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A STUDY ON THE NUMERICAL SOLUTION OF THE THIRD-ORDER DISPERSIVE EQUATIONS WITH HOMOTOPY PERTURBATION METHOD

ABSTRACT

In this paper, a new approach for solving [12] third-order dispersive partial differential equation in one-and higher-dimensional spaces is proposed. This method is an effective procedure to obtain for approximate solutions in applied mathematics. The study outlines the significant features of the method .The analysis have illustrated by investigating third-order dispersive equation model problem. This paper is particularly concerned about a numerical computation with the Homotopy perturbation method. The numerical results demonstrate that the new method are quite accurate and readily implemented.
Keywords: Third-Order Dispersive Equations, Homotopy Perturbation Method, Partial Differential Equations, One-And Higher-Dimensional Spaces, Higher-Dimensional Spaces

ÜÇ BOYUTLU DİSPERSİVE DENKLEMİNİN HOMOTOPİ PERTÜRBASYON METODU İLE NÜMERİK ÇÖZÜMLERİ ÜZERİNE BİR ÇALIŞMA

ÖZET

Bu çalışmada, bir boyutlu ve yüksek boyutlu uzaylarda üç boyutlu dispersive diferensiyel denklemin çözümü için yeni bir yaklaşım [12] önerilmektedir. Uygulamalı matematikteki bu metod yaklaşık çözüm elde etmek için etkili bir yöntemdir. Çalışma bu metodun önemli özelliklerini ana hatları ile göstermektedir. Analizler, üçboyutlu dispersive denkleminin model problemi incelenerek örneklendirilmiştir. Bu makale özellikle Homotopi Pertürbasyon metodunun nümerik bir sayısal hesaplaması ile ilgilidir. Nümerik sonuçlar yeni metodun oldukça doğru ve hızlı uygulanabilir olduğunu göstermektedir.
Anahtar Kelimeler: Üçboyutlu Dispersive Denklemi, Homotopi Pertürbasyon Denklemi, Kısmi Diferensiyel Denklemler, Bir Boyutlu Uzay, Yüksek Boyutlu Uzaylar

1. INTRODUCTION (GİRİŞ)

In this paper, we use the homotopy perturbation method (HPM) [1, 2, 3, and 4] in order to find the analytic solutions of third-order dispersive equation [5 and 6]. The method in applied mathematics can be an effective procedure to obtain the analytic and approximate solutions. It is too important to find analytic solutions of third-order dispersive equations. This equation is a mathematical model of complex physical occurrences that arise in engineering, chemistry, biology, mechanics and physics. A novel approach to linear or nonlinear problems [7 and 8] is particularly valuable as a tool for Scientists and applied Mathematicians.

The technique has many advantages over the classical techniques [9 and 10]. Because the method does not need linearization or weak nonlinearity assumptions. It is providing an efficient explicit solution with high accuracy and minimal calculation. It does also not require discretization and consequently massive computation. In this method, the perturbation equation can be easily constructed by homotopy in topology and the initial approximation can also be freely selected. HPM [16 and 17] is the most effective and convenient on effor both linear and nonlinear equations. This method does not depend on a small parameter. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p\in[0,1]$, which is considered as a "small parameter".

HPM has been shown to effectively, easily and accurately solve a large class of linear and nonlinear problems [13] with components converging rapidly to accurate solution. HPM was first proposed by He [1, 2 and 3] and was succesfully applied to various engineering problems [2 and 3].

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this study, we implemented homotopy perturbation method with symbolic computation to construct new complex solutions for third-order dispersive equations.

3. METHOD AND ITS APPLICATIONS (YÖNTEM VE UYGULAMALARI)

Before starting to give HPM, we will give a simple description of HPM at the one-and higher-dimensional spaces.

3.1. The One-Dimensional Dispersive Equation (Bir Boyutlu Dispersive Denklemi)

The linear, third-order dispersive partial differential equation in one space, in its simplest form, is given by

$$\frac{\partial u}{\partial t}+\alpha\frac{\partial^3 u}{\partial x^3}=g(x,t),\ L_0<x<L_1,\ t>0,\alpha>0, \tag{1}$$

where $g(x,t)$ is a source term and initial conditions[13]

$$u(x,0)=f(x). \tag{2}$$

The first of all we must obtain form HMP. To illustrate the basic ideas of this method, we consider the following equation [1]:

$$A(u)-f(r)=0,\ r\in\Omega, \tag{3}$$

with boundary condition

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\ r\in\Gamma, \tag{4}$$

where A is a general differential operator, B is a boundary operator, f(r) is a known analytical function and Γ is the boundary of the domain Ω. A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Eq.(3) can therefore be rewritten as follows:

$$L(u)+N(u)-f(r)=0,\ r\in\Omega, \tag{5}$$

Homotopy perturbation structure is shown as follows:

$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0, \tag{6}$$

where

$$v(r,p):\Omega\times[0,1]\rightarrow R. \tag{7}$$

In eq. (6), $p\in[0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of eq. (6) can be written as a power series in p, as following:

$$v=v_0+pv_1+p^2v_2+p^3v_3+\cdots, \tag{8}$$

and the best approximation for solution is

$$u=\lim_{p\to 1}v=v_0+v_1+v_2+v_3+\cdots, \tag{9}$$

The above convergence is discussed in [1]. At the time, we can simply solve this third-order dispersive partial differential equation in one space with HPM [14].

