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Stochastic Modelling of Pilling Degree Changes During the Pilling Process of Wool Fabrics

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ABSTRACT

As a fabric surface defect, pilling gives clothes an unpleasant appearance and is often characterized with small, complex clusters of fibers attaching to the surface of the garment caused by the fiber migration from yarns to the fabric surface as the fabric rubs against itself, another fabric, or even the skin. In this study, a Markov chain model was built based on the pilling propensity of wool fabrics, evaluated with a scale ranging from 1 (severe pilling) to 5 (non-pilling). These degrees were defined as the state space of Markov chain. The numerical values of the transition probability matrix related to the pilling degrees were obtained by maximum likelihood estimation (MLE). Based on the matrix, it was intended to model the changes in the pilling process of woven wool fabrics. Furthermore, given that the fabric will eventually be in state 1, 2 or 3, accepted as unpleasant appearance; the conditional mean first passage times for any transient state to enter any recurrent state for the first time were determined.

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KEYWORDS

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1. INTRODUCTION

Defining and evaluating the concept of pilling is a widely defined problem in textile production. Pilling is a fabric surface defect characterized with small and complex fiber clusters attached to the fabric surface, giving an unpleasant appearance. Moreover, pill formation leads to shortening the life cycle of clothes due to wear off and it sometimes causes problems resulting in the formation of holes [1]. There is a dynamic equilibrium in terms of the number of pills on a fabric surface between the pill formation and pill wear off, due to the abrasion, loose fibers are transferred to the fabric surface and a layer of fuzz is formed, then with the applied abrasion forces, these loose fibers are entangled. As the abrasion proceeds, the anchor fibers are eventually broken, and the pills break off. When pill formation rates are equal to pill wear off rates, an equilibrium is maintained as shown in Figure 1 [2-5].

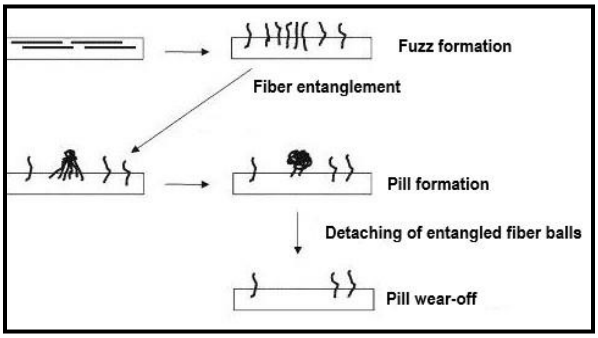

Figure 1. Schematic pilling formation [5]

The first studies on fabric pilling property were carried out between 1970 and 1990 and they were mostly on the pilling formation mechanism and modelling. Since then, the studies focused on fiber and yarn properties affecting pilling propensity as well as the effects of finishing treatments and pilling tendencies of different fabric

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structures and pilling test methods. By 2000`s, these studies generally investigated the objective evaluation of pilling with image analysis. However, only a few studies have been reported concerning the stochastic modelling of the pilling characteristics. The pill formation on fabric surface occurs in multiple states and transitions between states depend only on the last state of the fabric. The transition from the current state to the next state involves uncertainty and therefore, pilling process has a stochastic structure. Markov chains have an important role in describing this kind of systems indicating stochastic behaviour that changes over time [6]. Modelling of pilling and determining the transitions from every stage of pilling process is important in terms of predicting the pilling propensity of the fabrics and obtaining a probabilistic structure of pilling formation especially in the fabric development phases and before the fabric is reached to the consumer. By considering this situation, the pilling formation on wool fabrics was investigated in this study as a stochastic process and assumed to be a Markov chain.

Application of Markov principle is widely used for predictions in medical sciences, engineering, and financial management. Transition probability matrix handles the behaviour of a Markov chain and its elements determine the probability of moving from one state to the other state in a time interval. In studies based on observational data and using Markov chains to model any system behaviour, transition probability matrix is estimated from observational data using MLE. There are many studies using Markov models and focusing on estimating parameters by means of MLE, such as disease progression modelling, engineering, and financial area. However, there has not been that much research in modelling any process related to usage life of fabrics based on Markov chains.

