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HOMOTOPY PERTURBATION SUMUDU TRANSFORM METHOD FOR ONE-TWO-THREE DIMENSIONAL INITIAL VALUE PROBLEMS

ABSTRACT

In this paper, we study to obtain solutions of one-two-three dimensional initial value problems (IVP) by using homotopy perturbation sumudu transform method (HPSTM). We draw graphics of exact solutions for these equations by means of programming language Mathematica.

Keywords: Sumudu Transform Method,
Homotopy Perturbation Sumudu Transform Method,
One Dimensional IVP, Two Dimensional IVP,
Three Dimensional IVP

BİR-İKİ-ÜÇ BOYUTLU BAŞLANGIÇ DEĞER PROBLEMLERİ İÇİN HOMOTOPİ PERTÜRBASYON SUMUDU DÖNÜŞÜM METODU

ÖZET

Bu çalışmada, homotopi pertürbasyon sumudu dönüşüm metodu aracılığıyla bir-iki-üç boyutlu başlangıç değer problemlerinin çözümlerini elde etmeye çalıştık. Mathematica programı aracılığıyla bu denklemler için tam çözümlerinin grafiklerini çizdik.

Anahtar Kelimeler: Sumudu Dönüşüm Metodu,
Homotopi Pertürbasyon Sumudu Dönüşüm Metodu,
Bir Boyutlu Başlangıç Değer Problemi,
İki Boyutlu Başlangıç Değer Problemi,
Üç Boyutlu Başlangıç Değer Problemi

1. INTRODUCTION (GİRİŞ)

Sumudu transform method (STM) was firstly proposed by Watugala who was successfully applied to various linear differential equations [1 and 3]. Belgacem and Karaballi introduced fundamental properties of sumudu transform [4 and 5]. It was given that a number of studies in this area [6 and 14].

Homotopy perturbation method (HPM) was firstly proposed by He. He applied to various engineering problems [15 and 21]. HPM has been used to solve a large class of linear and nonlinear problems [22 and 23].

In this paper, we use HPSTM which involve the STM and HPM so as to find exact solutions of the one-two-three dimensional initial value problems (IVP) [24].

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this study, we implemented that homotopy perturbation sumudu transform method to obtain solutions of one-two-three dimensional initial value problems.

3. METHOD AND ITS APPLICATIONS (METOD VE UYGULAMALARI)

3.1. The Homotopy Perturbation Method (Homotopi Perturbasyon Metodu)

To illustrate the basic ideas of this method, we consider the following equation

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{1}$$

with boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma, \tag{2}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω.

A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Eq.(1) can be rewritten as following;

$$L(u) + N(u) - f(r) = 0, \quad r \in \Omega. \tag{3}$$

Homotopy perturbation structure is shown as following;

$$\mathrm{H}(v, p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0, \tag{4}$$

where

$$v(r, p) : \Omega \times [0,1] \rightarrow \Re. \tag{5}$$

In Eq.(4), $p \in [0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq.(4) can be written as a power series in p, as following;

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots, \tag{6}$$

and the best approximation for solution is

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots. \tag{7}$$

The convergence of series Eq.(7) has been proved by He in his paper [16]. This technique can have full advantage of the traditional perturbation techniques. Convergence rate of the series Eq.(7) depends on the nonlinear operator $A(v)$. The following opinions are suggested by He [16].

- The second derivative of $N(v)$ with respect to v must be small because the parameter may be relatively large, i.e., $p \rightarrow 1$.
- The norm of $L^{-1}(\partial N / \partial v)$ must be smaller than one so that the series converges.