In order to solve eq.(1), using HPM, we can construct a homotopy for this equation

$$(1-p)[\dot{Y}-\dot{u}_0]+p[\dot{Y}+\alpha Y'''-g(x,t)]=0, \tag{10}$$

$$\dot{Y}-\dot{u}_0-p\dot{Y}+p\dot{u}_0+p\dot{Y}+p\alpha Y'''-pg(x,t)=0, \tag{11}$$

where $\dot{Y}=\frac{\partial Y}{\partial t}$, $Y'''=\frac{\partial^3 Y}{\partial x^3}$ and $p\in[0,1]$. With initial approximation

$$Y_0=u_0(x,0)=f(x),$$

suppose the solution of eq. (10) has the form:

$$Y=Y_0+pY_1+p^2Y_2+p^3Y_3+\cdots=\sum_{n=0}^{\infty}p^nY_n(x,t), \tag{12}$$

$$Y=Y_0+pY_1+p^2Y_2+p^3Y_3+\cdots,$$

$$\dot{Y}=\dot{Y}_0+p\dot{Y}_1+p^2\dot{Y}_2+p^3\dot{Y}_3+\cdots,$$

$$Y'''=Y_0'''+pY_1'''+p^2Y_2'''+p^3Y_3'''+\cdots,$$

$$\left(\dot{Y}_0+p\dot{Y}_1+p^2\dot{Y}_2+p^3\dot{Y}_3\right)-\dot{u}_0+p\dot{u}_0+p\alpha\left(Y_0'''+pY_1'''+p^2Y_2'''+p^3Y_3'''\right)-pg(x,t)=0,$$

$$\dot{Y}_0+p\dot{Y}_1+p^2\dot{Y}_2+p^3\dot{Y}_3-\dot{u}_0+p\dot{u}_0+p\alpha Y_0'''+p^2\alpha Y_1'''+p^3\alpha Y_2'''-pg(x,t)=0,$$

Then, substituting eq. (12) into eq. (11), and rearranging based on powers of p-terms, we obtain:

$$p^0:\dot{Y}_0-\dot{u}_0=0, \tag{13}$$

$$p^1:\dot{Y}_1+\dot{u}_0+\alpha Y_0'''-g(x,t)=0, \tag{14}$$

$$p^2:\dot{Y}_2+\alpha Y_1'''=0, \quad (15)$$

$$p^3:\dot{Y}_3+\alpha Y_2'''=0, \quad (16)$$

$$\vdots$$

with solving eqs. (13);

$$p^0:\dot{Y}_0-\dot{u}_0=0\Rightarrow \dot{Y}_0=\dot{u}_0\Rightarrow Y_0=u_0=f(x)\Rightarrow Y_0=f(x), \quad (17)$$

$$p^1:\dot{Y}_1+\dot{u}_0+Y_0'''-g(x,t)=0\Rightarrow \dot{Y}_1=-\dot{u}_0-\alpha Y_0'''+g(x,t),$$
$$\Rightarrow Y_1=\int_0^t\left[-\dot{u}_0-\alpha Y_0'''+g(x,t)\right]dt, \quad (18)$$

$$p^2:\dot{Y}_2+\alpha Y_1'''=0\Rightarrow \dot{Y}_2=-\alpha Y_1'''\Rightarrow Y_2=\int_0^t\left[-\alpha Y_1'''\right]dt, \quad (19)$$

$$p^3:\dot{Y}_3+\alpha Y_2'''=0\Rightarrow \dot{Y}_3=-\alpha Y_2'''\Rightarrow Y_3=\int_0^t\left[-\alpha Y_2'''\right]dt, \quad (20)$$

$$\vdots$$

the above terms of the series (11) could be calculated. When we consider the series (11) with the terms (17)-(20) and suppose $p=1$, we obtain approximation solution of eq. (10) as following:

$$u(x,t)=Y_0+Y_1+Y_2+Y_3+\cdots, \quad (21)$$

$$u(x,t)=f(x)+\int_0^t\left[-\dot{u}_0-\alpha Y_0'''+g(x,t)\right]dt+\int_0^t\left[-\alpha Y_1'''\right]dt+\int_0^t\left[-\alpha Y_2'''\right]dt$$

As a result, the components $Y_0,Y_1,Y_2,Y_3,\cdots$ are identified and the series solution thus entirely determined [11, 12, 14 and 15].

