAŞLANGIÇ AKIM DERİNLİĞİ)

After calculation of initial elevation of spillway channel (chute) is carried out by way of Equation (4) and the energy equation for section 3, initial flow depth (d_n) of spillway channel on point B is determined (see Figure 2). In Figure 2, h_f is the head loss between sections 1 and 3 which can be taken as approximately 10% of the velocity head at section 1.

The elevation of point B in downstream channel is:

$$(\text{Elevation})_B = \text{MWL} - (1.6)\ H_o \qquad (4)$$

$$f(d_n) = [\text{MWL} - (\text{Elevation})_B] - (d_n\ .\ \cos\alpha\ ) - 1.1\ (\frac{Q^2}{d_n^{\,2}\ L^2\ 2g}) \qquad (5)$$

where MWL is maximum water level, $(\text{Elevation})_B$ is elevation of point B in downstream channel, and α is angle of spillway channel.

Initial flow depth (d_n) of spillway channel on point B is solved with Equation (5). This equation is nonlinear as Equation (3) and can be solved by utilizing an iterative technique.

7. NEWTON-RAPHSON TECHNIQUE AND CHEBYSHEV SOLUTION (NEWTON-RAPHSON METODU VE CHEBYSHEV ÇÖZÜMÜ)

Newton-Raphson technique is an iterative procedure. The procedure is based on (i) guessing a first approximation and (ii) using an iterative process to successively refine the solution to as many degrees of accuracy as required. Suppose an expression y=f(x) is known and it is desired to find the value of x which will make y=0.

The basic iteration formula of this method derived from Taylor series is given by;

$$x_{(k+1)} = x_k - \frac{f(x_k)}{f'(x_k)} \qquad (6)$$

where the subscript k denotes the number of iterations. Also, the prime (')denotes a derivative (i.e., $f'(x_k) = df(x_k)/dx_k$). After choosing an appropriate starting value, x_k, and applying Equation (6) successively, the value of $x_{(k+1)}$ will be found. Convergence is attained when

$$\left|x_{(k+1)} - x_k\right| < E \qquad (7)$$

where E is error tolerance.

The Chebyshev appoximation is way to speed up convergence of the Newton's method. It has advantage of requiring less iteration. The Chebyshev iteration formula is:

$$x_{(k+1)} = x_k - \frac{f(x_k)}{f'(x_k)} - \left[\frac{f^2(x_k)\ f''(x_k)}{f'^3(x_k)}\right] \qquad (8)$$

The term in brackets is the modification of the Newton-Raphson form. In Equation (8), the prime (") denotes a second derivative (i.e., $f''(x_k) = d^2 f(x_k)/dx_k^2$).

8. APPLICATION OF CHEBYSHEV SOLUTION (CHEBYSHEV ÇÖZÜMÜNÜN UYGULANMASI)

Equation (3) is in the form f(d)=0. Therefore, application of Chebyshev solution can readily be utilized to solve for unknown, d. Thus,

$$d_{(k+1)} = d_k - \frac{f(d_k)}{f'(d_k)} - \left[\frac{f^2(d_k)\ f''(d_k)}{f'^3(d_k)}\right] \qquad (9)$$

where $f(d_k)$ is defined by Equation (3).

($d_k > P$) must be for choosing an appropriate starting value. For the Chebyshev solution, the following terms are required. Differentiating Equation (3) with respect to d_k once and then a second time, one can write as:

$$f'(d_k) = \left[1 - \frac{2\ Q^2}{d_k^{\ 3}\ L^2\ \ 2g}\right] \qquad (10)$$

$$f''(d_k) = \left[\frac{6\ Q^2}{d_k^{\ 4}\ L^2\ \ 2g}\right] \qquad (11)$$

Also, Equation (5) is in the form $f(d_n)=0$ for flow depth of spillway channel on point B. Therefore, application of the Newton-Raphson technique can readily be utilized to solve for unknown, d_n. Thus,

$$d_{(n+1)} = d_n - \frac{f(d_n)}{f'(d_n)} - \left[\frac{f^2(d_n)\ f''(d_n)}{f'^3(d_n)}\right] \qquad (12)$$

where $f(d_n)$ is defined by Equation (5). The expressions of $f'(d_n)$ and $f''(d_n)$ are given as:

$$f'(d_n) = \left[\frac{2.2\ Q^2}{d_n^3\ L^2\ 2g} - \cos\alpha\right] \quad (13)$$

$$f''(d_n) = \left[-\frac{6.6\ Q^2}{d_n^4\ L^2\ 2g}\right] \quad (14)$$

9. ILLUSTRATIVE EXAMPLE (AÇIKLAYICI ÖRNEK)

An example provided in this section illustrates the use of the proposed iterative method.