3.2. The Homotopy Perturbation Sumudu Transform Method (Homotopi Perturbasyon Sumudu Dönüşüm Metodu)

To illustrate the basic ideas of this method, we consider a general linear form of one-two-three dimensional homogeneous partial differential equations;

$$\frac{\partial u(x,t)}{\partial t} = \Phi(x,t)\frac{\partial^2 u(x,t)}{\partial x^2} \tag{8}$$

with subject to initial condition

$$\Phi(x,0) = f(x), \tag{9}$$

where $f(x)$ is a known analytical function. We write the sumudu transform [1 and 14] of Eq.(8) as following;

$$\frac{\partial^2 F(x,u)}{\partial x^2} - \frac{1}{u\,\Phi(x,u)}F(x,u) + \frac{1}{u\,\Phi(x,u)}f(x,0) = 0. \tag{10}$$

According to HPM, we construct a homotopy in the form as following;

$$(1-p)\left[\frac{\partial^2 F}{\partial x^2} - \frac{\partial^2 f(x,0)}{\partial x^2}\right] + p\left[\frac{\partial^2 F(x,u)}{\partial x^2} - \frac{F(x,u)}{u\,\Phi(x,u)} + \frac{f(x,0)}{u\,\Phi(x,u)}\right] = 0 \tag{11}$$

where

$$u(x,0) = u_0 = f(x,0) \tag{12}$$

is initial condition of Eq.(8). Therefore, the sumudu transform of Eq.(8) is

$$F(x,u) = \sum_{n=0}^{\infty} p^n F_n(x,u).$$

We can assume that the solution of Eq.(8) can be written as a power series of p,

$$\begin{aligned} F(x,u) &= \sum_{n=0}^{\infty} p^n F_n(x,u) \\ &= F_0(x,u) + pF_1(x,u) + p^2F_2(x,u) + p^3F_3(x,u) + \cdots. \end{aligned} \tag{13}$$

When the limit get for $p \to 1$, the solution obtain as following;

$$\begin{aligned} F(x,u) &= \lim_{p\to 1}\left[F_0(x,u) + pF_1(x,u) + p^2F_2(x,u) + p^3F_3(x,u) + \cdots\right] \\ &= F_0(x,u) + F_1(x,u) + F_2(x,u) + F_3(x,u) + \cdots \\ &= \sum_{n=0}^{\infty} F_n(x,u). \end{aligned} \tag{14}$$

4. EXAMPLES (ÖRNEKLER)

4.1. Example 1 (Örnek 1)

The one-dimensional initial value problem [24] is given by

$$u_t = u_{xx}, \quad 0 < x < \pi,\ t > 0, \tag{15}$$

and initial condition [24] for Eq.(15) is

$$u(x,0) = u_0 = \cos(x). \tag{16}$$

We construct a sumudu transform [1 and 14] for Eq.(15) as following;

$$S\left[\frac{\partial u}{\partial t}\right] = \frac{1}{u}\left[F(x,u) - f(x,0)\right],$$

$$\Rightarrow \frac{1}{u}\left(F(x,u) - f(x,0)\right) = F'',$$

$$\Rightarrow F'' - \frac{1}{u}F + \frac{1}{u}\cos(x) = 0, \tag{17}$$

where are $F'' = \frac{\partial^2 F}{\partial x^2}$ and $\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 F(x,u)}{\partial x^2}$. According to HPM, we construct a homotopy for Eq.(17)

$$(1-p)\left[F''-u_0''\right]+p\left[F''-\frac{1}{u}F+\frac{1}{u}\cos(x)\right]=0$$

$$F''-u_0''+pu_0''-\frac{p}{u}F+\frac{1}{u}p\cos(x)=0 \tag{18}$$

where

$$F=F_0+pF_1+p^2F_2+p^3F_3+\cdots=\sum_{n=0}^{\infty}p^nF_n(x,u), \tag{19}$$

$$F''=F_0''+pF_1''+p^2F_2''+p^3F_3''+\cdots=\sum_{n=0}^{\infty}p^nF_n''(x,u).$$

Then, by substituting Eq.(19) into Eq.(18) and rearranging according to powers of p terms, we obtain

$$\left[F_0''+pF_1''+p^2F_2''+p^3F_3''\right]-u_0''+pu_0''+\frac{p}{u}\cos(x)-\frac{p}{u}\left[F_0+pF_1+p^2F_2+p^3F_3\right]=0$$

