Variational Iteration and Homotopy Perturbation Method for Solving a Three-Species Food Chain Model

Mehmet MERDAN[1], Tahir KHANİYEV[2]

[1] *Karadeniz Technical University, Engineering Faculty of Gümüşhane ,Civil Engineering, 29000, Gümüşhane*
[2] *TOBB University of economics and technology Faculty of Letters and Sciences Department of mathematics, 06000, Ankara, Turkey*

Abstract

In this article, homotopy perturbation method and variational iteration method are implemented to give approximate and analytical solutions of nonlinear ordinary differential equation systems such as a three-species food chain model. Homotopy perturbation is compared the variational iteration method for a three-species food chain model. The variational iteration method is predominant than the other non-linear methods, such as perturbation method. In this method, in general Lagrange multipliers are constructed by correction functionals for the systems. Multipliers can be identified by the variational theory. Some plots are presented to show the reliability and simplicity of the methods.

Key Words: Variational iteration method; Homotopy perturbation method; a three-species food chain model

Üç Canlı Türünün oluşturduğu besin zinciri modelinin varyasyonel iterasyon ve homotopy perturbation yöntemi ile çözümü

Özet

Bu makalede, üç canlı türünün oluşturduğu besin zincri modeli gibi lineer olmayan adi diferensiyel denklem sistemlerinin yaklaşık analitik çözümlerini elde edebilmek için homotopy perturbation ve varyasyonel iterasyon yöntemleri uygulandı. Homotopy perturbation yöntemi varyasyonel iterasyon yöntemi ile mukayese edildi. Varyasyonel iterasyon yöntemi perturbation yöntemi olarak bilinen diğer non lineer yöntemlerden daha üstündür. Bu yöntemde genelde Lagrange çarpanları sistemler için düzeltme fonksiyoneli ile elde edildi. Çarpanlar varyasyonel teori ile belirlendi. Yöntemlerin doğruluğunu göstermek için birkaç tane grafik sunuldu.

Anahtar kelimeler: Varyasyonel iterasyon yöntemi, Homotopy perturbation yöntemi; üç canlı türünün oluşturduğu besin zinciri modeli

1. Introduction

Dynamics of chaotic dynamical systems as a three-species food chain model is examined at the study [2]. The components of the basic tree-component model are proportional to the population density of the lowest trophic level species (prey), the middle trophic level species (intermediate predator) and highest trophic level species (top predator)is denoted respectively by $x(t)$, $y(t)$ and $z(t)$ .These quantities satisfy

$$\begin{cases} \dfrac{dx}{dt} = x(a_1 - b_1 x - c_1 y) \\ \dfrac{dy}{dt} = y(-a_2 + c_2 x - d_1 z) \\ \dfrac{dz}{dt} = z(-a_3 + d_2 y) \end{cases} \quad (1)$$

with the initial conditions:

$$x(0) = N_1, \quad y(0) = N_2, \quad z(0) = N_3.$$

Throughout this paper, we set

$a_1 = 4, a_2 = 0.3, a_3 = 0.15, b_1 = 0.01,$
$c_1 = 14, c_2 = 0.3, d_1 = 0.1, d_2 = 0.05.$

The motivation of this paper is to extend the application of the analytic homotopy-perturbation method (HPM) and variational iteration method[3,6–14,19,21] to solve the Lorenz system (1). The homotopy perturbation method (HPM) was first proposed by Chinese mathematician He [16–22]. The variational iteration method, which was proposed originally by He [13] in 1999, has been proved by many authors to be a powerful mathematical tool for various kinds of nonlinear problems.

2. Variational iteration method

According to the variational iteration method [10], we consider the following differential equation:

$$Lu + Nu = g(x), \quad (2)$$

where L is a linear operator, N is a non-linear operator, and $g(x)$ is an inhomogeneous term. Then, we can construct a correct functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda \begin{Bmatrix} Lu_n(s) \\ +N\tilde{u}_n(s) - g(s) \end{Bmatrix} ds, \quad (3)$$

where λ is a general Lagrangian multiplier [9–11], which can be identified optimally via variational theory. The second term on the right is called the correction and $\tilde{u}_n$ is considered as a restricted variation, i.e., $\delta \tilde{u}_n = 0$.

