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EXACT COMPLEX SOLUTIONS FOR SOME NONLINEAR PARTIAL DIFFERANTIAL EQUATIONS

ABSTRACT

In this paper, we obtain complex solutions of the seventh degree Lax KDV equation, (2+1) dimensional Konopelchenka-Dubravsky (KD) equation by using direct algebraic method which is obtained by Zhang [1].

Keywords: Seventh Degree Lax KDV Equation, (2+1) Dimensional Konopelchenka-Dubravsky (KD) Equation, Direct Algebraic Method, Complex Solutions

BAZI NONLİNEER KISMİ DİFERANSİYEL DENKLEMLER İÇİN TAM KARMAŞIK ÇÖZÜMLER

ÖZET

Bu makalede Zhang [1]daki direk cebirsel metodlar kullanılarak (2+1) boyutlu Konopelchenka-Dubravsky (KD) denklemi elde edildi,aynı zamanda yedinci mertebeden Lax KDV dalga hareket denkleminin çözümü elde edildi.

Anahtar Kelimeler: Yedinci Mertebeden Lax KDV Denklemi, (2+1) Boyutlu Konopelchenka-Dubravsky (KD) Denklemi, Direk Cebirsel Metod,Kompleks Çözümler

1. INTRODUCTION (GİRİŞ)

Solution of nonlinear differential equations have important role in mathematic and physics. Especially, traveling wave solutions of nonlinear equations have an crucial role in mathematical physics and solution theory [2]. Recently, it has been stuied traveling wave solutions of nonlinear differential equations through using symbolical computer programs such as Maple, Matlab, etc. [3-8]. There are many methods which find exact solutions of nonlinear partial differential equations. [9-14].

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this paper, we obtain complex solutions of the seventh degree Lax KDV equation, (2+1) dimensional Konopelchenka-Dubravsky (KD) equation by using direct algebraic method which is obtained by Zhang [1].

3. AN ANALYSIS OF THE METHOD AND APPLICATIONS (YÖNTEM VE UYGULAMALARIN BİR ANALİZİ)

Before starting to give a direct algebraic method, we will give a simple description of the direct algebraic method. For doing this, one can consider in a two variables general form of nonlinear PDE

$$Q(u,u_t,u_x,u_{xx},\ldots)=0\,, \qquad (2.1)$$

and transform Eq.(2.1) with $u(x,t)=u(\xi)$,$\xi=ik(x-ct)$,where k,c are real constants. After transformation, we get a nonlinear ODE for $u(\xi)$

$$Q'(u,-ikcu',iku',-k^2u'',\ldots)=0. \qquad (2.2)$$

we are looking for solution of the equation (2.2) as following

$$U(\xi)=\sum_{m=0}^{n}a_mF^m(\xi), \qquad (2.3)$$

where $\xi=ik(x-ct)$.

- n is a positive integer that can be determined by balancing the highest order derivate and with the highest nonlinear terms in equation.
- a_m and ξ can be determined. Substituting solution (2.3) into Eq. (2.2) yields a set of algebraic equations for F^m and $(m=0,1,2,\ldots)$.
- All coefficients of F^m have to vanish. After this separated algebraic equation, we could found coefficients a_0,a_m and ξ.

In this work, we obtain complex solutions of the Lax seventh-order KdV equation equation and (2+1) dimensional Konopelchenko-Dubrovsky (KD) equation by using the direct algebraic method which is introduced by Zhang [1].

$$F'(\xi)=b+F^2(\xi) \qquad (2.4)$$

where $F'=\dfrac{dF}{d\xi}$ and b is a constant. Some of the solutions of $F'(\xi)=b+F^2(\xi)$ are given in [1].

4. EXAMPLES (ÖRNEKLER)

- **Example:** Consider lax seventh-order KdV equation,

$$u_t+140u^3u_x+70u_x^3+280uu_xu_{xx}+70u^2u_{xxx}+70u_{xx}u_{xxx}+42u_xu_{xxxx}+14uu_{xxxxx}+u_{xxxxxxx}=0,$$

(3.1.1) We can use transformation with Eq. (2.1) then Eq. (3.1.1) become

$$-cu'+140u^3u'-70k^2\left(u'\right)^3-280k^2uu'u''-70k^2u^2u'''+70k^4u''u'''+42k^4u'u^{(4)}+14k^4uu^{(5)}-$$
$$-k^6u^{(7)}=0,$$

(3.1.2)

When balancing $u^3u',(u')^3,uu'u'',u^2u''',u''u''',u'u^{(4)}$ and $uu^{(5)}$ with $u^{(7)}$ then gives *n=2*. We may choose

$$u=a_0+a_1F+a_2F^2 .$$

(3.1.3)