$$F_0''+pF_1''+p^2F_2''+p^3F_3''-u_0''+pu_0''+\frac{p}{u}\cos(x)-\frac{1}{u}pF_0-\frac{1}{u}p^2F_1-\frac{1}{u}p^3F_2=0$$

$$p^0: F_0''-u_0''=0, \tag{20}$$

$$p^1: F_1''+u_0''+\frac{1}{u}\cos(x)-\frac{1}{u}F_0=0, \tag{21}$$

$$p^2: F_2''-\frac{1}{u}F_1=0 \tag{22}$$

$$p^3: F_3''-\frac{1}{u}F_2=0 \tag{23}$$

$$\vdots$$

with solving Eq.(20-23)

$$p^0: F_0''-u_0''=0 \quad \Rightarrow F_0=u_0=\cos(x), \tag{24}$$

$$\begin{aligned} p^1: F_1''+u_0''+\frac{1}{u}\cos(x)-\frac{1}{u}F_0=0 \ &\Rightarrow F_1=\int_0^x\int_0^x\left[-u_0''-\frac{1}{u}\cos(x)+\frac{1}{u}F_0\right]dx\,dx \\ &\Rightarrow F_1=-\cos(x), \end{aligned} \tag{25}$$

$$\begin{aligned} p^2: F_2''-\frac{1}{u}F_1=0 \Rightarrow F_2&=\int_0^x\int_0^x\left[\frac{1}{u}F_1\right]dx\,dx \\ \Rightarrow F_2&=\frac{1}{u}\cos(x), \end{aligned} \tag{26}$$

$$p^3: F_3''-\frac{1}{u}F_2=0 \Rightarrow F_3=\int_0^x\int_0^x\left[\frac{1}{u}F_2\right]dx\,dx$$

$$\Rightarrow F_3=-\frac{1}{u^2}\cos(x), \tag{27}$$

$$\vdots.$$

Because compounds of $F_4, F_5, F_6, \cdots$ have very little value, we can't consider and then can take into consideration only F_0, F_1, F_2, F_3 for solution by HPSTM. When we consider Eq.(19) and suppose $p=1$, we obtain sumudu transform of Eq.(18) as following;

$$
\begin{aligned}
F(x,u) &= F_0 + pF_1 + p^2F_2 + p^3F_3 + \cdots \\
&= \lim_{p\to 1}\left(F_0 + pF_1 + p^2F_2 + p^3F_3 + \cdots\right) \\
&= F_0 + F_1 + F_2 + F_3 + \cdots \\
&= \cos(x) - \cos(x) + \frac{1}{u}\cos(x) - \frac{1}{u^2}\cos(x) + \cdots \\
&= \cos(x) + \cos(x)\left[-\underbrace{\left(1 - \frac{1}{u} + \frac{1}{u^2}\cdots\right)}_{I_1}\right], \quad I_1 = \frac{1}{1-\frac{1}{u}} = \frac{u}{u-1} \\
&= \cos(x)\left[1 + \frac{u}{1-u}\right]
\end{aligned}
$$

$$
= \cos(x)\left[\frac{1}{1-u}\right]. \tag{28}
$$

Therefore, we obtain sumudu transform of Eq. (15) as following

$$
F(x,u) = \cos(x)\left[\frac{1}{1-u}\right]. \tag{29}
$$

When we take inverse sumudu transform of Eq. (29) by using inverse transform table in [4], we get solution of Eq.(15) by HPSTM as following;