Markov chains are classified according to time parameter and state space. If the state space and time parameter are discrete, the stochastic process is called discrete-time Markov chains. Fabric pilling formation is occurred at discrete time points and the pilling states of the fabric are discrete, therefore in this study, the fabric pilling process was modelled with discrete-time Markov chain. Discrete time Markov chains have been frequently used to model or evaluate chronic disease progression since chronic diseases can be described according to specific health states and Markov chains enable modelling transitions among health states. In these types of studies based on observational data, transition probability matrix is estimated using MLE [7-10]. In addition to disease progression modelling, there are various studies on engineering, finance and other fields using Markov chains focusing on the estimation of parameters with MLE. Malik and Thomas (2012) developed a Markov chain model based on behavioural scores of consumers to predict the credit risk of portfolios [11]. Shi et al. (2011) modelled driving cycles of vehicles using Markov theory and created a transition probability matrix using MLE based on experimental data [12]. Assuming that the next action of a user was only dependent on the last action of the user, Chierichetti et al. (2012), used Markov theory and MLE to model browsing behaviours of users on the web [13].

Considering the use of Markov chains on textile industry, Paras and Pal (2018) developed a model for counting the number of cycles that a garment can do in a reuse-based closed loop cycle. The proposed model was used to examine the textile waste flow in Scandinavian countries such as Denmark, Finland, Iceland, Norway, and Sweden [14]. Kumar, et.al (2018) studied fabric finishing system of a textile industry. A performance evaluating model of the system was obtained based on Markov-Birth-Death process with probabilistic method. The teaching-learning-based optimization (TLBO) was used as the optimization algorithm [15]. Baycan and Yildirim (2016) investigated the cyclical economic asymmetric behavioural dynamics of the Turkish textile and apparel industries. To this end, the hidden Markov regime switching models were used. The authors also showed the estimated Markov probabilities, average durations, and percentages of staying in the same state [16]. Badea, et.al (2016) designed a proper Markov chain to model data from a sub-branch of textile industry. The stochastic processes were used to optimize the time of the textile manufacturing process [17]. Kumar, et.al (2016) analysed the performance modelling and the availability of the fabric finishing system that consisted of four main subsystems of textile industry. Markov-Birth-Death process was used to evaluate the performance and analysis of availability [18]. Afrinaldi (2020), modelled the lifecycle of a product using the Markov chain and validated with the analysis of plastic lifecycle. The number of trips and duration of stay of a product in a particular lifecycle stage, the number of products visiting a specific lifecycle stage, the probability of a product being discarded and incinerated, and the expected total environmental impact of the product are predicted [19].

In this study, the pilling process of woollen fabrics, characteristically prone to pill was investigated by using stochastic modelling. Woollen fabrics are inherently staining resistant, fire retardant, temperature regulating and have wonderful acoustic properties however they are more prone to pilling, therefore the pilling process was carried out with 100% woven fabrics in this study. The pilling propensity on the surface of the fabric can be modelled as a Markov chain. This is because the pilling degrees indicated in Table 2 depend on the previous degrees. Estimates made using the Markov chain are important in terms of revealing the service life of the fabric. For this purpose, 58 different woven, 100% wool fabrics were subjected to pilling process in different pilling cycles to observe the changing of pilling degree by the time and the stochastic model was applied to this experimental data. In order to eliminate the subjectivity of pilling evaluation, PillGrade instrument was used for the assessment of pilling degree to the tested fabrics. Although there are current studies on the pilling mechanism, the transition between the pilling states has not been investigated in terms of stochastic approach before. Therefore, the main purpose of this study is to model the changes in terms of pilling degree of woollen fabrics during pilling process based on the generated Markov probability transition matrix. Accordingly, transition probability matrix of the pilling degrees was estimated with MLE. In addition, the conditional mean first passage times for any transient state to enter any recurrent state for the first time were determined, assuming that the fabric will eventually be in unpleasant appearance.

2. MATERIAL AND METHOD

2.1 Material

For the study, 58 different woven 100% wool fabrics in various woven constructions were used. The weight per unit area values were ranged between 150-300 g/m^2 and thickness values were in the range of 0.18 mm to 1.57 mm. The mass per unit area, thickness, warp and weft yarn count, warp and weft density values of the fabrics are given in Table 1. Besides the structural properties, photos of the fabrics taken by Leica S8APO Streo Microscobe are given in the Table 1, as well.

