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NEW MULTIPLE SOLUTION TO THE SHALLOW WATER WAVE EQUATION

ABSTRACT

In this paper by considering an improved tanh function method, we found some exact solution of Shallow Water Wave equation. The main idea of this method is to take full advantage of the Riccati equation which has more new solutions.
Keywords: Tanh Function Method, Riccati Equation, Shallow Water Wave Equation.

SHALLOW WATER WAVE DENKLEMİ İÇİN YENİ ÇOK YÖNLÜ ÇÖZÜM

ÖZET

Bu çalışmada, geliştirilmiş tanh metodu göz önüne alınarak Shallow Water Wave denkleminin bazı kesin sonuçları elde edilmiştir. Metodun temel amacı Riccati denkleminin bütün avantajlarını kullanarak daha yeni çözümler elde etmektir.
Anahtar Kelimeler: Tanh Fonksiyon Metodu, Riccati Denklemi, Shallow Water Wave Denklemi

1. INTRODUCTION (GİRİŞ)

In recent years, nonlinear phenomena play a crucial role in applied mathematics and physics. Directly searching for exact solutions of nonlinear partial differential equations (PDEs) has become more and more attractive partly due to the availability of computer symbolic systems like mathematica or maple that allow us to perform some complicated and tedious algebraic calculation on a computer as well as helping us to find exact solutions of PDEs [1, 2, 3 4 and 5] now.

Many explicit exact methods have been introduced in literature [6, 7, 8, 9, 10, 11, 12, 13, 14 and 15]. Some of them are Painleve method, homogeneous balance method, similarity reduction method, sine-cosine method, Darboux transformation, Cole-Hopf transformation, Generalized Miura transformation, tanh method, Backlund transformation and others methods [16 and 17].

One of the most effectively straightforward method constructing exact solution of PDEs is the extended tanh function method [18]. Let us simply describe the tanh function. For doing this, one can consider in two variables general form of nonlinear PDE

$$H(u,\ u_{tt},\ u_x,\ u_{xx},\dots)=0 \qquad\qquad (1.1)$$

and transform Equation (1.1) with

$$u(x,t)=u(\xi), \qquad \xi=k(x-\lambda t)$$

where, k and λ are the wave number and wave speed respectively. After the transformation, we get a nonlinear ODE for $u(\xi)$

$$H'\ (u',u'',u''',\dots)=0 \qquad\qquad (1.2)$$

The fact that the solutions of many nonlinear equations can be expressed as a finite series of tanh functions motivates us to seek for the solutions of Eq. (1.2) in the form

$$u(x,t)=\ u(\xi)=\sum a_i\tanh^i\ (\xi)=\sum a_i F^i \qquad\qquad (1.3)$$

where $F^i=\tanh^i(\xi)$, an equation for $F(\xi)$ is obtained. m is a positive integer that can be determined by balancing the linear term of highest order with the nonlinear term in Eq.(1.1) and k, λ, $a_1,a_2,\dots a_m$ are parameters to be determined.

Substituting solution (1.3) into Eq.(2.1) yields a set of algebraic equations for F^i, then all coefficients of F^i have to vanish. From these relations k, λ, $a_1,a_2,\dots a_m$ can be determined.

In this work, we will consider to solve Shallow Water Wave equation by using the improved tanh function method which is introduced by Chen and Zhang [19 and 20].

2. METHOD AND ITS APPLICATIONS (YÖNTEM VE UYGULAMALARI)

The main idea of this method is to take full advantage of the Riccati equation that tanh function satisfies and uses its solutions F to replace tanhξ. The required Riccati equation is given as

$$F'=A+BF+CF^2 \qquad\qquad (2.1)$$

where $F=dF/d\xi$ and A,B,C are constant. In the following, Chen and Zhang [19] have given several cases to get solution of Eq(2.1) in the form of finite series of tanh functions (1.3).

Case 1. If C=0, B≠0, then (2.1) has the solutions $(\exp(B\xi)-A)/B$

Case 2. If A=1/2, B=0, C=-1/2, then (2.1) has the solutions
$\coth\xi\pm\operatorname{csch}\xi$, $\tanh\xi\pm i\operatorname{sech}\xi$ ($i^2=-1$)

Case 3. If A=C=±1/2, B=0, then (2.1) has the solutions $\sec\xi\pm\tan\xi$, $\csc\xi\pm\cot\xi$.

Case 4. If A=1, B=0, C=-1, then (2.1) has the solutions $\tanh\xi$, $\coth\xi$.

Case 5. If A=C=1, B=0, then (2.1) has the solutions $\tan\xi$.

Case 6. A=C=-1, B=0, then (2.1) has the solutions $\cot\xi$.

Case 7. A=B=0, C≠0, then (2.1) has the solutions $-1/(c\xi+c_0)$.

We illustrate the method by considering the Shallow Water Wawe equation.