$$
u(x,t) = \cos(x)\, e^{-t}. \tag{30}
$$

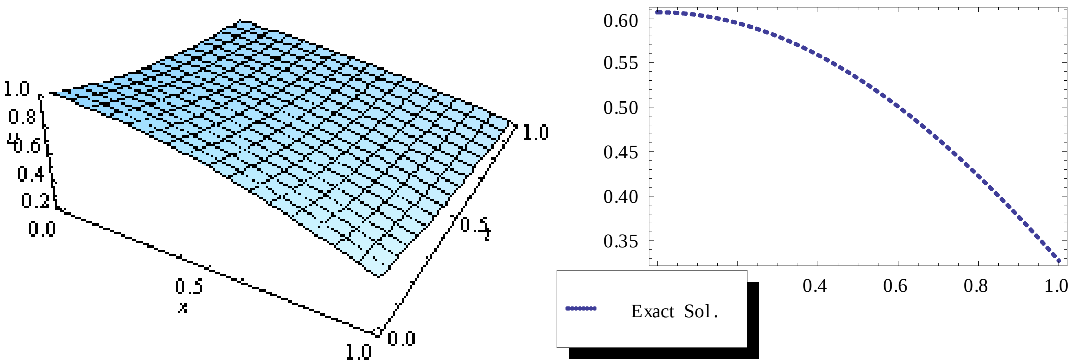


Figure 1. The 2D and 3D graphics of the exact solution $u(x,t)$ for Eq.(15) by means of HPSTM when $t = 0.5$

(Şekil 1. HPSTM aracılığıyla (15) denkleminin $t = 0.5$ için $u(x,t)$ tam çözümünün iki ve üç boyutlu grafikleri)

4.2. Example 2 (Örnek 2)

The two-dimensional initial value problem [24] is given by

$$
u_t = u_{xx} + u_{yy}, \quad 0 < x, y < \pi, \quad t > 0, \tag{31}
$$

and initial condition [24] for Eq.(31) is

$$
u(x,y,0) = u_0 = \sin(x+y). \tag{32}
$$

We construct a sumudu transform [1 and 14] for Eq.(31) as following;

$$
S\left[\frac{\partial u}{\partial t}\right] = \frac{1}{u}\left[F(x,y,u) - f(x,y,0)\right]
$$

$$
\Rightarrow \frac{1}{u}\left(F(x,y,u) - f(x,y,0)\right) = F'' + \ddot{F}
$$

$$\Rightarrow \frac{1}{u}F - \frac{1}{u}\sin(x+y) = F'' + \ddot{F} \Rightarrow F'' + \ddot{F} - \frac{1}{u}F + \frac{1}{u}\sin(x+y) = 0 \quad (33)$$

where are $F'' = \dfrac{\partial^2 F(x,y,u)}{\partial x^2}$ and $\ddot{F} = \dfrac{\partial^2 F(x,y,u)}{\partial y^2}$. According to HPM, we construct a homotopy for Eq.(33)

$$(1-p)\left[F'' - u_0''\right] + p\left[F'' + \ddot{F} - \frac{1}{u}F + \frac{1}{u}\sin(x+y)\right] = 0$$

$$F'' - u_0'' + pu_0'' + p\ddot{F} - \frac{1}{u}pF + \frac{1}{u}p\sin(x+y) = 0 \quad (34)$$

where

$$\ddot{F} = \ddot{F}_0 + p\ddot{F}_1 + p^2\ddot{F}_2 + p^3\ddot{F}_3 + \cdots = \sum_{n=0}^{\infty} p^n \ddot{F}_n(x,y,u),$$

$$F'' = F_0'' + pF_1'' + p^2F_2'' + p^3F_3'' + \cdots = \sum_{n=0}^{\infty} p^n F_n''(x,y,u), \quad (35)$$

$$F = F_0 + pF_1 + p^2F_2 + p^3F_3 + \cdots = \sum_{n=0}^{\infty} p^n F_n(x,y,u).$$

Then, by substituting Eq.(35) into Eq.(34) and rearranging according to powers of p terms, we obtain

$$F_0'' + pF_1'' + p^2F_2'' + p^3F_3'' - u_0'' + pu_0'' + p\ddot{F}_0 + p^2\ddot{F}_1 + p^3\ddot{F}_2 - \frac{1}{u}pF_0 - \frac{1}{u}p^2F_1$$
$$-\frac{1}{u}p^3F_2 + \frac{1}{u}p\sin(x+y) = 0.$$

$$p^0 : F_0'' - u_0'' = 0, \quad (36)$$

$$p^1 : F_1'' + u_0'' + \ddot{F}_0 - \frac{1}{u}F_0 + \frac{1}{u}\sin(x+y) = 0, \quad (37)$$