Table 1. The physical and structural properties of the fabrics used in the study

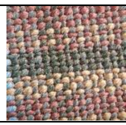
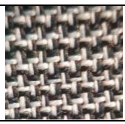
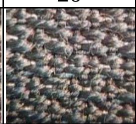
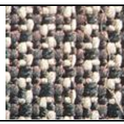
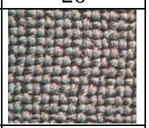
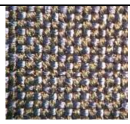
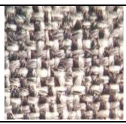
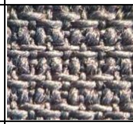
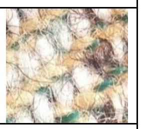
Fabric codes							
	F1	**F2**	**F3**	**F4**	**F5**	**F6**	**F7**
Mass per unit area (g/m^2)	251	167	187	163	165	166	192
Thickness (mm)	0,53	0,21	0,26	0,23	0,18	0,18	0,30
Warp yarn count (tex)	26	28	32	20	30	26	26
Weft yarn count (tex)	32	26	32	20	28	30	36
Warp yarn density (ends/cm)	50	33	29	42	32	31	32
Weft yarn density (picks/cm)	39	30	28	30	27	26	29
Fabric codes							
	F8	**F9**	**F10**	**F11**	**F12**	**F13**	**F14**
Mass per unit area (g/m^2)	185	156	174	174	210	290	175
Thickness (mm)	0,22	0,18	0,21	0,19	0,55	0,86	0,27
Warp yarn count (tex)	24	28	26	26	62	96	28
Weft yarn count (tex)	20	28	32	24	60	92	34
Warp yarn density (ends/cm)	40	26	30	37	18	16	34
Weft yarn density (picks/cm)	38	25	26	32	15	14	25
Fabric codes							
	F15	**F16**	**F17**	**F18**	**F19**	**F20**	**F21**
Mass per unit area (g/m^2)	250	155	200	171	204	298	170
Thickness (mm)	0,75	0,62	0,42	0,35	0,46	0,75	0,51
Warp yarn count (tex)	52	50	22	48	88	52	24
Weft yarn count (tex)	54	48	14	46	18	50	28
Warp yarn density (ends/cm)	19	18	40	21	19	26	32
Weft yarn density (picks/cm)	19	16	39	19	18	26	26
Fabric codes							
	F22	**F23**	**F24**	**F25**	**F26**	**F27**	**F28**
Mass per unit area (g/m^2)	211	172	233	194	178	284	274
Thickness (mm)	0,60	0,26	0,35	0,35	0,33	0,83	0,81
Warp yarn count (tex)	972	26	36	54	34	40	36
Weft yarn count (tex)	878	28	38	56	34	48	22
Warp yarn density (ends/cm)	3	37	26	19	40	27	27
Weft yarn density (picks/cm)	3	31	21	18	32	24	25

Table 1 (Continued)

Fabric codes							
	F29	**F30**	**F31**	**F32**	**F33**	**F34**	**F35**
Mass per unit area (g/m²)	299	237	229	267	280	293	282
Thickness (mm)	1,00	0,83	0,74	1,57	1,09	0,91	0,89
Warp yarn count (tex)	30	32	20	56	28	32	34
Weft yarn count (tex)	22	26	36	54	26	44	26
Warp yarn density (ends/cm)	27	30	29	25	39	28	35
Weft yarn density (picks/cm)	27	28	27	22	38	27	32
Fabric codes							
	F36	**F37**	**F38**	**F39**	**F40**	**F41**	**F42**
Mass per unit area (g/m²)	245	224	233	154	155	155	160
Thickness (mm)	0,75	0,73	0,80	0,58	0,57	0,58	0,69
Warp yarn count (tex)	30	32	24	34	22	34	112
Weft yarn count (tex)	20	26	22	24	22	32	116
Warp yarn density (ends/cm)	43	33	38	33	38	30	13
Weft yarn density (picks/cm)	38	25	34	31	30	25	13
Fabric codes							
	F43	**F44**	**F45**	**F46**	**F47**	**F48**	**F49**
Mass per unit area (g/m²)	164	150	156	156	154	157	152
Thickness (mm)	0,70	0,55	0,60	0,55	0,57	0,56	0,62
Warp yarn count (tex)	118	106	124	88	98	160	118
Weft yarn count (tex)	108	104	168	78	108	180	116
Warp yarn density (ends/cm)	13	13	9	17	15	10	14
Weft yarn density (picks/cm)	12	10	9	14	12	8	12
Fabric codes							
	F50	**F51**	**F52**	**F53**	**F54**	**F55**	**F56**
Mass per unit area (g/m²)	156	162	151	155	155	251	296
Thickness (mm)	0,72	0,74	0,65	0,66	0,68	0,70	0,74
Warp yarn count (tex)	118	86	70	38	62	54	100
Weft yarn count (tex)	130	108	74	38	64	62	88
Warp yarn density (ends/cm)	12	14	16	32	25	20	17
Weft yarn density (picks/cm)	11	14	16	23	23	20	15
Fabric codes							
	F57	**F58**					
Mass per unit area (g/m²)	239	226					
Thickness (mm)	0,68	0,65					
Warp yarn count (tex)	82	70					
Weft yarn count (tex)	76	68					
Warp yarn density (ends/cm)	17	16					
Weft yarn density (picks/cm)	16	14					