3. Homotopy perturbation method

To illustrate the homotopy perturbation method (HPM) for solving non-linear differential equations, He [21, 22] considered the following non-linear differential equation:

$$A(u) = f(r), \quad r \in \Omega \quad (4)$$

subject to the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \quad (5)$$

where A is a general differential operator, B is a boundary operator, f(r) is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal vector drawn outwards from Ω. The operator A can generally be divided into two parts M and N. Therefore, (3) can be rewritten as follows:

$$M(u) + N(u) = f(r), \quad r \in \Omega \quad (6)$$

He [22,23] constructed a homotopy $v(r, p) : \Omega x[0,1] \to \Re$ which satisfies

$$\begin{aligned} H(v, p) &= (1-p)\left[M(v) - M(u_0)\right] \\ &+ p\left[A(v) - f(r)\right] = 0, \end{aligned} \quad (7)$$

which is equivalent to

$$\begin{aligned} H(v, p) &= M(v) - M(u_0) \\ &+ pM(v_0) + p\left[N(v) - f(r)\right] = 0, \end{aligned} \quad (8)$$

where $p \in [0,1]$ is an embedding parameter, and u_0 is an initial approximation of (4). Obviously, we have

$$\begin{aligned} H(v, 0) &= M(v) - M(u_0) = 0, \\ H(v, 1) &= A(v) - f(r) = 0. \end{aligned} \quad (9)$$

The changing process of p from zero to unity is just that of H(v,p) from $M(v)-M(v_0)$ to $A(v)-f(r)$. In topology, this is called deformation and $M(v)-M(v_0)$ and $A(v)-f(r)$ are called homotopic. According to the homotopy perturbation method, the parameter p is used as a small parameter, and the solution of Eq. (7) can be expressed as a series in p in the Form

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \ldots \qquad (10)$$

When $p \to 1$, Eq. (7) corresponds to the original one, Eqs. (6) and (10) become the approximate solution of Eq. (6), i.e.,

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \ldots \qquad (11)$$

The convergence of the series in Eq. (11) is discussed by He in [21, 22].

4. Applications

In this section, we will apply the homotopy perturbation method to nonlinear ordinary differential systems (1).

4.1. Homotopy Perturbation Method to a Three-Species Food Chain Model

According to homotopy perturbation method, we derive a correct functional as follows:

$$\begin{aligned}
&(1-p)(\dot v_1 - \dot x_0) + p(\dot v_1 + v_1(-a_1 + b_1 v_1 + c_1 v_2)) = 0,\\
&(1-p)(\dot v_2 - \dot y_0) + p(\dot v_2 + v_2(a_2 - c_2 v_1 + d_1 v_3)) = 0, \quad (12)\\
&(1-p)(\dot v_3 - \dot z_0) + p(\dot v_3 + v_3(a_3 - d_2 v_2)) = 0,
\end{aligned}$$

where "dot" denotes differentiation with respect to t, and the initial approximations are as follows:

$$\begin{aligned}
&v_{1,0}(t) = x_0(t) = x(0) = N_1,\\
&v_{2,0}(t) = y_0(t) = y(0) = N_2, \qquad (13)\\
&v_{3,0}(t) = z_0(t) = z(0) = N_3.
\end{aligned}$$

and

$$\begin{aligned}
&v_1 = v_{1,0} + pv_{1,1} + p^2 v_{1,2} + p^3 v_{1,3} + \ldots,\\
&v_2 = v_{2,0} + pv_{2,1} + p^2 v_{2,2} + p^3 v_{2,3} + \ldots, \quad (14)\\
&v_3 = v_{3,0} + pv_{3,1} + p^2 v_{3,2} + p^3 v_{3,3} + \ldots,
\end{aligned}$$

where $v_{i,j}, i, j = 1,2,3,\ldots$ are functions yet to be determined. Substituting Eqs.(13) and (14) into Eq. (12) and arranging the coefficients of "p" powers, we have