$$p^2 : F_2'' + \ddot{F}_1 - \frac{1}{u}F_1 = 0, \quad (38)$$

$$p^3 : F_3'' + \ddot{F}_2 - \frac{1}{u}F_2 = 0, \quad (39)$$

$$\vdots$$

with solving Eq.(36-39)

$$p^0 : F_0'' - u_0'' = 0 \Rightarrow F_0 = u_0 = \sin(x+y), \quad (40)$$

$$p^1 : F_1'' + u_0'' + \ddot{F}_0 - \frac{1}{u}F_0 + \frac{1}{u}\sin(x+y) = 0$$

$$\Rightarrow F_1 = \int_0^x\int_0^x \left[-u_0'' - \ddot{F}_0 + \frac{1}{u}F_0 - \frac{1}{u}\sin(x+y)\right] dx\,dx \quad (41)$$

$$\Rightarrow F_1 = -2\sin(x+y),$$

$$p^2 : F_2'' + \ddot{F}_1 - \frac{1}{u}F_1 = 0 \Rightarrow F_2 = \int_0^x\int_0^x \left[-\ddot{F}_1 + \frac{1}{u}F_1\right] dx\,dx$$

$$\Rightarrow F_2 = 2\left(1+\frac{1}{u}\right)\sin(x+y), \quad (42)$$

$$p^3 : F_3'' + \ddot{F}_2 - \frac{1}{u}F_2 = 0 \Rightarrow F_3 = \int_0^x\int_0^x \left[-\ddot{F}_2 + \frac{1}{u}F_2\right] dx\,dx$$

$$\Rightarrow F_3 = -2\left(1+\frac{1}{u}\right)^2\sin(x+y), \quad (43)$$

$$\vdots$$

When we consider Eq.(35) and suppose $p=1$, we obtain sumudu transform of Eq.(31) for Eq.(40-43) as following

$$\begin{aligned}F(x,y,u) &= F_0(x,y,u)+pF_1(x,y,u)+p^2F_2(x,y,u)+p^3F_3(x,y,u)+\cdots\\ &= \lim_{p\to1}\left(F_0(x,y,u)+pF_1(x,y,u)+p^2F_2(x,y,u)+p^3F_3(x,y,u)+\cdots\right)\\ &= F_0(x,y,u)+F_1(x,y,u)+F_2(x,y,u)+F_3(x,y,u)+\cdots \qquad (44)\\ &= \sin(x+y)-2\sin(x+y)+2\left(1+\frac{1}{u}\right)\sin(x+y)\\ &\quad -2\left(1+\frac{1}{u}\right)^2\sin(x+y)+\cdots.\end{aligned}$$

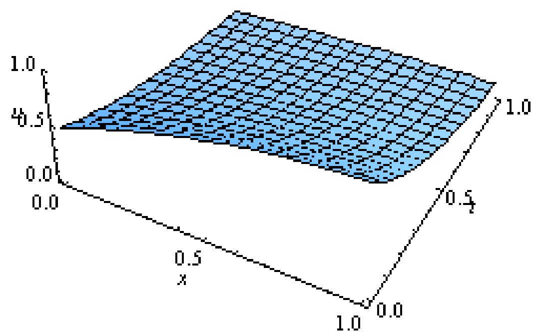
Therefore, we obtain sumudu transform of Eq.(31) as following

$$\begin{aligned}F(x,y,u) &= \sin(x+y)\left[1-2+2\left(1+\frac{1}{u}\right)-2\left(1+\frac{1}{u}\right)^2+\cdots\right]\\ &= \sin(x+y)\left[1-2+2\left(1+\frac{1}{u}\right)\left\{1+\left(-1-\frac{1}{u}\right)+\left(-1-\frac{1}{u}\right)^2+\cdots\right\}\right],\\ &= \sin(x+y)\left[1-2+2\left(1+\frac{1}{u}\right)\frac{u}{1+2u}\right]=\sin(x+y)\left[-1+2\left(\frac{1+u}{u}\right)\frac{u}{1+2u}\right]\\ &= \sin(x+y)\left[-1+\frac{2+2u}{1+2u}\right]=\sin(x+y)\left[\frac{1}{1+2u}\right]\end{aligned}$$