2.2 Experimental Method

The pilling propensities of fabrics were determined by using a laboratory test device simulating the usage conditions of fabrics. Although there are many instruments for pilling formation, the Martindale method gives nearly similar results compared to the real usage conditions. Test results obtained from Martindale pilling method provide us with critical information about a textile’s durability and suitability for certain applications. Therefore, the fabrics used in the study were subjected to pilling process by using James Heal Nu-Martindale Pilling and Abrasion Tester instrument (Figure 2a).

In pilling tests, the fabrics were fixed on the Martindale Tester, and the face of the test specimen was rubbed against the face of the same attached fabric in the form of a geometric figure, lissajous figure given in Figure 2b, that is, a straight line which became a gradually widening ellipse until it formed another straight line in the opposite direction and followed the same pattern again for a specific number of movements under the weight of 415 g, which is also a recommended weight for the pilling test of woven fabrics in EN ISO 12945-2 standard method [20]. In this study, after 250, 500, 750, 1000, 1250, 1500, 1750 and 2000 cycles, the fabric samples were evaluated in terms of pilling degree by using PillGrade objective pilling evaluation system. Generally pilling evaluation is performed by skilled expert operators comparing the specimens, after a predefined number of cycles performed by the testing equipment, with visual standards (which may be actual fabrics samples or photographs). On the basis this comparison, the experts define the resistance to pilling using the so called “degree of pilling” i.e. an index varying on a scale ranging from 5 which means no pilling to 1 which means very severe pilling, given also in Table 1 [21]. However, as this assessment is based on the operator, the repeatability is low. Therefore, instead of making pilling assessment by the textile experts, pilling estimation was carried out objectively using PillGrade instrument (Figure 2c).

The PillGrade system is an automated pilling assesment system used for objective and repeatable pilling evaluation. After the pilled fabric speciemen is scanned by a camera and each pill in the center area of the specimen is detected and measured, PillGrade uses the PillGrade Grading Formula to calculate a pilling degree between 1 and 5 [22]. Although PillGrade instrument measures various pilling related properties, in this study only pilling degree values were used since the aim of using PillGrade instrument was to eliminate the subjectivity of pilling evaluation, therefore the other parameters measured by the instrument were not considered, besides the pilling formula calculated by the instrument contains these parameters inherently. Table 2 presents the instructions about pilling degrees and Figure 3 indicates the woven fabrics with different pilling degree values. Pilling degree 5 and pilling degree 4 generally represent good surface characteristics of the fabrics, however the lower pilling degrees (1, 2, 3) are evaluated as unpleasant.

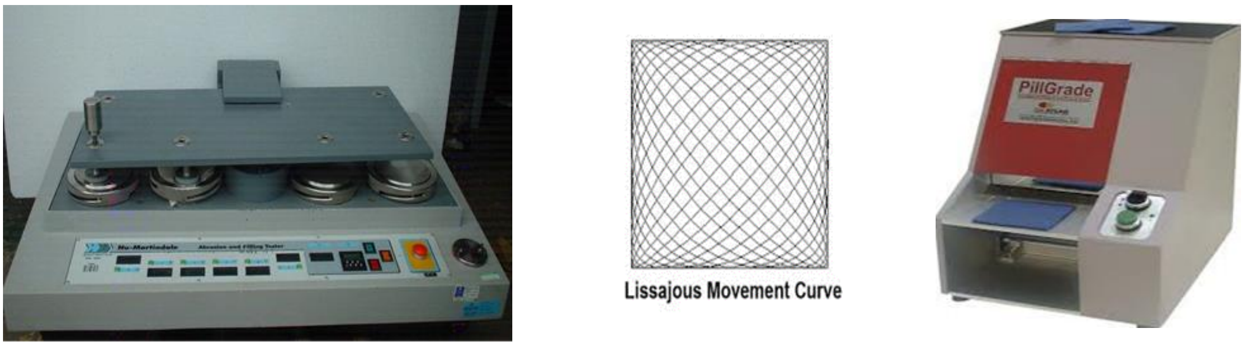


Figure 2. a) Martindale pilling and abrasion tester b) Lissajous movement curve c) PillGrade objective pilling evaluation system