$$\begin{aligned}
&\left(\dot v_{1,1} + N_1\left(-a_1 + b_1 N_1 + c_1 N_2\right)\right)p +\\
&\left(\dot v_{1,2} - a_1 v_{1,1} + 2b_1 N_1 + c_1\left(N_1 v_{2,1} + N_2 v_{1,1}\right)\right)p^2\\
&+\begin{pmatrix} \dot v_{1,3} - a_1 v_{1,2} + b_1\left(2N_1 v_{1,2} + {v_{1,1}}^2\right) + \\ c_1\left(N_1 v_{2,2} + v_{1,1} v_{2,1} + N_2 v_{1,2}\right) \end{pmatrix} p^3 + \ldots = 0,\\
&\left(\dot v_{2,1} + a_2 N_2 - c_2 N_1 N_2 + d_1 N_2 N_3\right)p\\
&+\begin{pmatrix} \dot v_{2,2} + a_2 v_{2,1} - c_2\left(N_1 v_{2,1} + v_{1,1} N_2\right) \\ +d_1\left(N_3 v_{2,1} + v_{3,1} N_2\right) \end{pmatrix} p^2\\
&+\begin{pmatrix} \dot v_{2,3} + a_2 v_{2,2} - c_2\left(N_1 v_{2,2} + v_{1,2} N_2 + v_{1,1} v_{2,1}\right) \\ +d_1\left(N_3 v_{2,2} + v_{3,2} N_2 + v_{3,1} v_{2,1}\right) \end{pmatrix} p^3 + \ldots = 0, \qquad (15)\\
&\left(\dot v_{3,1} + a_3 N_3 - d_2 N_2 N_3\right)p +\\
&\left(\dot v_{3,2} + a_3 v_{3,1} - d_2\left(N_3 v_{2,1} + N_2 v_{3,1}\right)\right)p^2\\
&+\left(\dot v_{3,3} + a_3 v_{3,2} - d_2\left(N_3 v_{2,2} + N_2 v_{3,2} + v_{3,1} v_{2,1}\right)\right)p^3 + \ldots = 0,
\end{aligned}$$

In order to obtain the unknowns $v_{i,j}(t), i, j = 1,2,3,$ we must construct and solve the following system which includes nine equations with nine unknowns, considering the initial conditions $v_{i,j}(0) = 0, i, j = 1,2,3,$

$$
\begin{aligned}
&\dot{v}_{1,1} + N_1\left(-a_1 + b_1 N_1 + c_1 N_2\right) = 0,\\
&\dot{v}_{1,2} - a_1 v_{1,1} + 2b_1 N_1 + c_1\left(N_1 v_{2,1} + N_2 v_{1,1}\right) = 0,\\
&\dot{v}_{1,3} - a_1 v_{1,2} + b_1\left(2N_1 v_{1,2} + v_{1,1}{}^2\right)\\
&\quad + c_1\left(N_1 v_{2,2} + v_{1,1}v_{2,1} + N_2 v_{1,2}\right) = 0,\\
&\dot{v}_{2,1} + a_2 N_2 - c_2 N_1 N_2 + d_1 N_2 N_3 = 0,\\
&\dot{v}_{2,2} + a_2 v_{2,1} - c_2\left(N_1 v_{2,1} + v_{1,1} N_2\right)\\
&+d_1\left(N_3 v_{2,1} + v_{3,1} N_2\right) = 0,\\
&\dot{v}_{2,3} + a_2 v_{2,2} - c_2\left(N_1 v_{2,2} + v_{1,2} N_2 + v_{1,1} v_{2,1}\right)\\
&+d_1\left(N_3 v_{2,2} + v_{3,2} N_2 + v_{3,1} v_{2,1}\right) = 0,\\
&\dot{v}_{3,1} + a_3 N_3 - d_2 N_2 N_3 = 0,\\
&\dot{v}_{3,2} + a_3 v_{3,1} - d_2\left(N_3 v_{2,1} + N_2 v_{3,1}\right) = 0,\\
&\dot{v}_{3,3} + a_3 v_{3,2} - d_2\left(N_3 v_{2,2} + N_2 v_{3,2} + v_{3,1} v_{2,1}\right) = 0.
\end{aligned}
\tag{16}
$$

From Eq. (11), if the three terms approximations are sufficient, we will obtain:

$$
\begin{aligned}
x(t) &= \lim_{p\to 1} v_1(t) = \sum_{k=0}^{2} v_{1,k}(t),\\
y(t) &= \lim_{p\to 1} v_2(t) = \sum_{k=0}^{2} v_{2,k}(t),\\
z(t) &= \lim_{p\to 1} v_3(t) = \sum_{k=0}^{2} v_{3,k}(t),
\end{aligned}
\tag{17}
$$

therefore

$$
x(t) = N_1 + N_1(a_1 - b_1N_1 - c_1N_2 - 2b_1N_1)t
+\frac{1}{2}\begin{bmatrix} -c_1(N_1(-a_2N_2 + c_2N_1N_2 - d_1N_2N_3 \\ +N_2N_1(a_1 - b_1N_1 - c_1N_2)) \\ a_1N_1(a_1 - b_1N_1 - c_1N_2) \end{bmatrix} t^2
$$

