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FINDING EXACT SOLUTION BY USING A NEW AUXILIARY EQUATION FOR FRACTIONAL RLW BURGES EQUATION

ABSTRACT

In this study a new method with a different auxiliary equation from the Riccati equation is used for constructing exact solutions of fractional nonlinear partial differential equations. The main idea of this method is to take full advantage of a different auxiliary equation from the Riccati equation which has more new solutions. Finally, more new solutions have been obtained for the fractional RLW Burgers equation.

Keywords: Fractional Nonlinear Partial Differential Equations, Fractional RLW Burgers Equation, Modified Riemann-Liouville Derivative, Nonlinear Partial Differential Equations, Riemann-Liouville Derivative

YENİ BİR YARDIMCI DENKLEM KULLANARAK KESİRLİ RLW BURGES DENKLEMİ İÇİN TAM ÇÖZÜM BULMA

ÖZET

Bu çalışmada kesirli lineer olmayan kısmi diferensiyel denklemlerin tam çözümlerinin oluşturulması için Riccati denkleminden farklı bir yardımcı denklem ile yeni bir metod kullanılmıştır. Bu metodun ana fikri, Riccati denkleminden farklı olarak yeni çözümlere sahip yeni bir yardımcı denklemden en iyi şekilde yararlanmaktır. Sonuç olarak, kesirli RLW Burgers denklemi için birçok yeni çözüm elde edilmiştir.

Anahtar Kelimeler: Kesirli Lineer Olmayan Kısmi Diferensiyel Denklemler, Kesirli RLW Burgers Denklemi, Modified Riemann-Liouville Türevi, Lineer Olmayan Kısmi Diferensiyel Denklemler, Riemann-Liouville Türevi

1. INTRODUCTION (GİRİŞ)

In the recent years, remarkable progress has been made in the construction of the approximate solutions for fractional nonlinear partial differential equations (fnPDE)[1, 2 and 3]. In particular fractional differential equations could be helpful to understand the behavior of the physical problems. Therewithal reaching to the exact solutions of fractional differential equations is very important. In this stage, it is not possible to solve the fnPDE before converting these equations into integer-order differential equations, doing this conversion we need to have a variable transformation by using a kind of fractional derivative and some useful formulas such as a modified Riemann-Liouville derivative which are proposed by Jumarie [4, 5 and 6].

Many explicit exact methods and analytic methods have been introduced in literature [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 and 21]. In our present work, we implement relatively new method and balance term definition [22] with a different auxiliary equation from the Riccati equation is used for constructing exact solutions of fnPDE. We also intend to investigate for the first time the applicability and effectiveness of the method on fnPDE. We can therefore easily convert fnPDE into nPDE or ODE using suitable transformation, so that everyone familiar with advanced calculus can deal with fractional calculus without any difficulty.

In this article, the first section presents the scope of the study as an introduction. In the second section contains some basic definitions of the modified Riemann-Liouville derivative, analyze of a new method and balance term definition. In the third section, we will obtain exact solutions of fractional RLW Burgers equation by using a new auxiliary equation. In the last section, we implement the conclusion.

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this article, we have presented a new method and balance term definition and used it to solve the fractional RLW Burgers equation. In fact, this method is readily applicable to a large variety of fnPDEs.

3. PRELIMINARIES AND NOTATIONS (TEMEL KAVRAMLAR VE GÖSTERIMLER)

In this part of the paper, it would be helpful to give some definitions and properties of the fractional calculus theory. For an introduction to the classical fractional calculus we refer the reader to [1, 2 and 3]. Here, we briefly review the modified Riemann-Liouville derivative from the recent fractional calculus proposed by Jumarie [4, 5 and 6]. Let f: [0, 1]→R be a continuous function and α∈(0, 1). The Jumarie modified fractional derivative of order α and f may be defined by expression of [9] as follows:

$$D_x^{\alpha} f(x) = \begin{cases} \frac{1}{\Gamma(-\alpha)} \int_0^x (x-\xi)^{-\alpha-1} [f(\xi)-f(0)]d\xi, & \alpha < 0, \\ \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_0^x (x-\xi)^{-\alpha} [f(\xi)-f(0)]d\xi, & 0<\alpha<1, \\ \left(f^{(n)}(x)\right)^{(\alpha-n)}, & n \le \alpha \le n+1, n \ge 1. \end{cases} \quad (1)$$