Design discharge,	$Q=202.00\ m^3/s$
Channel width,	$L=35.00$ m
Total head,	$H_o=1.96$ m
Height of crest,	$P=1.00$ m
Longitudinal slope of the downstream,	$S=0.09$ ($\cos\alpha=0.99597$)
Selected error tolerance,	E= 0.000001

i) Calculate discharge coefficient for vertical crest face.
ii) Calculate the water depth, d, in approach channel.
iii) Calculate K and n coefficients of ogee profile.
iv) Calculate the initial water depth, d_n, in downstream channel of spillway.

Solution (Çözüm)

- $\frac{P}{H_o}=0.51$, i.e., $\frac{P}{H_o} < 1.2$, therefore, Table 1 is applied for discharge coefficient of vertical faced ogee crest, then C_o= 2.098 from line 1 or number 1.
- Using Equation (9) depending on Equations (3), (10), (11) and (7), then d= 2.73 m.
- $V=\frac{Q}{A}$ and $V=\frac{Q}{(d\ L)}$= 2.11 m/s, thus h_a = $(\frac{V^2}{2g})$=0.23 m. Using h_a, if Tables 3 and 4 are applied, then K= 0.508 and n=1.831 from line 1 or number 1.
- Using Equation (5) depending on Equations (12), (13), (14) and (7), then d_n= 0.88 m.

9. CONCLUSIONS (SONUÇLAR)

In this paper, a numerical computation model based on utilizing modified Newton-Raphson technique with Chebyshev approximation to determine main water depths in ogee-crested spillway structure is introduced. An example for given the numerical computation model is presented. In practical, using of presented model poses new challenges to the designer (e.g., engineer, student) related to spillway design in dam engineering practice.

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NOTATIONS (SİMGELER)

B	Initial point of spillway channel (chute)
C_o	Spillway discharge coefficient
C_m	Spillway discharge coefficient of inclined ogee crest
d	Upstream flow depth, m
d_b	Flow depth in point B, m
d_n	Normal flow depth, m
E	Error tolerance
$(\text{Elevation})_B$	Elevation of point B in downstream channel
$f(x_k)$	Function
$f'(x_k)$	First derivative
$f''(x_k)$	Second derivative
g	Gravitational acceleration
H_o	Project head, m
h_a	Flow velocity head in upstream, m
h_f	Losses of friction, m
h_v	Flow velocity head in downstream, m
K	Equation coefficient for parabolic geometry
L	Effective length of spillway crest, m
MWL	Maximum water level
n	Equation coefficient for parabolic geometry
P	Height of crest, m
R	Radius of circle between parabola and chute ($R \geq 5\ d_n$)
S	Slope of spillway channel
Q	Design discharge, m^3/s
x	Total horizontal length of parabolic geometry, m
V	Flow velocity, m/s
y	Total vertical length of parabolic geometry, m
α	Angle of channel, deg.

APPENDIX-I (EK-1)

For determination of main water depths in ogee-crested spillway structure, a computer program which has Visual Basic (VBA) code may be prepared under the Microsoft Excel Macro.

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'Computer program for modified Newton-Raphson technique with Chebyshev approximation
'f (d)=the root function, f '(d)=the fist derivative, f ''(d)=the second derivative of function
'd=unknown variable, d1=equation root
'N=iteration number, Er=error tolerance
    Er=0.00001
    d=0.01
    For N = 1 To 20
    A= f (d)
    B= f '(d)
    C= f ''(d)
    d1= d - (A / B)-[(A^2-C)/(B)^3]
    If Abs(d1 - d) <= Er Then Exit For
    d= d1
    Next N
    Range("a10").Select
    ActiveCell.FormulaR1C1 = "Iteration number, N="
    Range("c10").Select
    ActiveCell.FormulaR1C1 = N
    Range("a12").Select
    ActiveCell.FormulaR1C1 = "Result, d ="
    Range("c12").Select
    ActiveCell.FormulaR1C1 = d1
    End Sub
```