$$
y(t) = N_2 + \left(-a_2N_2 + c_2N_1N_2 - d_1N_2N_3\right)t
+\frac{1}{2}\begin{bmatrix} -a_2\left(-a_2N_2 + c_2N_1N_2 - d_1N_2N_3\right) \\ c_2(N_1(-a_2N_2 + c_2N_1N_2 - d_1N_2N_3 \\ +N_2N_1(a_1 - b_1N_1 - c_1N_2)) \\ d_1\begin{pmatrix} N_2(-\mathrm{a}_3N_3 + \mathrm{d}_2N_2N_3) \\ +N_3\left(-a_2N_2 + c_2N_1N_2 - d_1N_2N_3\right) \end{pmatrix} \end{bmatrix} t^2
$$

$$
z(t) = N_3 + (-\mathrm{a}_3N_3 + \mathrm{d}_2N_2N_3)t
+\frac{1}{2}\begin{bmatrix} -a_3(-\mathrm{a}_3N_3 + \mathrm{d}_2N_2N_3) \\ d_2\begin{pmatrix} N_2(-\mathrm{a}_3N_3 + \mathrm{d}_2N_2N_3) \\ +N_3\left(-a_2N_2 + c_2N_1N_2 - d_1N_2N_3\right) \end{pmatrix} \end{bmatrix} t^2
\tag{18}
$$

.
.
.

Here

$N_1 = 0.05,\ N_2 = 0.1$ and $N_3 = 0.5$ for the three-component model.

A few first approximations for $x(t),\ y(t)$ and $z(t)$ are calculated and presented below:

Three terms approximations:

$$
\begin{aligned}
x(t) &= 0.05 + .128975t + .179393t^2 + .1749542617t^3,\\
y(t) &= 0.1 - .335t + .007908375000t^2 + .0003896485417t^3,\\
z(t) &= 0.5 - 0.0725t + .0048375t^2 - .0001273052083t^3,
\end{aligned}
\tag{19}
$$

Four terms approximations:

$$x(t) = 0.05 + .128975t + .179393t^2 + .1749542617t^3 + .1310726688t^4,$$
$$y(t) = 0.1 - .335t + .007908375000t^2 + .0003896485417t^3 + .0009215028272t^4,$$
$$z(t) = 0.5 - 0.0725t + .0048375t^2 - .0001273052083t^3 - .000002156144535t^4,$$
(20)

Five terms approximations:

$$x(t) = 0.05 + .128975t + .179393t^2 + .1749542617t^3 + .1310726688t^4 + .1170764399t^5,$$
$$y(t) = 0.1 - .335t + .007908375000t^2 + .0003896485417t^3 + .0009215028272t^4 - .0003176445010t^5,$$
$$z(t) = 0.5 - 0.0725t + .0048375t^2 - .0001273052083t^3 - .000002156144535t^4 + .0000002081296537t^5,$$
(21)

Six terms approximations:

$$x(t) = 0.05 + .128975t + .179393t^2 + .1749542617t^3 + .1310726688t^4 + .1170764399t^5 + .07454676901t^6,$$
$$y(t) = 0.1 - .335t + .007908375000t^2 + .0003896485417t^3 + .0009215028272t^4 - .0003176445010t^5 + .00008868254926t^6,$$
$$z(t) = 0.5 - 0.0725t + .0048375t^2 - .0001273052083t^3 - .000002156144535t^4 + .0000002081296537t^5 + .00000001050446549t^6,$$
(22)