In addition to this expression, we may give a summary of the fractional modified Riemann-Liouville derivative properties which are used further in this paper. Some of the useful formulas are given as

$$D_x^{\alpha} k = 0, \quad (k \text{ is a constant}) \quad (2)$$

$$D_x^{\alpha} x^{\mu} = \begin{cases} 0, & (\mu \le \alpha - 1), \\ \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)} x^{\mu-\alpha}, & (\mu > \alpha - 1). \end{cases} \quad (3)$$

Similar to integer-order differentiation, the Jumarie' modified fractional differentiation is a linear operation:

$$D_x^\alpha(\beta u(x)+\gamma v(x)) = \beta D_x^\alpha u(x) + \gamma D_x^\alpha v(x), \quad (4)$$

where β and γ are constants, and satisfies the so called Leibniz rule for the Jumarie' modified fractional derivative is equal to the standard one:

$$D_x^\alpha(u(x)v(x)) = v(x)D_x^\alpha u(x) + u(x)D_x^\alpha v(x) = \sum_{k=0}^{\infty}\binom{n}{k}u^{(k)}(x)D_x^{\alpha-k}v(x), \quad (5)$$

if v(x) is continuous in [0,x] and u(x) has continuous derivative in [0,x]. The last properties is

$$D_x^\alpha[f(u(x))] = f_u'(u)D_x^\alpha u(x) = D_u^\alpha f(u)(u_x')^\alpha, \quad (6)$$

which are direct consequences of the equality $d^\alpha x(t) = \Gamma(1+\alpha)dx(t)$.

In last, let us consider the time fractional differential equation with independent variables $x = (x_1, x_2, ..., x_m, t)$ and adependent variable u,

$$G(u, D_x^\alpha u, w_{x_1}, u_{x_2}, u_{x_3}, D_x^{2\alpha}u, u_{x_1x_1}, u_{x_2x_2}, u_{x_3x_3}, ...) = 0. \quad (7)$$

Let us give a variable transformation for Eq. (7) as

$$u(x_1, x_2, ..., x_m, t) = u(\xi), \ \xi = x_1 + l_1x_2 + ... + l_{m-1}x_m + \frac{ct^\alpha}{\Gamma(\alpha+1)}, \quad (8)$$

where l_i and $\kappa$are constants to be determined later; after the transformation the fractional differential equation (7) is reduced to an ordinary differential equation

$$H(u(\xi), u'(\xi), u''(\xi), ...), \quad (9)$$

where$' = \frac{d}{d(\xi)}$.

For more information on the mathematical properties of fractional derivatives can consult the mentioned references.

4. ANALYSIS OF THE METHOD (METODUN ANALİZİ)

Let us simply describe the method. Consider a given time fractional partial differential equation in two variables and a dependent variable u,

$$G(u, D_t^\alpha u, u_x, u_{xx}, u_{xxx}, ...) = 0. \quad (10)$$

Let us give a variable transformation for Eq. (7) as

$$u(x,t) = u(\xi), \quad \xi = x + \frac{ct^\alpha}{\Gamma(\alpha+1)}, \quad (11)$$

where$\kappa$is constant to be determined later; after the transformation the fractional differential equation (10) is reduced to an ordinary differential equation

$$H(u(\xi), u'(\xi), u''(\xi), ...). \quad (12)$$

The fact that the solutions of many nonlinear equations can be expressed as a finite series of solutions of the auxiliary equation motivates us to seek for the solutions of Eq. (10) in the form

$$u(x,t) = u(\xi) = \lambda\sum_{i=-m}^{m} a_i F(\xi)^i \quad (13)$$

where $\xi = x - \frac{ct^\alpha}{\Gamma(\alpha+1)}$, c and λ are constants, m is a positive integer that can be determined by balancing the linear term of highest order with the nonlinear term in Eq. (9), λ is balancing coefficient that will be defined in a new "Balance term" definition and $a_0, a_1, a_2, ...$ are parameters to be determined. Substituting (13) into Eq. (12) yields a set of algebraic equations for $a_0, a_1, a_2, ...$because all coefficients of F have to vanish. From these relations $a_0, a_1, a_2, ...$ can be determined. The main idea of our method is to take full advantage of the new auxiliary equation. The desired auxiliary equation presents as following

$$F' = \frac{A}{F} + BF + CF^3 \tag{14}$$

where $\frac{dF}{d\xi} = F'$, *A*, *B* and *C* are constants.