Table 2. The meaning of pilling degrees

Pilling degree	Description
5	No surface change
4	Very small amount of pill and fuzz
3	Moderate fuzz and pill
2	Clearly visible amount of pill and fuzz
1	Very dense amount of fuzz and pill

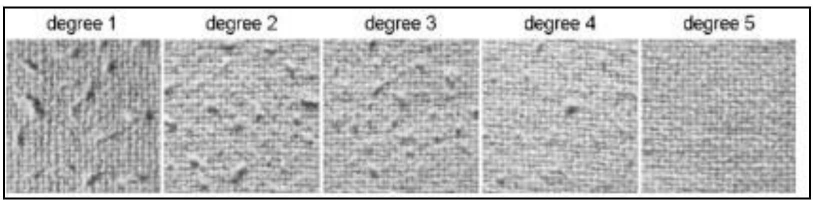


Figure 3. Woven fabrics in different pilling degrees [23]

2.3 Modelling Procedure

A Markov chain model was built based on the pilling propensity of wool fabrics in this study since the pilling occurrence on fabric surface had a multiple state structure and the transitions from the current state to the next state involve uncertainty. The related theoretical framework and its implementation are given in the following.

Let $\{X_n, x_n \in S, n \geq 0\}$ represent a stochastic process. In this stochastic process, $S = \{1, 2, 3, ..., s\}$ represents finite state space of the process and $n \geq 0$ denotes discrete time points. Let X_n be the value of the system characteristics at time *n* and generally it is not known with certainty before time point *n.* If this stochastic process has the following property

Then it can be said that the stochastic process $\{X_n, x_n \in S, n \geq 0\}$ has Markov property and this property means that the transitions in the past do not have any effects on the current transitions. Above, probability p_{ij} is called transition probability and is the probability of going from state *i* at time *n* to state *j* at time *n*+1. In other words, at any time *n*, the future state X_{n+1} can only be determined by the current state X_n and independent of all other previous states $X_0, ..., X_{n-1}$ and the value of $X_n \in S$ [24].

Let ***P*** represent the transition probability matrix of a Markov chain and it can be given in (2):

$$P(X_{n+1} = j \mid X_0 = x_0, X_1 = x_1, ..., X_{n-1} = x_{n-1}, X_n = i) = P(X_{n+1} = j \mid X_n = i) = p_{ij} \quad (1)$$

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1s} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2s} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ p_{s1} & p_{s2} & p_{s3} & \cdots & p_{ss} \end{bmatrix} = \left[p_{ij}\right] \quad (2)$$

At any state, sum of all transition probabilities equals 1, $\sum_{j \in S} p_{ij} = 1, i \in S$.

In order to determine the transition probability matrix, maximum likelihood estimation (MLE) method can be used. Let $\boldsymbol{Q} = [q_1, q_2, ..., q_s]$ be initial probability distribution of the process and $q_{x_0} = P(X_0 = x_0), x_0 \in S$. Then, the probability of any sequence of states, say $X_{0:M}$, can be computed as follows:

$$P(X_{0:M} \mid \boldsymbol{P}, \boldsymbol{Q}) = P(X_M = x_M \mid X_{M-1} = x_{M-1}, ..., X_1 = x_1, X_0 = x_0)$$

$$= q_{x_0} \prod_{k=1}^{M} p_{x_{k-1} x_k} \quad (3)$$

An indicator function can be used to determine transition counts:

$$I(x_k = j, x_{k-1} = i) = \begin{cases} 1, & \text{If there is a transition from state i to state j at time k} \\ 0, & \text{otherwise} \end{cases}$$

Suppose that at time *k*-1, the state of the process is x_{k-1} and at time *k*, the state of the process is x_k . Then the transition probability is as follows:

$$P(X_k = x_k \mid X_{k-1} = x_{k-1}) = p_{x_{k-1} x_k} = p_{11}^{I(x_k = 1, x_{k-1} = 1)} p_{12}^{I(x_k = 2, x_{k-1} = 1)} ... p_{ss}^{I(x_k = s, x_{k-1} = s)} \quad (4)$$