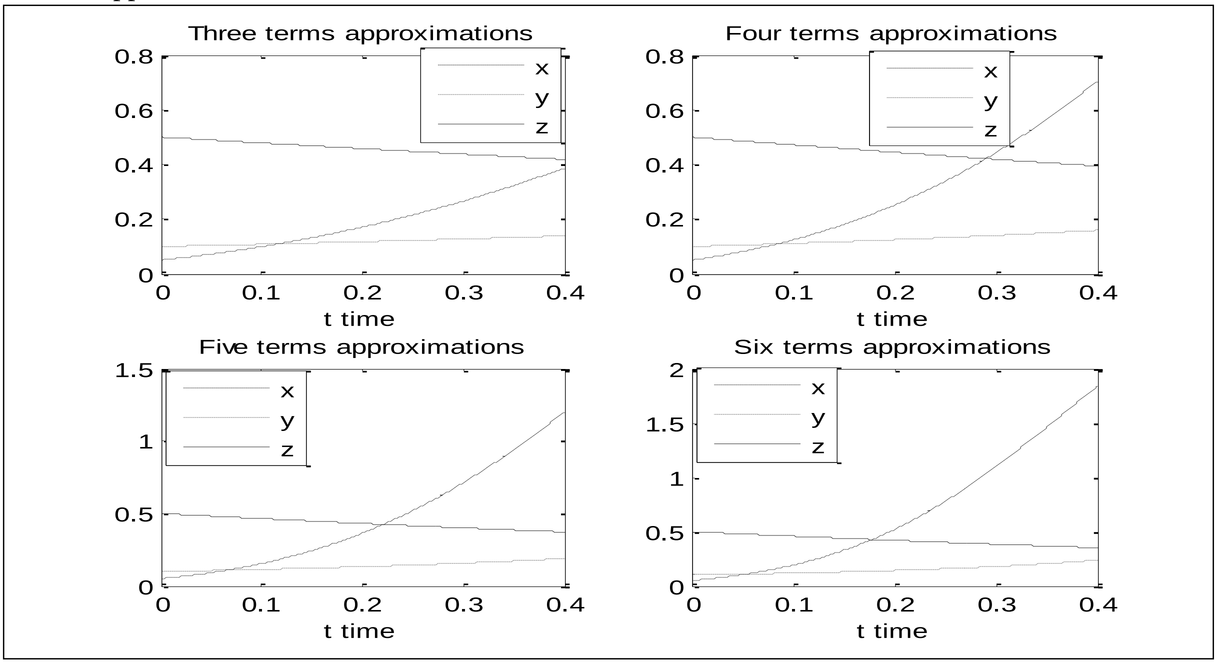

Figure. 1. Plots of three, four five and six terms approximations for a three-species food chain model

In this section, we will apply the variational iteration method to nonlinear ordinary differential systems (1).

4.2. Variational Iteration Method to a Three-Species Food Chain Model

According to the variational iteration method, we derive a correct functional as follows:

$$x_{n+1}(t)=x_n(t)+\int_0^t \lambda_1 \left\{ \begin{array}{l} x_n'(\xi) \\ -\tilde{x}_n(\xi)\begin{pmatrix} a_1-c_1\tilde{y}_n(\xi) \\ -b_1\tilde{x}_n(\xi)\end{pmatrix}\end{array}\right\} d\xi,$$

$$y_{n+1}(t)=y_n(t)+\int_0^t \lambda_2 \left\{ \begin{array}{l} y_n'(\xi) \\ -\tilde{y}_n(\xi)\begin{pmatrix} a_2-d_1\tilde{z}_n(\xi) \\ +c_2\tilde{x}_n(\xi)\end{pmatrix}\end{array}\right\} d\xi,$$

$$z_{n+1}(t)=z_n(t)+\int_0^t \lambda_3 \left\{ \begin{array}{l} v_n'(\xi) \\ -\tilde{z}_n(\xi)\begin{pmatrix} -a_3 \\ +d_2\tilde{y}_n(\xi)\end{pmatrix}\end{array}\right\} d\xi,$$

(23)

where λ_1, λ_2 and λ_3 are general Lagrange multipliers, $\tilde{x}_n(\xi), \tilde{x}_n(\xi)\tilde{y}_n(\xi), \tilde{x}_n(\xi)\tilde{z}_n(\xi), \tilde{y}_n(\xi)$ and $\tilde{z}_n(\xi)$ denote restricted variations, i.e.

$$\delta\tilde{x}_n(\xi)=\delta\,\tilde{x}_n(\xi)\tilde{y}_n(\xi)=\delta\tilde{x}_n(\xi)\tilde{z}_n(\xi)=\delta\tilde{y}_n(\xi)=\delta\,\tilde{z}_n(\xi)=0$$

Making the above correction functionals stationary, we can obtain following stationary conditions:

$$\begin{aligned} &\lambda_1'(\xi)=0,\\ &1+\lambda_1(\xi)\big|_{\xi=t}=0,\\ &\lambda_2'(\xi)=0,\\ &1+\lambda_2(\xi)\big|_{\xi=t}=0,\\ &\lambda_3'(\xi)=0,\\ &1+\lambda_3(\xi)\big|_{\xi=t}=0, \end{aligned} \qquad (24)$$