3.2. The Higher-Dimensional Dispersive Equation (Yüksek Boyutlu Dispersive Denklemi)

The linear, third-order dispersive partial differential equation in higher-dimensional space, in its simplest form, is given by

$$u_t+au_{xxx}+bu_{yyy}+cu_{zzz}=g(x,y,z,t),\ L_0<x,y,z<L_1,\ t>0,\ \alpha,b,c>0, \quad (22)$$

where $g(x,y,z,t)$ is a source term and initial conditions

$$u(x,y,z,0)=f(x,y,z), \quad (23)$$

In order to solve eq.(22), using HPM, we can construct a homotopy for this equation.

$$(1-p)[\dot{Y}-\dot{u}_0]+p[\dot{Y}+\alpha Y'''+b\overset{'''}{Y}+c\overset{\cdots}{Y}-g(x,y,z,t)]=0, \quad (24)$$

$$\dot{Y}-\dot{u}_0-p\dot{Y}+p\dot{u}_0+p\dot{Y}+\alpha pY'''+pb\overset{'''}{Y}+cp\overset{\cdots}{Y}-pg(x,y,z,t)=0, \quad (25)$$

where $Y'''=\frac{\partial^3 y}{\partial x^3}$, $\dot{Y}=\frac{\partial y}{\partial t}$, $\overset{'''}{Y}=\frac{\partial^3 y}{\partial y^3}$, $\overset{\cdots}{Y}=\frac{\partial^3 y}{\partial z^3}$ and $p\in[0,1]$. With initial approximation $u_0(x,y,z,0)=f(x,y,z,)$ suppose the solution of eq. (22) has the form:

$$Y = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots = \sum_{n=0}^{\infty} p^n Y_n(x,t), \tag{26}$$

$$Y = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots,$$

$$\dot{Y} = \dot{Y}_0 + p\dot{Y}_1 + p^2\dot{Y}_2 + p^3\dot{Y}_3 + \cdots, \tag{27}$$

$$Y''' = Y_0''' + pY_1''' + p^2Y_2''' + p^3Y_3''' + \cdots,$$

$$\dddot{Y} = \dddot{Y}_0 + p\dddot{Y}_1 + p^2\dddot{Y}_2 + p^3\dddot{Y}_3 + \cdots,$$

$$\overset{'''}{Y} = \overset{'''}{Y}_0 + p\overset{'''}{Y}_1 + p^2\overset{'''}{Y}_2 + p^3\overset{'''}{Y}_3 + \cdots,$$

Then, substituting eq. (27) into eq. (25), and rearranging based on powers of p-terms, we obtain:

$$\dot{Y} - \dot{u}_0 - p\dot{Y} + p\dot{u}_0 + p\dot{Y} + \alpha pY''' + pb\overset{'''}{Y} + cp\dddot{Y} - pg(x,y,z,t) = 0,$$

$$\left(\dot{Y}_0 + p\dot{Y}_1 + p^2\dot{Y}_2 + p^3\dot{Y}_3\right) - \dot{u}_0 - p\left(\dot{Y}_0 + p\dot{Y}_1 + p^2\dot{Y}_2 + p^3\dot{Y}_3\right) + p\dot{u}_0$$

$$+p\left(\dot{Y}_0 + p\dot{Y}_1 + p^2\dot{Y}_2 + p^3\dot{Y}_3\right) + \alpha p\left(Y_0''' + pY_1''' + p^2Y_2''' + p^3Y_3'''\right)$$

$$+pb\left(\overset{'''}{Y}_0 + p\overset{'''}{Y}_1 + p^2\overset{'''}{Y}_2 + p^3\overset{'''}{Y}_3\right) + cp\left(\dddot{Y}_0 + p\dddot{Y}_1 + p^2\dddot{Y}_2 + p^3\dddot{Y}_3\right) - pg(x,y,z,t) = 0,$$

$$\dot{Y}_0 + p\dot{Y}_1 + p^2\dot{Y}_2 + p^3\dot{Y}_3 - \dot{u}_0 - p\dot{Y}_0 - p^2\dot{Y}_1 - p^3\dot{Y}_2 + p\dot{u}_0 + p\dot{Y}_0 + p^2\dot{Y}_1$$

$$+p^3\dot{Y}_2 + apY_0''' + ap^2Y_1''' + ap^3Y_2''' + pb\overset{'''}{Y}_0 + p^2b\overset{'''}{Y}_1 + bp^3\overset{'''}{Y}_2 + cp\dddot{Y}_0 + cp^2\dddot{Y}_1$$