Substituting (3.1.3) into Eq. (3.1.2) yields a set of algebraic equations for a_0,a_1,a_2,b. These systems are finding as

$$140a_0^3a_1b-a_1bc-140a_0^2a_1b^2k^2-70a_1^3b^3k^2-560a_0a_1a_2b^3k^2+224a_0a_1b^3k^4+952a_1a_2b^4k^4-$$
$$-272a_1b^4k^6=0,$$
$$420a_0^2a_1^2b+280a_0^3a_2b-2a_2bc-840a_0a_1^2b^2k^2-1120a_0^2a_2b^2k^2-980a_1^2a_2b^3k^2-1120a_0a_2^2b^3k^2+$$
$$+1176a_1^2b^3k^4+3808a_0a_2b^3k^4+3584a_2^2b^4k^4-7936a_2b^4k^6=0,$$
$$140a_0^3a_1+420a_0a_1^3b+1260a_0^2a_1a_2b-a_1c-560a_0^2a_1bk^2-910a_1^3b^2k^2-6440a_0a_1a_2b^2k^2-$$
$$-2520a_1a_2^2b^3k^2+1904a_0a_1b^2k^4+16240a_1a_2b^3k^4-3968a_1b^3k^6=0,$$
$$420a_0^2a_1^2+280a_0^3a_2+140a_1^4b+1680a_0a_1^2a_2b+840a_0^2a_2^2b-2a_2c-2240a_0a_1^2bk^2-2800a_0^2a_2bk^2-$$
$$-7140a_1^2a_2b^2k^2-7840a_0a_2^2b^2k^2-1680a_2^3b^3k^2+5656a_1^2b^2k^4+17248a_0a_2b^2k^4+$$
$$+31136a_2^2b^3k^4-56320a_2b^3k^6=0,$$
$$420a_0a_1^3+1260a_0^2a_1a_2+700a_1^3a_2b+2100a_0a_1a_2^2b-420a_0^2a_1k^2-1890a_1^3bk^2-12880a_0a_1a_2bk^2-$$
$$-14420a_1a_2^2b^2k^2+3360a_0a_1bk^4+53648a_1a_2b^2k^4-12096a_1b^2k^6=0,$$
$$140a_1^4+1680a_0a_1^2a_2+840a_0^2a_2^2+1260a_1^2a_2^2b+840a_0a_2^3b-1400a_0a_1^2k^2-1680a_0^2a_2k^2-$$
$$-12460a_1^2a_2bk^2-13440a_0a_2^2bk^2-8400a_2^3b^2k^2+8008a_1^2bk^4+23520a_0a_2bk^4+81312a_2^2b^2k^4-$$
$$-129024a_2b^2k^6=0,$$
$$700a_1^3a_2+2100a_0a_1a_2^2+980a_1a_2^3b-1050a_1^3k^2-7000a_0a_1a_2k^2-22680a_1a_2^2bk^2+1680a_0a_1k^4+$$
$$+63056a_1a_2bk^4-13440a_1bk^6=0,$$
$$1260a_1^2a_2^2+840a_0a_2^3+280a_2^4b-6300a_1^2a_2k^2-6720a_0a_2^2k^2-12320a_2^3bk^2+3528a_1^2k^4+$$
$$+10080a_0a_2k^4+84000a_2^2bk^4-120960a_2bk^6=0,$$
$$980a_1a_2^3-10780a_1a_2^2k^2+24696a_1a_2k^4-5040a_1k^6=0,$$
$$280a_2^4-5600a_2^3k^2+30240a_2^2k^4-40320a_2k^6=0.$$

(3.1.4)

From the solutions of the system, we can found

- **Case 1:**

$$a_0=\frac{4bk^2}{3}+\frac{56\sqrt[3]{2}\left(1+i\sqrt{3}\right)\ b^2k^4}{3\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}-$$

$$-\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}{840\sqrt[3]{2}}$$

$a_1=0,\ a_2=2k^2,\ k\neq 0.$

(3.1.5)

- **Case 2:**

$$a_0=\frac{38bk^2}{5},\ a_1=0,\ a_2=2k^2,\quad c=\frac{867136b^3k^6}{25},\ k\neq 0.$$

(3.1.6)

- **Case 3:**

$$a_0=0,\ a_1=0,\ a_2=12k^2,\ b=0,\ c=0,\ k\neq 0.$$

(3.1.7)

- **Case 4:**

$$a_0=0,\ a_1=0,\ a_2=6k^2,\ b=0,\ c=736b^3k^6,\ k\neq 0.$$

(3.1.8)

Case 5.

$$a_0=\sqrt[3]{\frac{c}{140}},\ a_1=0,\ a_2=2k^2,\ b=0,\ k\neq 0.$$

(3.1.9)

With the aid of Mathematica substituting (3.1.5)-(3.1.6)-(3.1.7)-(3.1.8) and (3.1.9) into (3.1.3), we have obtained the following complex solutions of equation (3.1.1). These solutions are:

- **Family 1:**

$$u_1=\frac{4bk^2}{3}+\frac{56\sqrt[3]{2}\left(1+i\sqrt{3}\right)\ b^2k^4}{3\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}-$$

$$-\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}{840\sqrt[3]{2}}+$$

$$+2k^2\left(-\sqrt{-b}Tanh\left[\sqrt{-b}\left(ikx-ikct\right)\right]\right)^2.$$

(3.1.10)

where $b<0$, k is an arbitrary real constant.

$$u_2=\frac{4bk^2}{3}+\frac{56\sqrt[3]{2}\left(1+i\sqrt{3}\right)\ b^2k^4}{3\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}-$$

$$-\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}{840\sqrt[3]{2}}+$$

$$+2k^2\left(-\sqrt{-b}Coth\left[\sqrt{-b}\left(ikx-ikct\right)\right]\right)^2.$$

(3.1.11)

where $b<0$, k is an arbitrary real constant.

$$u_3=\frac{4bk^2}{3}+\frac{56\sqrt[3]{2}\left(1+i\sqrt{3}\right)\ b^2k^4}{3\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}-$$

$$-\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}{840\sqrt[3]{2}}+$$

$$+2k^2\left(\sqrt{b}Tan\left[\sqrt{b}\left(ikx-ikct\right)\right]\right)^2.$$

(3.1.12)

where $b>0$, k is an arbitrary real constant.

$$u_4=\frac{4bk^2}{3}+\frac{56\sqrt[3]{2}\left(1+i\sqrt{3}\right)\ b^2k^4}{3\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}-$$

$$-\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{529200c+1254400b^3k^6+\sqrt{15420489728000b^6k^{12}+\left(529200c+1254400b^3k^6\right)^2}}}{840\sqrt[3]{2}}+$$

$$+2k^2\left(-\sqrt{b}Cot\left[\sqrt{b}\left(ikx-ikct\right)\right]\right)^2.$$

(3.1.13)

where $b>0$, k is an arbitrary real constant.

- **Family 2:**

$$u_5=\frac{38bk^2}{5}+2k^2\left(-\sqrt{-b}Tanh\left[\sqrt{-b}\left(ikx-\frac{867136}{25}b^3k^7it\right)\right]\right)^2.$$

(3.1.14)

where $b<0$, k is an arbitrary real constant.

$$u_6=\frac{38bk^2}{5}+2k^2\left(-\sqrt{-b}Coth\left[\sqrt{-b}\left(ikx-\frac{867136}{25}b^3k^7it\right)\right]\right)^2.$$

(3.1.15)

where $b<0$, k is an arbitrary real constant.

$$u_7=\frac{38bk^2}{5}+2k^2\left(\sqrt{b}Tan\left[\sqrt{b}\left(ikx-\frac{867136}{25}b^3k^7it\right)\right]\right)^2.$$

(3.1.16)

where $b>0$, k is an arbitrary real constant.

$$u_8=\frac{38bk^2}{5}+2k^2\left(-\sqrt{b}Cot\left[\sqrt{b}\left(ikx-\frac{867136}{25}b^3k^7it\right)\right]\right)^2.$$

(3.1.17)

where $b>0$, k is an arbitrary real constant.

- **Family 3:**

$$u_9 = 12k^2\left(-\frac{1}{ikx}\right)^2.$$

(3.1.18)

where $b=0$, k is an arbitrary real constant.

- **Family 4:**

$$u_{10} = 6k^2\left(-\frac{1}{ikx-736b^3k^7it}\right)^2.$$

(3.1.19)

where $b=0$, k is an arbitrary real constant.

- **Family 5:**

$$u_{11} = \sqrt[3]{\frac{c}{140}} + 2k^2\left(-\frac{1}{ikx-ikct}\right)^2.$$

(3.1.20)

where $b=0$, k is an arbitrary real constant.

- **Example:** Consider (2+1) dimensional Konopelchenko-Dubrovsky (KD) equation,

$$u_t - u_{xxx} - 6cuu_x + \frac{3}{2}d^2u^2u_x - 3v_y + 3du_xv = 0,$$
$$u_y - v_x = 0.$$

(3.2.1)

where, c and d arbitrary real parameters, $\xi = ik(x-\alpha y-\beta t)$. We can use transformation with Eq. (2.1) then Eq. (3.2.1) become

$$-\beta u' + k^2u''' - 6cuu' + \frac{3}{2}d^2u^2u' + 3\alpha v' + 3dvu' = 0,$$
$$-\alpha u' - v' = 0.$$

(3.2.2)

When balancing u^2u' with u''' then gives $n_1 = 1$ and u' with v'then gives $n_2 = 1$. We may choose

$$u = a_0 + a_1F$$
$$v = b_0 + b_1F \; .$$

(3.2.3)