Case 1. If $A = -\frac{1}{4}, B = \frac{1}{2}, C = -\frac{1}{2}$ then (14) has the solution

$$F = \frac{1}{\sqrt{1 + \tan(\xi) + \sec(\xi)}}$$

Case 2. If $A = \frac{1}{4}, B = -\frac{1}{2}, C = 0$ then (14) has the solutions

$$F = \frac{1}{\sqrt{1 + \csc h(\xi) + \coth(\xi)}} \text{ or } F = \frac{1}{\sqrt{1 + i\sec h(\xi) + \tanh(\xi)}}$$

Case 3. If $A = \frac{1}{2}, B = 1, C = 0$ then (14) has the solutions $F = \frac{1}{\sqrt{1 + \cot h(\xi)}}$ or

$$F = \frac{1}{\sqrt{1 + \tanh(\xi)}}.$$

In the following we present a new approach to the "Balance term" definition;

Definition: When Eq. (1) is transformed with $u(x,t) = u(\xi),\ \xi = x - \frac{ct^\alpha}{\Gamma(1+\alpha)}$, where c is real constant, we get a nonlinear ordinary differential equation for $u(\xi)$ as following

$$H(u, cu', u'', u''', \ldots) = 0. \tag{15}$$

Let $u^{(p)}$ is the highest order derivative linear term and $u^q u^{(r)}$ is the highest nonlinear term in (15) and $F' = k_0 + k_1 F + k_2 F^2 + \ldots + k_n F^n$ is the auxiliary equation that is used to solve the fnPDE then the "Balance term" *m* can be decided by the balancing the nonlinear term $u^q u^{(r)}$ and the linear term $u^{(p)}$ with acceptances of $u \cong \lambda F^i$ and $F' \cong F^n$ where *n* is integer $(n \neq 1)$ and λ is the balance coefficient that can be determined later.

Example. We consider the fractional KdV equation of the form

$$\frac{\partial^\alpha u}{\partial t^\alpha} - 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0, t > 0, 0 < \alpha, \tag{16}$$

With the initial condition $u(x,0) = f(x)$. For the fractional KdV equation with the transform $u(x,t) = u(\xi),\ \xi = x - \frac{ct^\alpha}{\Gamma(1+\alpha)}$, we have the ordinary differential equation as following

$$-cu' + 6uu' + u''' = 0. \tag{17}$$

By the balancing linear term u''' with nonlinear term uu'

$$u' = \left(\lambda F^{m}\right)' = \lambda m F^{m-1} F' = \lambda m F^{m-1} F^{n} = \lambda m F^{m+n-1},$$

$$u'' = \left(\lambda m F^{m+n-1}\right)' = \lambda m\left(m+n-1\right) F^{m+n-2} F',$$
$$= \lambda m\left(m+n-1\right) F^{m+n-2} F^{n} = \lambda m\left(m+n-1\right) F^{m+2n-2},$$

$$u''' = \left(\lambda m\left(m+n-1\right) F^{m+2n-2}\right)' = \lambda m\left(m+n-1\right)\left(m+2n-2\right) F^{m+2n-3} F' \quad (18)$$
$$= \lambda m\left(m+n-1\right)\left(m+2n-2\right) F^{m+2n-3} F^{n}$$
$$= \lambda m\left(m+n-1\right)\left(m+2n-2\right) F^{m+3n-3},$$
$$uu' = \lambda F^{m} \lambda m F^{m+n-1} = \lambda^{2} m F^{2m+n-1},$$

we have the equations above and the equating uu' to u''',we obtain

$$\lambda^{2} m F^{2m+n-1} = \lambda m\left(m+n-1\right)\left(m+2n-2\right) F^{m+3n-3}.$$

From this point the balance coefficient λ can be calculated as

$$\lambda = \left(m+n-1\right)\left(m+2n-2\right) \text{ and } m = 2\left(n-1\right).$$

If it is noticed that our new balance term *m* $\left(m = 2\left(n-1\right)\right)$ is connected to *n*. Namely our new balance term definition is connected to chosen auxiliary equation [22].