The Lagrange multipliers, therefore, can be identified as

$$\lambda_1=\lambda_2=\lambda_3=-1. \qquad (25)$$

Substituting Eq. (25) into the correction functional Eq. (23) results in the following iteration formula:

$$x_{n+1}(t)=x_n(t)-\int_0^t \left\{ x_n'(\xi)-x_n(\xi)\begin{pmatrix} a_1-c_1y_n(\xi) \\ -b_1x_n(\xi)\end{pmatrix}\right\} d\xi,$$

$$y_{n+1}(t)=y_n(t)-\int_0^t \left\{ y_n'(\xi)-y_n(\xi)\begin{pmatrix} a_2-d_1z_n(\xi) \\ +c_2x_n(\xi)\end{pmatrix}\right\} d\xi,$$

$$z_{n+1}(t)=z_n(t)-\int_0^t \left\{ z_n'(\xi)-z_n(\xi)\begin{pmatrix} -a_3 \\ +d_2y_n(\xi)\end{pmatrix}\right\} d\xi,$$

(26)

We start with initial approximations $x_0(t)=N_1$, $y_0(t)=N_2$ and $z_0(t)=N_3$. By the above iteration formula, we can obtain a few first terms being calculated:

$$\begin{aligned} x_1(t)&=N_1+\left[N_1a_1-N_1N_2c_1-N_1^2b_1\right]t,\\ y_1(t)&=N_2+\left[N_2a_2-N_2N_3d_1+N_1N_2c_2\right]t, \quad (27)\\ z_1(t)&=N_3+\left[N_2N_3d_2-N_3a_3\right]t, \end{aligned}$$

$$x_2(t) = N_1 + \left[N_1a_1 - N_1N_2c_1 - N_1^2b_1\right]t$$
$$+\begin{bmatrix} a_1 - c_1\left[N_2 + \left(N_2a_2 - N_2N_3d_1 + N_1N_2c_2\right)t\right] \\ -b_1\left[N_1 + \left(N_1a_1 - N_1N_2c_1 - N_1^2b_1\right)t\right] \end{bmatrix}t$$
$$y_2(t) = N_2 + \left[N_2a_2 - N_2N_3d_1 + N_1N_2c_2\right]t$$
$$+\left[N_2 + \left(N_2a_2 - N_2N_3d_1 + N_1N_2c_2\right)t\right]$$
$$\left[a_2 - d_1\left(N_2N_3d_2 - N_3a_3\right)t\right]$$
$$+c_2\left[N_1 + \left(N_1a_1 - N_1N_2c_1 - N_1^2b_1\right)t\right]t$$
$$z_2(t) = N_3 + \left[N_2N_3d_2 - N_3a_3\right]t$$
$$+\left[N_3 + \left(N_2N_3d_2 - N_3a_3\right)t\right]$$
$$\left[-a_3 + d_2\left[N_2 + \left(N_2a_2 - N_2N_3d_1 + N_1N_2c_2\right)t\right]\right]t$$
(28)

.
.
.

Continuing in this manner, we can find the rest of components.

A five terms approximation to the solutions are considered

$$x(t) \approx x_4,$$
$$y(t) \approx y_4,$$
$$z(t) \approx z_4.$$
(29)

This was done with the standard parameter values given above and initial values $N_1 = 0.05,\ N_2 = 0.1$ and $N_3 = 0.5$ for the three-component model.

A few first approximations for $x(t),\ y(t)$ and $z(t)$ are calculated and presented below:

$$x_1(t) = 0.05 + .129975t,$$
$$y_1(t) = 0.1 + .0265t,$$
$$z_1(t) = 0.5 - .0725t,$$
(30)

$$x_2(t) = 0.05 + .129975t + .319255025t^2 - .4838966001t^3,$$
$$y_2(t) = 0.1 + .053t + .01164675t^2 + .00122542625t^3,$$
$$z_2(t) = 0.5 - .145t + .011175t^2 - .0000960625t^3,$$
(31)

$$x_3(t) = 0.05 + .389925t + .957765075t^2 + .5796427849t^3$$
$$-.4075555774t^4 - .02137819435t^5$$
$$+.02721995825t^6 + .0008067558425t^7,$$
$$y_3(t) = 0.1 + .0795t + .03494025t^2 + .01867942075t^3$$
$$+.004968085852t^4 + .0004469168796t^5$$
$$-.0000529651676t^6 - .00001777761613t^7,$$
$$z_3(t) = 0.05 - .2175t + .033525t^2 - .00180951875t^3$$
$$-.00001026046875t^4 - .000002631284375t^5$$
$$+.0000006287661211t^6 - .5885875457\text{e-}8t^7,$$
(32)