The indicator function $I(.)$ provides that only one transition can occur at time *k*. Using this function in (3), the following results can be drawn:

$$P(X_{0:M} \mid \boldsymbol{P},\boldsymbol{Q}) = q_{x_0} \prod_{k=1}^{M}\prod_{i}\prod_{j} p_{ij}^{I(x_k=j,x_{k-1}=i)} = q_{x_0}\prod_{i}\prod_{j} p_{ij}^{\sum_{k=1}^{M} I(x_k=j,x_{k-1}=i)} \quad (5)$$

Let n_{ij} denote the number of transitions from state i to state j at time k. Then, the value of n_{ij} can be determined using the indicator function as follows:

$$n_{ij} = \sum_{k=1}^{M} I(x_k = j, x_{k-1} = i) \quad (6)$$

The number of times that the process starts in state i can be given as follows:

$$n_i = I(x_0 = i) \quad (7)$$

Then the likelihood of the sequence $X_{0:M}$ can be given as follows [25]:

$$P(X_{0:M} \mid \boldsymbol{P},\boldsymbol{Q}) = L(x) = q_{x_0}\prod_{i}\prod_{j} p_{ij}^{n_{ij}} \quad (8)$$

In this function, one of the transition probabilities should be written according to the other transition probabilities using the fact that the sum of all transition probabilities equals 1:

$$p_{ij} = 1 - \sum_{k \neq j} p_{ik}$$

After substituting this probability in (8), to maximize function $L(x)$, first, the logarithms should be taken:

$$\log L(x) = \log q_{x_0} + \sum_{i,\, j\neq k} n_{ij}\log p_{ij} + n_{ik}\log(1-\sum_{k\neq j} p_{ik}) \quad (9)$$

After taking derivatives of this function and equating them to zero, the result is drawn:

$$p_{ij} = \frac{n_{ij}}{\sum_{j} n_{ij}} \quad (10)$$

A Markov chain having both recurrent states and transient states is a reducible chain and it can be partitioned into two closed classes consisting of transient and recurrent states [6]. For this type of Markov chain, the canonical form of the transition matrix can be generated and the matrix in question, $\boldsymbol{P}$, is represented as follows:

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{S} & \boldsymbol{0} \\ \boldsymbol{D} & \boldsymbol{Q} \end{bmatrix} \quad (11)$$

where the square matrix $\boldsymbol{S}$ is the class of recurrent states, the matrix $\boldsymbol{D}$ and $\boldsymbol{Q}$ consist of the transitions from transient states to recurrent states and the transitions among transient states, respectively.

If the closed class of recurrent states $\boldsymbol{S}$ is represented by $\boldsymbol{C}$,

$$\boldsymbol{P} = \begin{matrix} (\text{Recurrent States})\, \boldsymbol{C} \\ (\text{Transient States}) \end{matrix} \begin{bmatrix} \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{D_C} & \boldsymbol{Q} \end{bmatrix} \quad (12)$$

where $\boldsymbol{Q}$ is transient matrix and the column vector $\boldsymbol{D_C}$ is rearranged form of the structure of matrix $\boldsymbol{D}$ which is obtained summing concerned probabilities.

Since the Markov chain is reducible, if it is assumed that the chain begins in a transient state, the expected (total) number of times the chain visits a (all) transient state(s) before it is absorbed in a recurrent state can be calculated. To achieve this, the inverse of the difference of $\boldsymbol{I}$ which is identity matrix and $\boldsymbol{Q}$ of rearranged matrix $\boldsymbol{P}$ in the canonical form exists and is called fundamental matrix, denoted by $\boldsymbol{U}$ and it is defined as follows:

$$\boldsymbol{U} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} \quad (13)$$

Let u_{ij} in $\boldsymbol{U}$ denote the expected number of times the chain visits transient state j before the chain enters an absorbing state, given that the chain begins in transient state i.

Suppose that the matrix of eventual passage probabilities from the transient states to the recurrent states is denoted by $\boldsymbol{F_C}$ and it is calculated using the formula $\boldsymbol{F_C} = \boldsymbol{UD}$. In addition, suppose that the matrix of eventual passage probabilities from the transient states to the one of the recurrent states is denoted by $\boldsymbol{f_C}$ and it is calculated using the formula $\boldsymbol{f_C} = \boldsymbol{UD_C}$ [26].