5. APPLICATION OF THE METHOD (METODUN UYGULAMASI)

5.1. Example 1 (Örnek 1)

Let’s consider the fractional RLW Burgers equation

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} + \frac{\partial u}{\partial x} + 12u\frac{\partial u}{\partial x} - \frac{\partial^{2}}{\partial x^{2}}\left(\frac{\partial u}{\partial t}\right) = 0,\ t > 0,\ 0 < \alpha, \quad (19)$$

with the transform (11) we have the following equation.

$$-cu' + u' + 12uu' - u'' + cu''' = 0. \quad (20)$$

From the Definition 2.1 we have the balance term of RLW Burgers by using the auxiliary equation (14), is equal to 4 and the balance coefficient could find 48. If the values of the *m* and λ is written into (13). Therefore, we may choose the following ansatz:

$$u\left(x,t\right) = u(\xi) = 48\left(a_{-4}F^{-4} + a_{-3}F^{-3} + a_{-2}F^{-2} + a_{-1}F^{-1} + a_{0} + a_{1}F + a_{2}F^{2} + a_{3}F^{3} + a_{4}F^{4}\right). \quad (21)$$

Substituting (20) into (17) along with Eq. (14) and using Mathematica yields a system of equations [22]. Setting the coefficients of F^{i} in the obtained system of equations to zero, we can deduce the following set of algebraic polynomials with the respect unknowns $a_0, a_1, a_2, \ldots$ namely: $u\left(x,t\right) = u\left(\xi\right),\ \xi = x - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}$

$$3ca_{-4} + 12a_{-4}^{2} = 0, \left(\frac{105ca_{-3}}{64} + 21a_{-4}a_{-3}\right) = 0, \left(-\frac{3a_{-4}}{2} - \frac{27ca_{-4}}{2} - 24a_{-4}^{2} + 9a_{-3}^{2} +\right.$$
$$\left.\frac{3ca_{-2}}{4} + 18a_{-4}a_{-2}\right) = 0, \left(-\frac{15a_{-3}}{16} - \frac{225ca_{-3}}{32} - 42a_{-4}a_{-3} + 15a_{-3}a_{-2} + \frac{15ca_{-1}}{64} + 15a_{-4}a_{-1}\right) = 0,$$
$$\left(6a_{-4} + 28ca_{-4} + 24a_{-4}^{2} - 18a_{-3}^{2} - \frac{a_{-2}}{2} - 3ca_{-2} - 36a_{-4}a_{-2} + 6a_{-2}^{2} + 12a_{-3}a_{-1} + 12a_{-4}a_{0}\right) = 0,$$