$$
\begin{aligned}
x_4(t) = {} & 0.05 + .5199t + 1.91553015t^2 + 2.6089091t^3 \\
& -.1783383111t^4 - 2.313402444t^5 - .1685959067t^6 \\
& +.01504844683t^7 - .0752112378t^8 + .02741692556t^9 \\
& +.003377382416t^{10} - .000416217648t^{11} \\
& -.0001826910779t^{12} - .8079245347\text{e-}5t^{13} \\
& +.1231768053\text{e-}5t^{14} + .1942821896\text{e-}6t^{15}, \\
y_4(t) = {} & 0.1 + .106t + .0698805t^2 + .0673651255t^3 \\
& +.05474885791t^4 + .01588933851t^5 \\
& +.001779947402t^6 - .7581689025\text{e-}5t^7 \\
& -.1439143923\text{e-}2t^8 - .6191604829\text{e-}3t^9 \\
& -.7706325467\text{e-}4t^{10} + .9092589199\text{e-}5t^{11} \\
& +.4080603973\text{e-}5t^{12} + .1789294670\text{e-}6t^{13} \\
& -.2733507997\text{e-}7t^{14} - .4302669168\text{e-}8t^{15}, \\
z_4(t) = {} & 0.5 - .29t + .06705t^2 - .0066617t^3 \\
& +.4723919250\text{e-}3t^4 - .2870431373\text{e-}4t^5 \\
& -.1373536750\text{e-}4t^6 - .1373536750\text{e-}4t^7 \\
& +.4203809187\text{e-}6t^8 + .6018226760\text{e-}7t^9 \\
& -.2531359036\text{e-}7t^{10} + .1727511554\text{e-}8t^{11} \\
& +.2867688755\text{e-}10t^{12} + .5422381735\text{e-}12t^{13} \\
& -.5433108178\text{e-}12t^{14} + .5231841723\text{e-}14t^{15},
\end{aligned}
$$

(33)

These results obtained by homotopy perturbation method, three, four, five and six terms approximations for $x(t)$, $y(t)$ and $z(t)$ are calculated and presented follow. These results are plotted in Figure 2. The Homotopy perturbation method was tested by comparing the results with the results of the Variational iteration method.

These results are plotted in Figure 2.

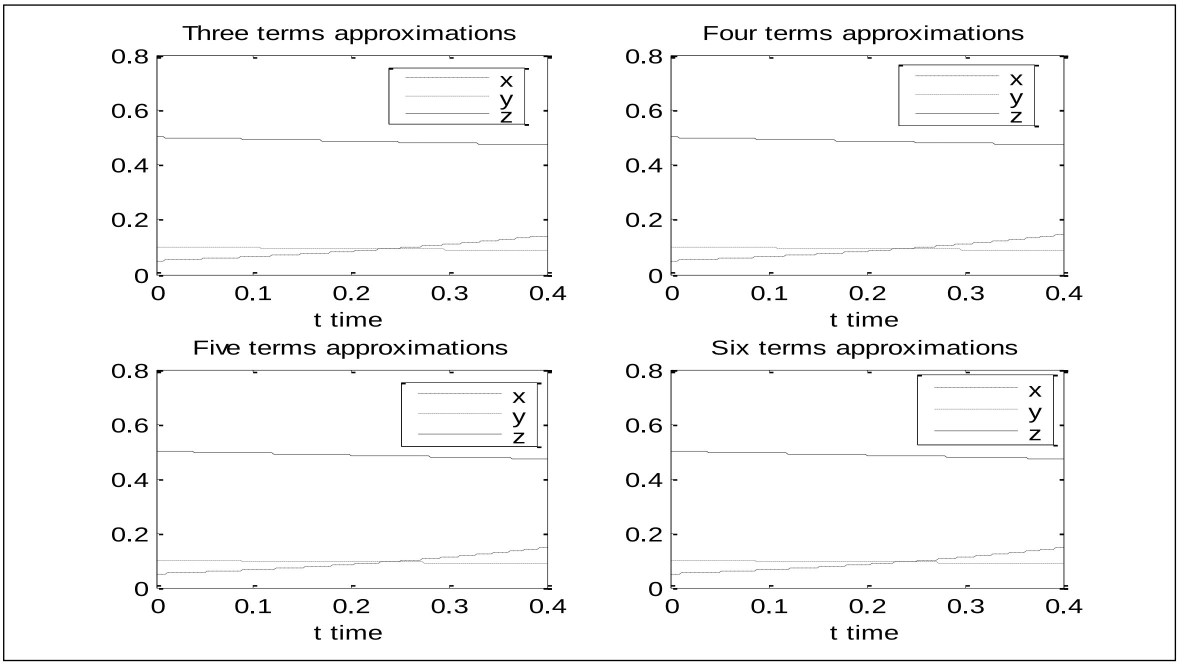

Figure. 2. Plots of three, four five and six terms approximations for a three-species food chain model

5. Conclusions

In this paper, homotopy perturbation method was used for finding the solutions of nonlinear ordinary differential equation systems such as a three-species food chain model. We demonstrated the accuracy and efficiency of these methods by solving some ordinary differential equation systems. We apply He's homotopy perturbation method to calculate certain integrals. It is easy and very beneficial tool for calculating certain difficult integrals or in deriving new integration formula.