Assume that $p_{ij}^{(n)}$ represents an n-step transition probability for the recurrent closed class $\boldsymbol{D_C}$ consisting of N states, where i and j are two recurrent states existing in the same recurrent closed class in question. Let

$$\lim_{n\to\infty} p_{ij}^{(n)} = \pi_j$$

where π_j is the steady state probability for state j belonging to the corresponding recurrent class. Then the limiting probability vector for $\boldsymbol{D_C}$ can be calculated using the equalities obtained by the following equation:

$$\boldsymbol{\pi} = \boldsymbol{\pi S} \tag{14}$$

where $\boldsymbol{\pi} = [\pi_1, \pi_2, ..., \pi_N]$, $\sum_{j=1}^{N} \pi_j = 1$ and $\boldsymbol{S}$ is the class of recurrent states in Equation (11).

The limiting probability of a transition from a transient state i to a recurrent state j in $\boldsymbol{C}$ is equal to the product of f_{iC}, the probability of eventual passage from the transient states to the one of the recurrent states in question, and π_j, the steady state probability for state j belonging to the corresponding recurrent class. This can be formulated as follows:

$$\lim_{n\to\infty} p_{ij}^{(n)} = f_{iC}\pi_j \tag{15}$$

If the initial state is transient state i, the limiting distribution of the process $\boldsymbol{\pi}'$ can be obtained as follows:

$$\boldsymbol{\pi}' = \left[\underbrace{f_{iC}\pi_1 \;\; f_{iC}\pi_2 \;\; \cdots \;\; f_{iC}\pi_N}_{\text{The limiting probabilities of recurrent states}} \quad \underbrace{0 \;\; \cdots \;\; 0}_{\text{The limiting probabilities of transient states}} \right]$$

For an irreducible Markov chain, let m_{ij} expected number of transitions before the chain first reaches state j, given that it is currently in state i; m_{ij} is called the mean first passage time from state i to state j. Assume that the chain is currently in state i. Then with probability p_{ij}, it will make one transition to go from state i to state j. For $k \neq j$, the chain next goes with probability p_{ik} to state k. In this case, it will make an average of $1 + m_{kj}$ transitions to go from i to j. This reasoning implies that [24]

$$m_{ij} = p_{ij}(1) + \sum_{k\neq j} p_{ik}(1 + m_{kj}) \tag{16}$$

Since $$p_{ij} + \sum_{k\neq j} p_{ik} = 1$$

$$m_{ij} = 1 + \sum_{k\neq j} p_{ik} m_{kj}$$

$$m_{ii} = \frac{1}{\pi_i}$$

The Markov chain given as application in this study is a reducible Markov chain and consists of one irreducible chain including recurrent states, the calculation of conditional mean first passage times is appropriate for each transient state. In order to calculate the conditional mean first passage time from transient state i to any target recurrent state h, the following formula can be used.

$$m_{ih} = m_{iC} \sum_{j=1}^{R} \frac{f_{ij}}{f_{iC}} + \sum_{\substack{j=1 \\ j\neq h}}^{R} \frac{f_{ij}}{f_{iC}} m_{jh} = m_{iC} + \sum_{\substack{j=1 \\ j\neq h}}^{R} \frac{f_{ij}}{f_{iC}} m_{jh} \tag{17}$$

where R is non-absorbing recurrent states [27].

3. RESULTS AND DISCUSSION

According to the theoretical framework given in Section 2, the state transition diagram of pilling process must firstly be determined. With this aim, the states of the stochastic process were defined based on the results of the pilling degrees measured by the PillGrade instrument given in Figure 4. Each number indicated in the figure shows the states. According to the assumed model, the states were constructed as follows:

$$\text{States} = \begin{cases} 5, & \text{pilling degree} \geq 4.5 \\ 4, & 4 \leq \text{pilling degree} < 4.5 \\ 3, & 3 \leq \text{pilling degree} < 4 \\ 2, & 2 \leq \text{pilling degree} < 3 \\ 1, & \text{pilling degree} < 2 \end{cases}$$

The forward arrows point out the deterioration of the fabric surface in terms of pilling and reverse arrows indicate the recovery due to the detaching of pills on the fabric surface. The stochastic process can remain on the current state since the pilling degree of the fabric may not change. If the pilling degree decreases by one or more degrees, the process can make a transition to the mentioned states and this situation shows that the fabric surface is deteriorated. On the other hand, as the detaching of pills on the fabric surface is more difficult to occur, except for the state 5, the process can move to at most one higher state.