$$(\frac{15a_{-3}}{4}+\frac{429ca_{-3}}{32}+42a_{-4}a_{-3}-30a_{-3}a_{-2}-\frac{3a_{-1}}{16}-\frac{27ca_{-1}}{32}-30a_{-4}a_{-1}+9a_{-2}a_{-1}+9a_{-3}a_0-$$
$$\frac{3ca_1}{64}+9a_{-4}a_1)=0,(8a_{-4}+20ca_{-4}-3a_{-2}-4ca_{-2}+12a_{-2}^2+24a_{-3}a_{-1}-6a_{-1}^2+24a_{-4}a_0-$$
$$12a_{-2}a_0-12a_{-3}a_1-12a_{-4}a_2)=0,(-10a_{-4}-32ca_{-4}+18a_{-3}^2+2a_{-2}+5ca_{-2}+36a_{-4}a_{-2}-$$
$$12a_{-2}^2-24a_{-3}a_{-1}+3a_{-1}^2-24a_{-4}a_0+6a_{-2}a_0+6a_{-3}a_1+6a_{-4}a_2)=0,$$
$$(-6a_{-3}-\frac{27ca_{-3}}{2}+30a_{-3}a_{-2}+\frac{3a_{-1}}{4}+\frac{35ca_{-1}}{32}+30a_{-4}a_{-1}-18a_{-2}a_{-1}-18a_{-3}a_0+3a_{-1}a_0+$$
$$\frac{a_1}{16}+\frac{3ca_1}{32}-18a_{-4}a_1+3a_{-2}a_1+3a_{-3}a_2+\frac{3ca_3}{64}+3a_{-4}a_3)=0,$$
$$(\frac{9a_{-3}}{2}+\frac{105ca_{-3}}{16}-a_{-1}-\frac{ca_{-1}}{2}+18a_{-2}a_{-1}+18a_{-3}a_0-6a_{-1}a_0-\frac{a_1}{4}+\frac{ca_1}{32}+18a_{-4}a_1-6a_{-2}a_1-$$
$$3a_0a_1-6a_{-3}a_2-3a_{-1}a_2-\frac{3a_3}{16}+\frac{9ca_3}{32}-6a_{-4}a_3-3a_{-2}a_3-3a_{-3}a_4)=0,$$
$$(-2a_{-4}-6ca_{-4}+2a_{-2}+ca_{-2}+6a_{-1}^2+12a_{-2}a_0+12a_{-3}a_1-3a_1^2-\frac{ca_2}{2}+12a_{-4}a_2-6a_0a_2-$$
$$6a_{-1}a_3-\frac{a_4}{2}+\frac{3ca_4}{2}-6a_{-2}a_4)=0,(-\frac{3a_{-3}}{4}-\frac{9ca_{-3}}{8}+\frac{a_{-1}}{2}-\frac{ca_{-1}}{16}+6a_{-1}a_0+\frac{ca_1}{2}+6a_{-2}a_1+6a_0a_1+$$
$$6a_{-3}a_2+6a_{-1}a_2-9a_1a_2+\frac{3a_3}{4}-\frac{105ca_3}{32}+6a_{-4}a_3+6a_{-2}a_3-9a_0a_3+6a_{-3}a_4-9a_{-1}a_4)=0,$$
$$(6a_1^2-a_2+4ca_2+12a_0a_2-6a_2^2+12a_{-1}a_3-12a_1a_3+2a_4-10ca_4+12a_{-2}a_4-12a_0a_4)=0,$$
$$(-\frac{3}{8}ca_{-3}+\frac{a_{-1}}{4}-\frac{3ca_{-1}}{8}+\frac{a_1}{2}-\frac{35ca_1}{16}-6a_0a_1-6a_{-1}a_2+18a_1a_2-3a_3+\frac{27ca_3}{2}-6a_{-2}a_3+$$
$$18a_0a_3-15a_2a_3-6a_{-3}a_4+18a_{-1}a_4-15a_1a_4)=0,$$
$$(-6a_1^2+2a_2-10ca_2-12a_0a_2+12a_2^2-12a_{-1}a_3+24a_1a_3-9a_3^2-6a_4+32ca_4-$$
$$12a_{-2}a_4+24a_0a_4-18a_2a_4)=0,(-\frac{105ca_3}{8}-42a_3a_4)=0,$$
$$(\frac{3ca_{-1}}{8}-\frac{3a_1}{4}+\frac{27ca_1}{8}-18a_1a_2+\frac{9a_3}{2}-\frac{429ca_3}{16}-18a_0a_3+30a_2a_3-18a_{-1}a_4+30a_1a_4$$
$$-21a_3a_4)=0,(-\frac{15ca_1}{8}-\frac{15a_3}{4}+\frac{225ca_3}{8}-30a_2a_3-30a_1a_4+42a_3a_4)=0,(-24ca_4-24a_4^2)=0,$$
$$(-2a_2+12ca_2-12a_2^2-24a_1a_3+18a_3^2+8a_4-56ca_4-24a_0a_4+36a_2a_4-12a_4^2)=0,$$
$$(-6ca_2-18a_3^2-6a_4+54ca_4-36a_2a_4+24a_4^2)=0. \qquad (22)$$