$$F(x,y,u)=\sin(x+y)\left[\frac{1}{1+2u}\right]. \qquad (45)$$

When we take inverse sumudu transform of Eq.(45) by using inverse transform table in [4], we get solution of Eq.(31) by HPSTM as following

$$u(x,y,t)=\sin(x+y)\,e^{-2t}. \qquad (46)$$



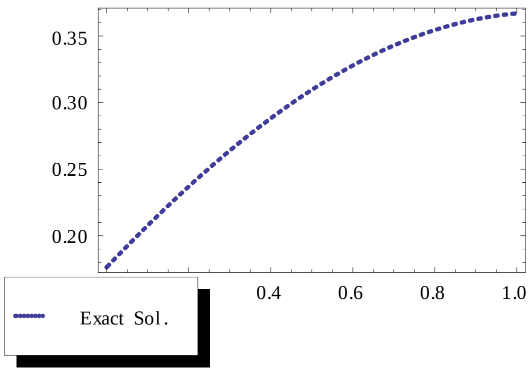


Figure 2. The 2D and 3D graphics of the exact solution $u(x,y,t)$ for Eq.(31) by means of HPSTM when $t=y=0.5$
(Şekil 2. HPSTM aracılığıyla (31) denkleminin $t=y=0.5$ için $u(x,y,t)$ tam çözümünün iki ve üç boyutlu grafikleri)

4.3. Example 3 (Örnek 3)

The three-dimensional initial value problem [24] is given

$$u_t=2\left(u_{xx}+u_{yy}+u_{zz}\right),\quad 0<x,y,z<\pi,\quad t>0, \qquad (47)$$

and initial condition [24] for Eq.(47) is

$$u(x,y,z,0)=u_0=\sin(x)\cos(y)\cos(z). \qquad (48)$$

We construct a sumudu transform [1 and 14] for Eq.(31) as following

$$
\begin{aligned}
&S\left[\frac{\partial u}{\partial t}\right] = \frac{1}{u}\left[F(x,y,z,u) - f(x,y,z,0)\right] \\
&\Rightarrow \frac{1}{u}\left(F(x,y,z,u) - f(x,y,z,0)\right) = 2\left(F'' + \ddot{F} + \overset{\approx}{F}\right) \\
&\Rightarrow \frac{1}{u}F - \frac{1}{u}\sin(x)\cos(y)\cos(z) = 2(F'' + \ddot{F} + \overset{\approx}{F}) \\
&\Rightarrow F'' + \ddot{F} + \overset{\approx}{F} - \frac{1}{2u}F + \frac{1}{2u}\sin(x)\cos(y)\cos(z) = 0
\end{aligned}
\tag{49}
$$

where are $F'' = \dfrac{\partial^2 F(x,y,z,u)}{\partial x^2}$, $\ddot{F} = \dfrac{\partial^2 F(x,y,z,u)}{\partial y^2}$ and $\overset{\approx}{F} = \dfrac{\partial^2 F(x,y,z,u)}{\partial z^2}$.

According to HPM, we construct a homotopy for Eq.(49)

$$
(1-p)\left[F'' - u_0''\right] + p\left[F'' + \ddot{F} + \overset{\approx}{F} - \frac{1}{2u}F + \frac{1}{2u}\sin(x)\cos(y)\cos(z)\right] = 0
$$

$$
F'' - u_0'' + pu_0'' + p\ddot{F} + p\overset{\approx}{F} - \frac{1}{2u}pF + \frac{1}{2u}p\sin(x)\cos(y)\cos(z) = 0 \tag{50}
$$