Substituting (3.2.3) into Eq. (3.2.2) yields a set of algebraic equations for a_0, a_1, b_0, b_1. These systems are finding as

$$-6a_0a_1bc + 3a_1bb_0d + \frac{3}{2}a_0^2a_1bd^2 + 2a_1b^2k^2 + 3bb_1\alpha - a_1b\beta = 0,$$
$$-6a_1^2bc + 3a_1bb_1d + 3a_0a_1^2bd^2 = 0,$$
$$-6a_0a_1c + 3a_1b_0d + \frac{3}{2}a_0^2a_1d^2 + \frac{3}{2}a_1^3bd^2 + 8a_1bk^2 + 3b_1\alpha - a_1\beta = 0,$$

(3.2.4)

$$-6a_1^2c + 3a_1b_1d + 3a_0a_1^2d^2 = 0,$$

$$\frac{3}{2}a_1^3d^2+6a_1k^2=0,$$
$$-bb_1-a_1b\alpha=0,$$
$$-b_1-a_1\alpha=0.$$

From the solutions of the system, we can found

- **Case 1:**

$$a_0=\frac{2c+d\alpha}{d^2},a_1=-\frac{2ik}{d},\ b_0=\frac{\frac{12c^2}{d^2}-4bk^2+3\alpha^2+2\beta}{6d},\ b_1=\frac{2ik\alpha}{d},d\neq 0,k\neq 0.$$

(3.2.5)

Case 2.

$$a_0=\frac{2c+d\alpha}{d^2},a_1=\frac{2ik}{d},\ b_0=\frac{\frac{12c^2}{d^2}+3\alpha^2+2\beta}{6d},\ b_1=-\frac{2ik\alpha}{d},d\neq 0,k\neq 0.$$

(3.2.6)

With the aid of Mathematica substituting (3.2.5) and (3.2.6) into (3.2.3), we have obtained the following complex solutions of equation (3.2.1). These solutions are:

- **Family 1:**

$$u_1=\frac{2c+d\alpha}{d^2}-\frac{2ik}{d}\left(-\sqrt{-b}Tanh\left[\sqrt{-b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$
$$v_1=\frac{\frac{12c^2}{d^2}-4bk^2+3\alpha^2+2\beta}{6d}+\frac{2ik\alpha}{d}\left(-\sqrt{-b}Tanh\left[\sqrt{-b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$

(3.2.7)

where $b<0$, k is an arbitrary real constant.

$$u_2=\frac{2c+d\alpha}{d^2}-\frac{2ik}{d}\left(-\sqrt{-b}Coth\left[\sqrt{-b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$
$$v_2=\frac{\frac{12c^2}{d^2}-4bk^2+3\alpha^2+2\beta}{6d}+\frac{2ik\alpha}{d}\left(-\sqrt{-b}Coth\left[\sqrt{-b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$

(3.2.8)

where $b<0$, k is an arbitrary real constant.

$$u_3=\frac{2c+d\alpha}{d^2}-\frac{2ik}{d}\left(\sqrt{b}Tan\left[\sqrt{b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$
$$v_3=\frac{\frac{12c^2}{d^2}-4bk^2+3\alpha^2+2\beta}{6d}+\frac{2ik\alpha}{d}\left(\sqrt{b}Tan\left[\sqrt{b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$

(3.2.9)

where $b>0$, k is an arbitrary real constant.

$$u_4=\frac{2c+d\alpha}{d^2}-\frac{2ik}{d}\left(-\sqrt{b}Cot\left[\sqrt{b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$

$$v_4 = \frac{\frac{12c^2}{d^2} - 4bk^2 + 3\alpha^2 + 2\beta}{6d} + \frac{2ik\alpha}{d}\left(-\sqrt{b}Cot\left[\sqrt{b}\,ik\left(x-\alpha y-\beta t\right)\right]\right)$$

(3.2.10)

where $b>0$, k is an arbitrary real constant.

- **Family 2:**

$$u_5 = \frac{2c+d\alpha}{d^2} + \frac{2ik}{d}\left(-\frac{1}{ik\left(x-\alpha y-\beta t\right)}\right)$$

$$v_5 = \frac{\frac{12c^2}{d^2} + 3\alpha^2 + 2\beta}{6d} - \frac{2ik\alpha}{d}\left(-\frac{1}{ik\left(x-\alpha y-\beta t\right)}\right)$$

(3.2.11)

where $b=0$, k is an arbitrary real constant.

5. CONCLUSIONS (SONUÇLAR)

In this paper, it is obtained solutions for (2+1) dimensional (KD) equation and seventh ordinary Lax KDV equations by using direct algebraic method. This method can be used to many other nonlinear equations or couple ones. In addition, this model is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

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