From the system of equations (22) we have

i) $a_0=-\frac{11}{60}-\frac{i}{12},\ a_1=a_2=a_3=a_4=a_{-1}=a_{-3}=0,\ a_{-2}=\frac{1}{10}+\frac{i}{10},\ a_{-4}=-\frac{i}{20},c=\frac{i}{5}$

$$u(x,t)=-\left(\frac{11+5i}{60}\right)+\left(\frac{1+i}{10}\right)\left(\frac{1}{\sqrt{1+sec[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]+tan[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]}}\right)^{-2} -\frac{i}{20}\left(\frac{1}{\sqrt{1+sec[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]+tan[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]}}\right)^{-4} \quad (23)$$

ii) $a_0=\frac{1+5i}{60}, c=-a_4, a_1=a_{-2}=a_3=a_{-4}=a_{-1}=a_{-3}=0, a_2=-\frac{1+i}{5}, a_4=\pm\frac{i}{5},$

$$u(x,t)=\frac{1+5i}{60}-\frac{1+i}{5}\sqrt{1+sec[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]+tan[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]}^{\,2} + \frac{i}{5}\left(\sqrt{1+sec[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]+tan[x-\frac{it^{\alpha}}{5\Gamma(1+\alpha)}]}\right)^{4} \quad (24)$$

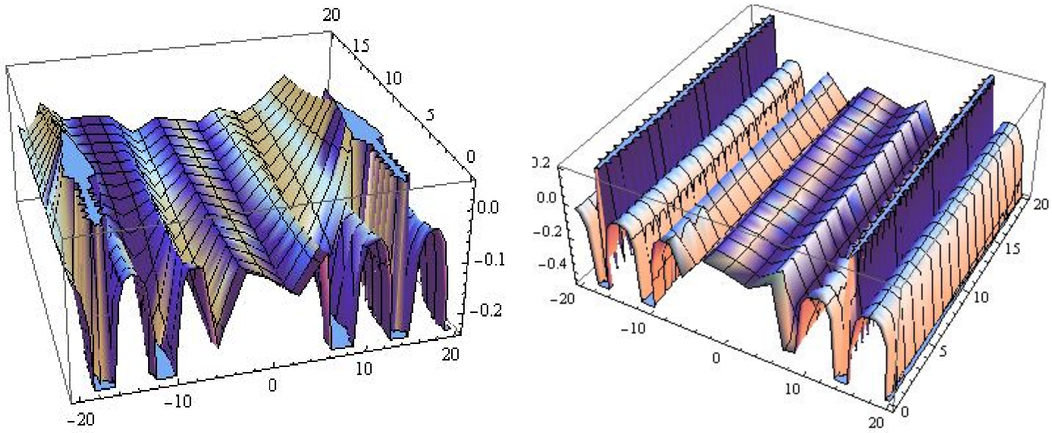

Figure 5.1: Graphs of the solution of the fractional RLW Burgers of (23) for i. $u(x,t)$ corresponding to the values $\alpha=0.5$ and $\alpha=0.1$
(Şekil.5.1 $\alpha=0.5$ ve $\alpha=0.1$ değerlerine karşılık gelen i. $u(x,t)$ için (23) kesirli RLW Burgers denkleminin çözümünün grafiği)

From the Definition 2.1 we have the balance term of the fractional RLW Burgers equation by using the auxiliary equation (2.3) for "Case 2", is equal to -4 then we have the following system of equations

$$-3ca_{-4}-12a_{-4}^2=0,(-\frac{3a_{-4}}{2}+\frac{27ca_{-4}}{2}+24a_{-4}^2-9a_{-3}^2-\frac{3ca_{-2}}{4}-18a_{-4}a_{-2})=0,$$
$$(-\frac{15a_{-3}}{16}+\frac{225ca_{-3}}{32}+42a_{-4}a_{-3}-15a_{-3}a_{-2}-\frac{15ca_{-1}}{64}-15a_{-4}a_{-1})=0,$$
$$(4a_{-4}-18ca_{-4}+18a_{-3}^2-\frac{a_{-2}}{2}+3ca_{-2}+36a_{-4}a_{-2}-6a_{-2}^2-12a_{-3}a_{-1}-12a_{-4}a_0)=0, \quad (25)$$
$$(\frac{9a_{-3}}{4}-\frac{135ca_{-3}}{16}+30a_{-3}a_{-2}-\frac{3a_{-1}}{16}+\frac{27ca_{-1}}{32}+30a_{-4}a_{-1}-9a_{-2}a_{-1}-9a_{-3}a_0)=0,$$
$$(-2a_{-4}+6ca_{-4}+a_{-2}-3ca_{-2}+12a_{-2}^2+24a_{-3}a_{-1}-3a_{-1}^2+24a_{-4}a_0-6a_{-2}a_0)=0,$$
$$(-\frac{3a_{-3}}{4}+\frac{15ca_{-3}}{8}+\frac{a_{-1}}{4}-\frac{9ca_{-1}}{16}+18a_{-2}a_{-1}+18a_{-3}a_0-3a_{-1}a_0)=0,$$
$$(-\frac{105}{64}ca_{-3}-21a_{-4}a_{-3})=0,(6a_{-1}^2+12a_{-2}a_0)=0,(\frac{a_{-1}}{4}-\frac{3ca_{-1}}{8}+6a_{-1}a_0)=0.$$