where

$$
\begin{aligned}
\ddot{F} &= \ddot{F}_0 + p\ddot{F}_1 + p^2\ddot{F}_2 + p^3\ddot{F}_3 + \cdots = \sum_{n=0}^{\infty} p^n \ddot{F}_n(x,y,z,u), \\
F'' &= F_0'' + pF_1'' + p^2F_2'' + p^3F_3'' + \cdots = \sum_{n=0}^{\infty} p^n F_n''(x,y,z,u), \\
F &= F_0 + pF_1 + p^2F_2 + p^3F_3 + \cdots = \sum_{n=0}^{\infty} p^n F_n(x,y,z,u), \\
\overset{\approx}{F} &= \overset{\approx}{F}_0 + p\overset{\approx}{F}_1 + p^2\overset{\approx}{F}_2 + p^3\overset{\approx}{F}_3 + \cdots = \sum_{n=0}^{\infty} p^n \overset{\approx}{F}_n(x,y,z,u).
\end{aligned}
\tag{51}
$$

Then, by substituting Eq.(51) into Eq.(50) and rearranging according to powers of p terms, we obtain

$$
\begin{aligned}
&F_0'' + pF_1'' + p^2F_2'' + p^3F_3'' - u_0'' + pu_0'' + p\ddot{F}_0 + p^2\ddot{F}_1 + p^3\ddot{F}_2 + p\overset{\approx}{F}_0 + p^2\overset{\approx}{F}_1 \\
&+p^3\overset{\approx}{F}_2 - \frac{1}{2u}pF_0 - \frac{1}{2u}p^2F_1 - \frac{1}{2u}p^3F_2 + \frac{1}{2u}p\sin(x)\cos(y)\cos(z) = 0,
\end{aligned}
$$

$$
p^0 : F_0'' - u_0'' = 0, \tag{52}
$$

$$
p^1 : F_1'' + u_0'' + \ddot{F}_0 + \overset{\approx}{F}_0 - \frac{1}{2u}F_0 + \frac{1}{2u}\sin(x)\cos(y)\cos(z) = 0, \tag{53}
$$

$$
p^2 : F_2'' + \ddot{F}_1 + \overset{\approx}{F}_1 - \frac{1}{2u}F_1 = 0, \tag{54}
$$

$$
p^3 : F_3'' + \ddot{F}_2 + \overset{\approx}{F}_2 - \frac{1}{2u}F_2 = 0, \tag{55}
$$

$$
\vdots
$$

with solving Eq.(52-55)

$$
p^0 : F_0'' - u_0'' = 0 \quad \Rightarrow F_0 = u_0 = \sin(x)\cos(y)\cos(z), \tag{56}
$$

$$p^1 : F_1'' + u_0'' + \ddot{F}_0 + \approx{F}_0 - \frac{1}{2u} F_0 + \frac{1}{2u} \sin(x)\cos(y)\cos(z) = 0$$

$$\Rightarrow F_1 = \int_0^x \int_0^x \left[-u_0'' - \ddot{F}_0 - \approx{F}_0 + \frac{1}{2u} F_0 - \frac{1}{2u} \sin(x)\cos(y)\cos(z) \right] dx\, dx \quad (57)$$

$$\Rightarrow F_1 = -6 \sin(x)\cos(y)\cos(z),$$

$$p^2 : F_2'' + \ddot{F}_1 + \approx{F}_1 - \frac{1}{2u} F_1 = 0 \Rightarrow F_2 = \int_0^x \int_0^x \left[-\ddot{F}_1 - \approx{F}_1 + \frac{1}{2u} F_1 \right] dx\, dx$$

$$\Rightarrow F_2 = 6\left(5 + \frac{1}{u}\right) \sin(x)\cos(y)\cos(z), \quad (58)$$

$$p^3 : F_3'' + \ddot{F}_2 + \approx{F}_2 - \frac{1}{2u} F_2 = 0 \Rightarrow F_3 = \int_0^x \int_0^x \left[-\ddot{F}_2 - \approx{F}_2 + \frac{1}{2u} F_2 \right] dx\, dx$$

$$\Rightarrow F_3 = -6\left(5 + \frac{1}{u}\right)^2 \sin(x)\cos(y)\cos(z), \quad (59)$$