The computations associated with the examples in this paper were performed using Maple 7 and Matlab 7

REFERENCES

1. Rafei.M., Daniali. H., Ganji.D.D., Variational iteration method for solving the epidemic model and the prey and predator problem, Applied Mathematics and Computation, 186,1701–1709, (2007).
2. Klebano. A., Hastings. A., Chaos in three species food chains, J Math Biol, 32,427–451, (1994).
3. He. J.H., A coupling method of homotopy technique and perturbation technique for nonlinear problems. Int J Non-Linear Mech, 35, 37–43, (2000).
4. Jordan. D.W., Smith P., Nonlinear Ordinary Differential Equations, thirded, Oxford University Press, (1999).
5. Biazar J., Solution of the epidemic model by Adomian decomposition method, Applied Mathematics and Computation ,173, 1101–1106, (2006).
6. Chowdhury. MSH, I. Hashim. I., Application of homotopy-perturbation method to Klein–Gordon and sine-Gordon equations, Chaos Solitons & Fractals, in press.
7. He. J.H., Homotopy perturbation method for solving boundary value problems, Phys Lett A ,350, 87–8, (2006).
8. Biazar. J., Ilie M., Khoshkenar A., A new approach to the solution of the prey and predator problem and comparison of the results with the Adomian method, Applied Mathematics and Computation, 171, 486–491, (2005).
9. He. J.H., A new approach to nonlinear partial differential equations, Communications in, Nonlinear Science and Numerical Simulation, 2, 230–235, (1997).
10. He. J.H, Wu .X.H., Construction of solitary solution and compacton-like solution by variational iteration method, Chaos, Solitons & Fractals, 29, 108–113, (2006).
11. He. J.H., Approximate analytical solution for seepage flow with fractional derivatives in porous media, Computer Methods in Applied Mechanics and Engineering, 167, 57–68, (1998).
12. He. J.H., Approximate solution of nonlinear differential equations with convolution product nonlinearities, Computer Methods in Applied Mechanics and Engineering, 167, 69–73, (1998).
13. He. J.H., Variational iteration method-a kind of nonlinear analytical technique: some examples, International Journal of Nonlinear Mechanics, 34, 699–708,(1999).
14. He. J.H, Some asymptotic methods for strongly nonlinear equations, International Journal of Modern Physics B, 20, 1141–1199, (2006).
15. Abdou M.A., Soliman A.A., Variational-iteration method for solving Burger's and coupled Burger's equations, Journal of Computational and Applied Mathematics, 181, 245–251, (2005).
16. He J.H., Homotopy perturbation method for solving boundary value problems, Phys Lett A, 350, 87–8, (2006).
17. Soliman A.A., A numerical simulation and explicit solutions of KdV–Burgers' and Lax's seventh-order KdV equations, Chaos,Solitons & Fractals, 29, 294–302, (2006).
18. Gakkhar. S, Naji RK., Chaos in three species ratio dependent food chain, Chaos Solitons & Fractals, 14,771–778, (2002).
19. J.H. He, Semi-inverse method of establishing generalized principles for fluid mechanics with emphasis on turbomachinery aerodynamics, International Journal of Turbo Jet-Engines, 14, 23–28, (1997).
20. Finlayson. B.A., The Method of Weighted Residuals and Variational Principles, Academic press, New York, (1972).
21. He. J.H., Homotopy perturbation technique. Comput Methods Appl Mech Engrg, 62,257–62, (1999).
22. He. J.H., A coupling method of a homotopy technique and a perturbation technique for non-linear problems, Int J Non-linear Mech ,35, 37–43, (2000).

23. Coskun. E., Merdan .M., Global stability and periodic solution of a viral dynamic model, Journal of Science of science and art faculty, 2, 256-267,(2007).

24. Merdan,M., 2009. Homotopy Perturbation Method for Solving Human T-cell Lymphotropic Virus I(HTLV-I) Infection of CD4+ T-cells Model, Mathematical and Computation Applications;14(2), 85-96.