$$+cp^3\dddot{Y}_2 - pg(x,y,z,t) = 0,$$

$$p^0: \dot{Y}_0 - \dot{u}_0 = 0, \tag{28}$$

$$p^1: \dot{Y}_1 + \dot{u}_0 + aY_0''' + b\overset{'''}{Y}_0 + c\dddot{Y}_0 - g(x,y,z,t) = 0, \tag{29}$$

$$p^2: \dot{Y}_2 + aY_1''' + b\overset{'''}{Y}_1 + c\dddot{Y}_1 = 0, \tag{30}$$

$$p^3: \dot{Y}_3 + aY_2''' + b\overset{'''}{Y}_2 + c\dddot{Y}_2 = 0, \tag{31}$$

$$\vdots$$

with solving eqs. (28);

$$p^0: \dot{Y}_0 - \dot{u}_0 = 0 \Rightarrow \dot{Y}_0 = \dot{u}_0 \Rightarrow Y_0 = u_0(x,y,z,o) = f(x,y,z,)$$
$$\Rightarrow Y_0 = f(x,y,z,), \tag{32}$$

$$p^1: \dot{Y}_1 + \dot{u}_0 + aY_0''' + b\overset{'''}{Y}_0 + c\dddot{Y}_0 - g(x,y,z,t) = 0$$
$$\Rightarrow Y_1 = -\int_0^t \left[\dot{u}_0 + aY_0''' + b\overset{'''}{Y}_0 + c\dddot{Y}_0 - g(x,y,z,t)\right] dt \tag{33}$$

$$p^2: \dot{Y}_2 + aY_1^{'''} + b\overset{'''}{Y_1} + c\overset{...}{Y_1} = 0$$
$$\Rightarrow Y_2 = -\int_0^t \left[aY_1^{'''} + b\overset{'''}{Y_1} + c\overset{...}{Y_1} \right] dt \qquad (34)$$
$$p^3: \dot{Y}_3 + aY_2^{'''} + b\overset{'''}{Y_2} + c\overset{...}{Y_2} = 0,$$
$$\Rightarrow Y_3 = -\int_0^t \left[aY_2^{'''} + b\overset{'''}{Y_2} + c\overset{...}{Y_2} \right] dt, \qquad (35)$$
$$\vdots$$

the above terms of the series (25) could be calculated. When we consider the series (25) with the terms (27) and suppose $p = 1$, we obtain approximation solution of eq. (22) as following:

$$u(x,y,z,t) = Y_0 + Y_1 + Y_2 + Y_3 + \cdots,$$
$$= f(x,y,z,t) - \int_0^t \left[\dot{u}_0 + aY_0^{'''} + b\overset{'''}{Y_0} + c\overset{...}{Y_0} - g(x,y,z,t) \right] dt$$
$$-\int_0^t \left[aY_1^{'''} + b\overset{'''}{Y_1} + c\overset{...}{Y_1} \right] dt - \int_0^t \left[aY_2^{'''} + b\overset{'''}{Y_2} + c\overset{...}{Y_2} \right] dt, \qquad (36)$$

As a result, the components $Y_0, Y_1, Y_2, Y_3, \cdots$ are identified and the series solution thus entirely determined.

EXAMPLE 1. (ÖRNEK 1)

We first discuss the simplest linear dispersive KdV equation

$$u_t + 2u_x + u_{xxx} = 0, \ t > 0, \qquad (37)$$

with initial condition

$$u(x,0) = \sin(x), \qquad (38)$$

In order to solve eq.(37), using HPM, we can construct a homotopy for this equation.

$$(1-p)[\dot{Y} - \dot{u}_0] + p[\dot{Y} + 2Y' + Y^{'''}] = 0, \qquad (39)$$

$$\dot{Y} - \dot{u}_0 - p\dot{Y} + p\dot{u}_0 + p\dot{Y} + 2pY' + pY^{'''} = 0, \qquad (40)$$

$$\dot{Y} - \dot{u}_0 + p\dot{u}_0 + 2pY' + pY^{'''} = 0, \qquad (41)$$

where $Y^{'''} = \frac{\partial^3 y}{\partial x^3}$, $\dot{Y} = \frac{\partial y}{\partial t}$, $Y' = \frac{\partial y}{\partial x}$, and $p \in [0,1]$. With initial approximation $u_0(x,0) = \sin(x)$ suppose the solution of eq. (37) has the form:

$$Y = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots = \sum_{n=0}^{\infty} p^n Y_n(x,t), \qquad (42)$$

$$Y = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots,$$
$$\dot{Y} = \dot{Y}_0 + p\dot{Y}_1 + p^2\dot{Y}_2 + p^3\dot{Y}_3 + \cdots, \qquad (43)$$
$$Y' = Y_0' + pY_1' + p^2Y_2' + p^3Y_3' + \cdots,$$
$$Y^{'''} = Y_0^{'''} + pY_1^{'''} + p^2Y_2^{'''} + p^3Y_3^{'''} + \cdots,$$