$\vdots$

When we consider Eq.(51) and suppose $p = 1$, we obtain sumudu transform of Eq.(47) for Eq.(56-59) as following

$$\begin{aligned} F(x,y,z,u) &= F_0(x,y,z,u) + pF_1(x,y,z,u) + p^2F_2(x,y,z,u) + p^3F_3(x,y,z,u) + \cdots \\ &= \lim_{p\to 1}\left(F_0 + pF_1 + p^2F_2 + p^3F_3 + \cdots\right) \\ &= F_0 + F_1 + F_2 + F_3 + \cdots \\ &= \sin(x)\cos(y)\cos(z) + 6\left(5+\frac{1}{u}\right)\sin(x)\cos(y)\cos(z) \\ &\quad - 6\sin(x)\cos(y)\cos(z) - 6\left(5+\frac{1}{u}\right)^2 \sin(x)\cos(y)\cos(z)\cdots. \end{aligned}$$

Therefore, we obtain sumudu transform of Eq.(47) as following

$$\begin{aligned} F(x,y,z,u) &= \sin(x)\cos(y)\cos(z) - 6\sin(x)\cos(y)\cos(z)\left[1 - \left(5+\frac{1}{u}\right) + \left(5+\frac{1}{u}\right)^2 + \cdots\right] \\ &= \sin(x)\cos(y)\cos(z) - 6\sin(x)\cos(y)\cos(z)\left[1 + \left(-5-\frac{1}{u}\right) + \left(-5-\frac{1}{u}\right)^2 + \cdots\right] \\ &= \sin(x)\cos(y)\cos(z) - 6\sin(x)\cos(y)\cos(z)\left[\frac{u}{1+6u}\right] \\ &= \sin(x)\cos(y)\cos(z)\left(1 - \frac{6u}{1+6u}\right) \\ &= \sin(x)\cos(y)\cos(z)\left(\frac{1}{1+6u}\right) \end{aligned}$$

$$F(x,y,z,u) = \sin(x)\cos(y)\cos(z)\left(\frac{1}{1+6u}\right). \quad (60)$$

When we take inverse sumudu transform of Eq.(60) by using inverse transform table in [4], we get solution of Eq.(47) by HPSTM as following

$$u(x,y,z,t) = \sin(x)\cos(y)\cos(z)\, e^{-6t}. \quad (61)$$

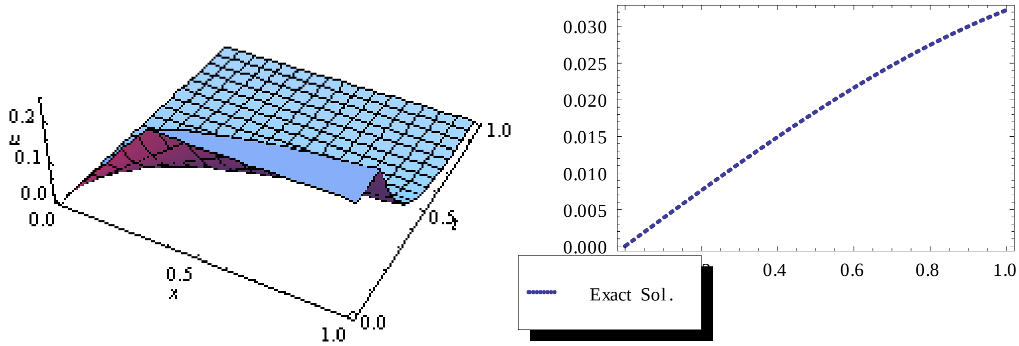


Figure 3. The 2D and 3D graphics of the exact solution $u(x,y,z,t)$ for Eq.(47) by means of HPSTM when $t = y = z = 0.5$
(Şekil 3. HPSTM aracılığıyla (47) denkleminin $t = y = z = 0.5$ için $u(x,y,z,t)$ tam çözümünün iki ve üç boyutlu grafikleri)

5. CONCLUSION (SONUÇ)

In this paper, we present the exact solutions of one-two-three dimensional initial value problems by using HPSTM. We draw graphics of exact solutions for these equations. STM can be used to solve variety initial value problems. Moreover, it performs to solve partial differential equations including engineering and applied sciences.

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