3. EXAMPLE (ÖRNEK)
Shallow Water Wave equation

$$u_{xxt}+au_x u_{xt}+B u_t u_{xx}- u_{xt}- u_{xx}=0 \tag{3.0}$$

If we accept that $\alpha=-1$, $\beta=-1$, m=2 we conclude (3.1) equation by (3.0) equation

$$U_{xxxt}-u_x u_{xt}-u_t u_{xx}-u_{xt}-u_{xx}=0 \tag{3.1}$$

for doing this example. We could use transformation with Eq.(1.1) for the equation. Let us Shallow Water Wawe consider equation solutions

$$u(x,t)=u(\xi), \quad \xi=kx-kwt \tag{3.2}$$

Substituting (3.2) into (3.1), we get

$$-k^4wu^{(4)}-\alpha k^3wu'u''-k^3w\beta u'u''+k^2wu''-k^2u''=0 \tag{3.3}$$
$$-k^4wu^{(4)}+2k^3wu'u''+ k^2wu''-k^2u''=0$$

an integrating (3.3) following equation and when we assume that integration constant is zero

$$-k^2wu^{(4)}+2kwu'u''+ wu''-u''=0 \tag{3.4}$$
$$-k^2wu'''+2k^3w(u')^2+ wu'-u'=0$$

when balancing (u.u') with (u''') then gives m=2. Therefore, we may choose
$$u=a_0+a_1F. \tag{3.5}$$

Substituting (3.5) into (3.4) along with Eq(2.1) and using Mathematica yields a system of equations w, t, F^M. Setting the

coefficients of F^M in the obtained system of equations to zero. We can deduce the following set of algebraic polynomials with the respect unknowns a_0, a_1, a_2 namely.

$u'=a_1A+a_1BF+a_1CF^2$

$u''=a_1AB+a_1B^2F+2a_1ACF+3a_1BCF^2+2a_1C^2F^3$

$u'''=a_1B^2(A+BF+CF^2)+2a_1AC(A+BF+CF^2)+6a_1BCF(A+BF+CF^2)+6a_1C^2F^2(A+BF+CF^2)$

$u'''=a_1AB^2+a_1B^3F+7a_1B^2CF^2+2a_1A^2C+8a_1ABCF+8a_1AC^2F^2+12a_1BC^2F^3+6a_1C^3F^4$

and

$F^0:-k^2wa_1A^2B-2k^2wa_1A^2C-a_1A+kwa_1{}^2A^2+a_1wA=0$

$F^1:-k^2wa_1B^3-8k^2wa_1ABC+a_1wB-a_1B+2kwa_1{}^2AB=0$

$F^2:-7k^2wa_1B^2C-8k^2wa_1AC^2+a_1wC-a_1C+2kwa_1{}^2AC+kwa_1B^2=0$ (3.6)

$F^3:-12k^2wa_1BC^2+2kwa_1BC=0$

$F^4:-6k^2a_1C^3+kwa_1{}^2C^2=0$

From the solutions of the system, we can find

$a_0=0$, $a_1=6kC$

$w=1/(4k^2AC+1)$

with the aid of Mathematica, we find
- When we choose A=1, B=0, C=1 in Eq(3.6) then
$a_0=0$, $a_1=6k$, $w=1/(4k^2+1)$
Therefore, the solution can be found as

$u(x,t)=6k\ \tan[k(x-(t/4k^2+1))]$

In the case, if we choose A=-1, B=0, C=-1 in Eq(3.6) then
$a_0=0$, $a_1=-6k$, $w=1/(4k^2+1)$

$u(x,t)=-6k\ \cot[k(x-(t/(4k^2+1)))]$

- Again, when we choose A=1, B=0, C=-1 then from the Eq(3.6)
$a_0=0$, $a_1=-6k$, $w=1/(-4k^2+1)$

$u(x,t)=-6k\ \tanh[k(x-(t/-4k^2+1))]$
$u(x,t)=-6k\ \coth[k(x-(t/-4k^2+1))]$

- When we choose A=1/2, B=0, C=1/2 then we can find the coefficients of Eq(3.6) as
$a_0=0$, $a_1=3k$, $w=1/(k^2+1)$
and using the coefficients, the solutions can be found as

$u(x,t)=3k[\sec[k(x-(t/k^2+1))]]+\tan[k(x-(t/k^2+1))]]$

$u(x,t)=3k[\csc[k(x-(t/k^2+1))]+\cot[k(x-(t/k^2+1))]]$

- When we choose A=-1/2, B=0, C=-1/2 then we can find the coefficients of Eq(3.6)
$a_0=0$, $a_1=-3k$, $w=1/(k^2+1)$

$u(x,\ t)=-3k[\sec[k(x-(t/k^2+1))]-\tan[k(x-(t/k^2+1))]]$

- When we choose A=1/2, B=0, C=-1/2 then we can find the coefficients of Eq(3.6)
$a_0=0$, $a_1=-3k$, $w=1/(-k^2+1)$
$u(x,t)=-3k[\coth[k(x+(t/k^2+1))]\pm\csch[k(x+(t/k^2+1))]]$
$u(x,t)=-3k[\tanh[k(x+(t/k^2+1))]\ \pm i.\sech[k(x+(t/k^2+1))]]$

4. CONCLUSION (SONUÇ)

We have presented a generalized tanh function method and used it to solve the Shallow Water Wave equation. In fact this method is readily applicable to a large variety of nonlinear PDEs.

Firstly, all the nonlinear PDEs whish can be solved by other tanh function method can be solved easily by this method. Secondly we have used only the special solutions of Eq(2.1). If we use only the special solutions of Eq(2.1), we can obtain more solutions. We are also aware of the fact that not all fundamental equations can be treated with the method.

We also obtain some new and more general solutions at same time. Furthermore, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

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