Then, substituting eq. (43) into eq. (41), and rearranging based on powers of p-terms, we obtain:

$$\dot{Y}-\dot{u}_0+p\dot{u}_0+2pY'+pY'''=0,$$

$$\left(\dot{Y}_0+p\dot{Y}_1+p^2\dot{Y}_2+p^3\dot{Y}_3\right)-\dot{u}_0+p\dot{u}_0$$

$$+2p\left(Y_0'+pY_1'+p^2Y_2'+p^3Y_3'\right)+p\left(Y_0'''+pY_1'''+p^2Y_2'''+p^3Y_3'''\right)=0,$$

$$\dot{Y}_0+p\dot{Y}_1+p^2\dot{Y}_2+p^3\dot{Y}_3-\dot{u}_0+p\dot{u}_0+2pY_0'+2p^2Y_1'+2p^3Y_2'+pY_0'''$$
$$+p^2Y_1'''+p^3Y_2'''=0,$$

$$p^0:\dot{Y}_0-\dot{u}_0=0, \tag{44}$$

$$p^1:\dot{Y}_1+\dot{u}_0+2Y_0'+Y_0'''=0, \tag{45}$$

$$p^2:\dot{Y}_2+2Y_1'+Y_1'''=0, \tag{46}$$

$$p^3:\dot{Y}_3+2Y_2'+Y_2'''=0, \tag{47}$$

$$\vdots$$

with solving eqs. (44);

$$p^0:\dot{Y}_0-\dot{u}_0=0\Rightarrow \dot{Y}_0=\dot{u}_0\Rightarrow Y_0=u_0(x,o)=\sin(x)\Rightarrow Y_0=\sin(x), \tag{48}$$

$$p^1:\dot{Y}_1+\dot{u}_0+2Y_0'+Y_0'''=0\Rightarrow \dot{Y}_1=-\dot{u}_0-2Y_0'-Y_0'''=0,$$

$$\Rightarrow \dot{Y}_1=-2\cos(x)+\cos(x)=-\cos(x), \tag{49}$$

$$\Rightarrow Y_1=\int_0^t\left[-\cos(x)\right]dt=-t\cos(x),$$

$$p^2:\dot{Y}_2+2Y_1'+Y_1'''=0\Rightarrow \dot{Y}_2=-2Y_1'-Y_1''',$$

$$\Rightarrow \dot{Y}_2=-2t\sin(x)+t\sin(x)=-t\sin(x), \tag{50}$$

$$\Rightarrow Y_2=\int_0^t\left[-t\sin(x)\right]dt=\frac{-t^2}{2!}\sin(x),$$

$$p^3:\dot{Y}_3+2Y_2'+Y_2'''=0\Rightarrow \dot{Y}_3=-2Y_2'-Y_2''',$$

$$\Rightarrow \dot{Y}_3=t^2\cos(x)-\frac{t^2}{2!}\cos(x)=\frac{t^2}{2!}\cos(x), \tag{51}$$

$$\Rightarrow Y_3=\int_0^t\left[\frac{t^2}{2!}\cos(x)\right]dt=\frac{t^3}{3!}\cos(x),$$

$$\vdots$$

the above terms of the series (42) could be calculated. When we consider the series (42) with the terms (43) and suppose $p=1$, we obtain approximation solution of Eq. (37) as following:

$$u(x,t) = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots,$$

$$u(x,t) = \lim_{p\to 1}\left(Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots\right),$$

$$= Y_0 + Y_1 + Y_2 + Y_3,$$

$$= \sin(x) - t\cos(x) - \frac{t^2}{2!}\sin(x) + \frac{t^3}{3!}\cos(x), \tag{52}$$

Exact solution of equation is $u(x,t)=\sin(x-t)$.

EXAMPLE 2.(ÖRNEK 2)

We closest consider the linear dispersive KdV equation in a two dimensional space

$$u_t + u_{xxx} + u_{yyy} = 0,\ t>0, \tag{53}$$

with initial condition

$$u(x,y,0)=\cos(x+y), \tag{54}$$

In order to solve eq.(53), using HPM, we can construct a homotopy for this equation.

$$(1-p)[\dot{Y}-\dot{u}_0]+p[\dot{Y}+Y''' + \overset{'''}{Y}]=0, \tag{55}$$

$$\dot{Y}-\dot{u}_0-p\dot{Y}+p\dot{u}_0+p\dot{Y}+pY'''+p\overset{'''}{Y}=0, \tag{56}$$

$$\dot{Y}-\dot{u}_0+p\dot{u}_0+pY'''+p\overset{'''}{Y}=0, \tag{57}$$

where $Y'''=\dfrac{\partial^3 y}{\partial x^3}$, $\dot{Y}=\dfrac{\partial y}{\partial t}$, $\overset{'''}{Y}=\dfrac{\partial^3 y}{\partial y^3}$, and $p\in[0,1]$. With initial approximation $u(x,y,0)=\cos(x+y)$, suppose the solution of eq. (53) has the form:

$$Y = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots = \sum_{n=0}^{\infty} p^n Y_n(x,t) \tag{58}$$

$$Y = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots,$$

$$\dot{Y} = \dot{Y}_0 + p\dot{Y}_1 + p^2\dot{Y}_2 + p^3\dot{Y}_3 + \cdots, \tag{59}$$

$$\overset{'''}{Y} = \overset{'''}{Y}_0 + p\overset{'''}{Y}_1 + p^2\overset{'''}{Y}_2 + p^3\overset{'''}{Y}_3 + \cdots,$$

$$Y''' = Y_0''' + pY_1''' + p^2Y_2''' + p^3Y_3''' + \cdots,$$

Then, substituting eq. (59) into eq. (57), and rearranging based on powers of p-terms, we obtain:

$$\dot{Y}-\dot{u}_0+p\dot{u}_0+pY'''+p\overset{'''}{Y}=0,$$

$$\left(\dot{Y}_0+p\dot{Y}_1+p^2\dot{Y}_2+p^3\dot{Y}_3\right)-\dot{u}_0+p\dot{u}_0$$

$$+p\left(Y_0'''+pY_1'''+p^2Y_2'''+p^3Y_3'''\right)+p\left(\overset{'''}{Y}_0+p\overset{'''}{Y}_1+p^2\overset{'''}{Y}_2+p^3\overset{'''}{Y}_3\right)=0,$$

$$\dot{Y}_0+p\dot{Y}_1+p^2\dot{Y}_2+p^3\dot{Y}_3-\dot{u}_0+p\dot{u}_0+pY_0'''+p^2Y_1'''+p^3Y_2'''+p\overset{'''}{Y}_0+p^2\overset{'''}{Y}_1+p^3\overset{'''}{Y}_2=0,$$

$$p^0 : \dot{Y}_0 - \dot{u}_0 = 0, \tag{60}$$

$$p^1 : \dot{Y}_1 + \dot{u}_0 + \overset{'''}{Y}_0 + Y_0''' = 0, \tag{61}$$

$$p^2 : \dot{Y}_2 + \overset{'''}{Y}_1 + Y_1''' = 0, \tag{62}$$

$$p^3 : \dot{Y}_3 + \overset{'''}{Y}_2 + Y_2''' = 0, \tag{63}$$

$$\vdots$$

with solving eqs. (60);

$$p^0 : \dot{Y}_0 - \dot{u}_0 = 0 \Rightarrow \dot{Y}_0 = \dot{u}_0,$$

$$\Rightarrow Y_0 = u_0(x,o) = \cos(x+y), \tag{64}$$

$$\Rightarrow Y_0 = \cos(x+y),$$

$$p^1 : \dot{Y}_1 + \dot{u}_0 + \overset{'''}{Y}_0 + Y_0''' = 0, \Rightarrow \dot{Y}_1 = -\dot{u}_0 - \overset{'''}{Y}_0 - Y_0''' = 0,$$

$$\Rightarrow \dot{Y}_1 = -2\sin(x+y), \tag{65}$$

$$\Rightarrow Y_1 = \int_0^t \left[-2\sin(x+y)\right]dt = (-2t)\sin(x+y),$$

$$p^2 : p^2 : \dot{Y}_2 + \overset{'''}{Y}_1 + Y_1''' = 0 \Rightarrow \dot{Y}_2 = -\overset{'''}{Y}_1 - Y_1''',$$

$$\Rightarrow \dot{Y}_2 = -4t\cos(x+y), \tag{66}$$

$$\Rightarrow Y_2 = \int_0^t \left[-4t\cos(x+y)\right]dt = \frac{-(2t)^2}{2!}\cos(x+y),$$

$$p^3 : \dot{Y}_3 + \overset{'''}{Y}_2 + Y_2''' = 0 \Rightarrow \dot{Y}_3 = -\overset{'''}{Y}_2 - Y_2'''$$

$$\Rightarrow \dot{Y}_3 = \frac{8t^2}{2!}\sin(x+y), \tag{67}$$

$$\Rightarrow Y_3 = \int_0^t \left[\frac{8t^2}{2!}\sin(x+y)\right]dt = \frac{8t^3}{3!}\sin(x+y),$$