From the system of equation (3.7) we have

i) $a_0=a_{-1}=0, a_{-2}=-\frac{1}{5}, a_{-3}=0, a_{-4}=\frac{1}{20}, c=-\frac{1}{5}$

$$u(x,t)=-\frac{1}{5}\left(\frac{1}{\sqrt{1+sec[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}]+tan[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}]}}\right)^{-2}+\frac{1}{20}\left(\frac{1}{\sqrt{1+sec[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}]+tan[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}]}}\right)^{-4} \quad (26)$$

or

$$u(x,t)=-\frac{1}{5}\left(\frac{1}{\sqrt{1+csch\left[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}\right]+coth\left[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}\right]}}\right)^{-2}+\frac{1}{20}\left(\frac{1}{\sqrt{1+csch\left[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}\right]+coth\left[x+\frac{t^\alpha}{5\Gamma(1+\alpha)}\right]}}\right)^{-4} \quad (27)$$

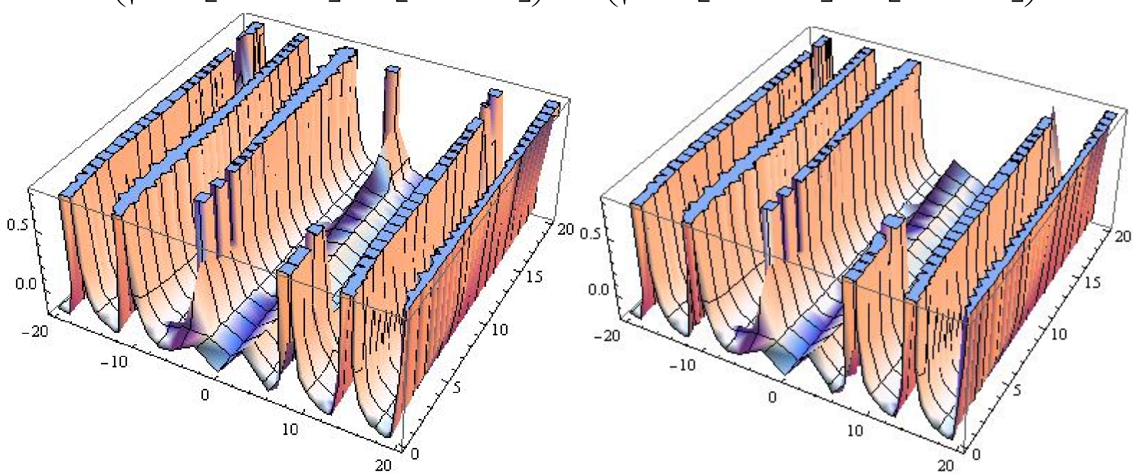

Figure 5.2 Graphs of the solution of the fractional RLW Burgers of (26) for i. $u(x, t)$ corresponding to the values $\alpha=0.5$ and $\alpha=0.1$

(Şekil.5.2 $\alpha=0.5$ ve $\alpha=0.1$ değerlerine karşılık gelen i. $u(x,t)$ için (26) kesirli RLW Burgers denkleminin çözümünün grafiği)

6. CONCLUSION (SONUÇ)

We have presented a new method and balance term definition and used it to solve the fractional RLW Burgers equation. In fact, this method is readily applicable to a large variety of fnPDEs. Firstly, all fnPDEs which can be solved by the other methods can be solved by our method. Secondly, we used only the special solutions of Eq.(14). If we use other solutions of Eq. (14), we can obtain more solitary wave solutions. Lastly, it is a computerizable method, which allow us to perform complicated and tedious algebraic calculation on computer and so our balance term definition is effectively useful for any to chosen auxiliary equation.

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