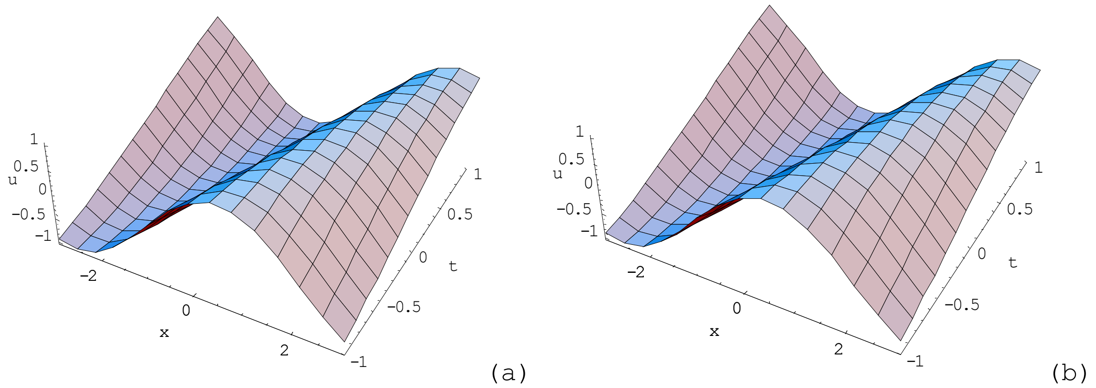
$$\vdots$$

the above terms of the series (60-63) could be calculated. When we consider the series (64-67) with the terms (57) and suppose $p = 1$, we obtain approximation solution of Eq. (53) as following:

$$\begin{aligned} u(x,t) &= Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots, \\ u(x,t) &= \lim_{p\to 1}\left(Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots\right), \\ &= Y_0 + Y_1 + Y_2 + Y_3, \\ &= \cos(x+y) + (-2t)\sin(x+y) - \frac{(2t)^2}{2!}\cos(x+y) + \frac{8t^3}{3!}\sin(x+y), \end{aligned} \tag{68}$$

Exact solution of equation is $u(x,y,t) = \sin(x+y+2t)$.

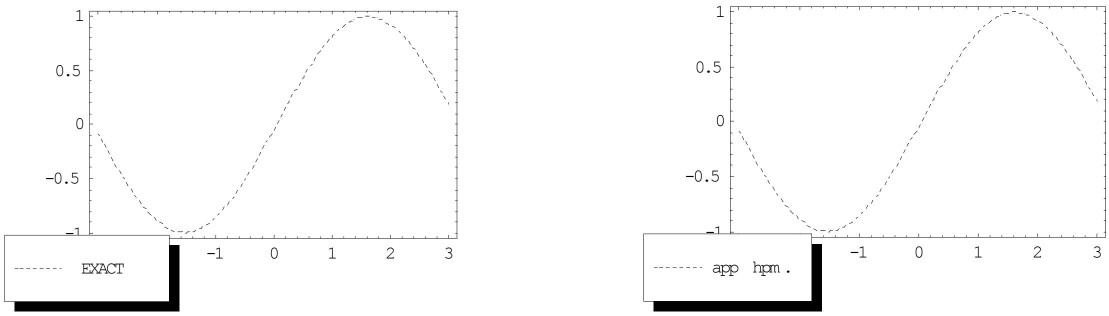
The approximation can be also obtained by y-direction or by alternating use of x-and y-directions iterations formula.



(a):Exact solution(kesin çözüm)
(b): Approximation solution (yaklaşık çözüm)

Figure 1. The plots 3D of the numerical results for Y_3 in comparison with the analytic solution $u(x,y)$ when t =0.05 with initial condition of eq.(37) by means of HPM

(Şekil 1. HPM vasıtasıyla başlanğıc şartı ile denklem (37)nin t =0.05 alındığı zaman $u(x,y)$ analitik çözümü ile Y_3ün sayısal sonuçlarının üç boyutlu grafiği)



(a):Exact solution(kesin çözüm)
(b): Approximation solution (yaklaşık çözüm)

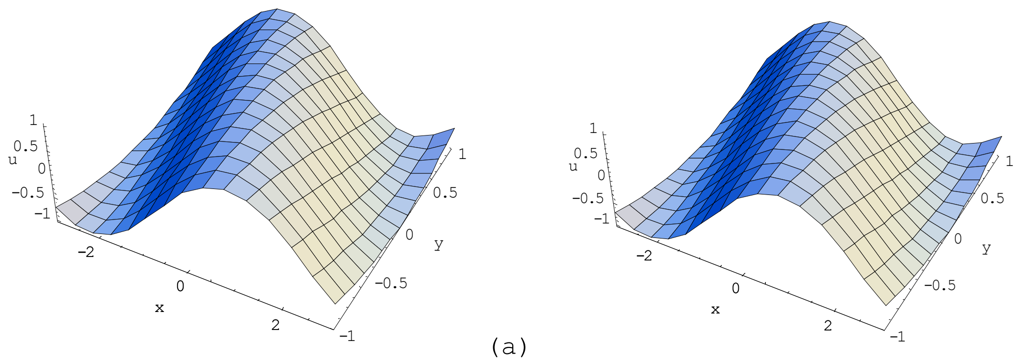
Figure 2. The plots of the numerical results for Y_3 in comparison with the analytic solution $u(x,y)$ when t =0.05 with initial condition of eq. (37) by means of HPM

(Şekil 2. HPM vasıtasıyla başlanğıc şartı ile denklem (37)nin t =0.05 alındığı zaman $u(x,y)$ analitik çözümü ile Y_3ün sayısal sonuçlarının grafiği)

Table 1. The numerical results for Y_3 in comparison with the analytic solution $u(x,y)$ when t =0.05 with initial condition of eq. (37) by means of HPM

(Tablo 1. HPM vasıtasıyla başlanğıc şartı ile denklem (37)nin t =0.05 alındığı zaman $u(x,y)$ analitik çözümü ile Y_3 ün sayısal sonuçlarının karşılaştırılması)

t\x	-10	-5
0.1	$-0,277355 \times 10^{-3}$	0.985246×10^{-4}
0.2	$-0,219907 \times 10^{-2}$	0.81952×10^{-3}
0.3	$-0,735163 \times 10^{-2}$	0.28699×10^{-2}
0.4	0.172517×10^{-1}	0.704476×10^{-2}
0.5	0.333391×10^{-1}	0.14223×10^{-1}



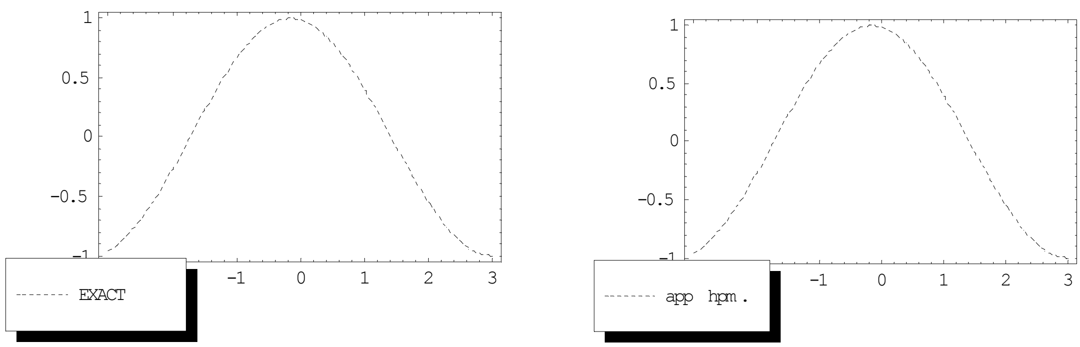
(a) (b)

(a): Exact solution (kesin çözüm)

(b): Approximation solution (yaklaşık çözüm)

Figure 3. The plots 3D of the numerical results for Y_3 in comparison with the analytic solution $u(x,y)$ when t =0.05 with initial condition of eq. (53) by means of HPM

Şekil 3. HPM vasıtasıyla başlanğıc şartı ile denklem (53)nin t =0.05 alındıgı zaman $u(x,y)$ analitik çözümü ile Y_3 ün sayısal sonuçlarını üç boyutlu grafiği



(a): Exact solution (kesin çözüm) (b): Approximation solution (yaklaşık çözüm)

Figure 4. The plots of the numerical results for Y_3 in comparison with the analytic solution $u(x,y)$ when t =0.05 with initial condition of eq. (53) by means of HPM

(Şekil 4. HPM vasıtasıyla başlanğıc şartı ile denklem (53)nin t =0.05 alındığı zaman $u(x,y)$ analitik çözümü ile Y_3 ün sayısal sonuçlarının grafiği)

Table 2. The numerical results for Y_3 in comparison with the analytic solution $u(x,y)$ when t =0.05 with initial condition of eq. (53) by means of HPM
(Tablo 2. HPM vasıtasıyla başlanğıc şartı ile denklem (53)nin t =0.05 alındığı zaman $u(x,y)$ analitik çözümü ile Y_3 ün sayısal sonuçlarının karşılaştırılması)

t\x	25	30
0.1	$0.663748x10^{-4}$	$0,132287\ x10^{-4}$
0.2	$0.106318x10^{-2}$	$0,252824\ x10^{-3}$
0.3	$0.537399x10^{-2}$	$0,148366x10^{-2}$
0.4	$0.169132x10^{-1}$	$0,531466x10^{-2}$
0.5	$0.410106x10^{-1}$	$0,144496x10^{-1}$

4. CONCLUSION (SONUÇ)

In this work, the HPM [17] was used for third-order dispersive partial differential equation in one space and in two space with initial conditions and obtained their analytic solutions. The analytic solutions of the nonlinear equations have a fundamental importance.

Various effective methods [7 and 8] have been developed to understand the mechanisms of these physical models, to help physicists and engineers and to ensure knowledge for physical problems and its applications. A clear conclusion can be draw from the numerical results that the HPM algorithm provides analytic solutions without spatial discretizations for the nonlinear partial differential equations.

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