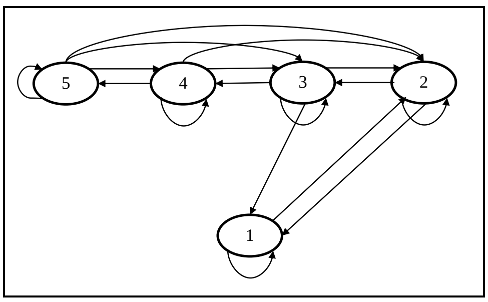


Figure 4. The state transition diagram of pilling process

Based on the state transition diagram of pilling process in Figure 4, the transition probability matrix can be formed as follows:

$$
\boldsymbol{P} = \begin{array}{c} \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{array}
\begin{array}{c} \begin{array}{ccccc} 5 & 4 & 3 & 2 & 1 \end{array} \\
\begin{bmatrix} p_{55} & p_{54} & p_{53} & p_{52} & 0 \\ p_{45} & p_{44} & p_{43} & p_{42} & 0 \\ 0 & p_{34} & p_{33} & p_{32} & p_{31} \\ 0 & 0 & p_{23} & p_{22} & p_{21} \\ 0 & 0 & 0 & p_{12} & p_{11} \end{bmatrix} \end{array}
$$

Maximum likelihood estimation (MLE) method is used to obtain the value of the probabilities in the matrix ***P***. According to the state transition diagram of pilling process, 58 different wool fabrics with 150-300 g/m^2 weight per unit area were pilled at every 250 cycles up to 2000 cycles. Then, it can be said that the number of discrete time points, $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, where the observations were obtained are 9.

The probabilities in the transition probability matrix are estimated by using MLE. If shown for one of all transitions, 12 wool fabrics make a transition from state 5 to state 3, and the number of fabrics remaining at state 5 at the end of 2000 cycles is 274, then the probability of the transition from state 5 to state 3 is calculated as follows:

$$p_{53} = \frac{12}{274} = 0.0438$$

After estimating the transitions of all the other states in the same way, the transition probability matrix ***P*** has the following probabilities:

$$
\boldsymbol{P} = \begin{array}{c} \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{array}
\begin{array}{c} \begin{array}{ccccc} 5 & 4 & 3 & 2 & 1 \end{array} \\
\begin{bmatrix} 0.8832 & 0.0511 & 0.0438 & 0.0219 & 0 \\ 0.12 & 0.6 & 0.28 & 0 & 0 \\ 0 & 0 & 0.6964 & 0.3036 & 0 \\ 0 & 0 & 0.0161 & 0.7742 & 0.2097 \\ 0 & 0 & 0 & 0.0426 & 0.9574 \end{bmatrix} \end{array}
$$

The Markov chain presented above has both recurrent states and transient states. For this reason, this Markov chain is a reducible one and it can be partitioned into two closed classes consisting of transient and recurrent states [6]. States 1, 2 and 3 are the members of recurrent class and states 4 and 5 are those of transient class. The reorganized form of the matrix ***P*** is

$$
\boldsymbol{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix} p_{11} & p_{12} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 \\ p_{31} & p_{32} & p_{33} & p_{34} & 0 \\ 0 & p_{42} & p_{43} & p_{44} & p_{45} \\ 0 & p_{52} & p_{53} & p_{54} & p_{55} \end{bmatrix} \end{array}
= \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\left[\begin{array}{ccc|cc} 0.9574 & 0.0426 & 0 & 0 & 0 \\ 0.2097 & 0.7742 & 0.0161 & 0 & 0 \\ 0 & 0.3036 & 0.6964 & 0 & 0 \\ \hline 0 & 0 & 0.28 & 0.6 & 0.12 \\ 0 & 0.0219 & 0.0438 & 0.0511 & 0.8832 \end{array}\right] \end{array}
$$

The elements of the transition matrix ***P*** given in (11) is represented as follows:

$$
\boldsymbol{S} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}
\begin{bmatrix} p_{11} & p_{12} & 0 \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}
= \begin{array}{c} 1 \\ 2 \\ 3 \end{array}
\begin{bmatrix} 0.9574 & 0.0426 & 0 \\ 0.2097 & 0.7742 & 0.0161 \\ 0 & 0.3036 & 0.6964 \end{bmatrix}
$$

$$
\boldsymbol{D} = \begin{array}{c} 4 \\ 5 \end{array}
\begin{bmatrix} 0 & p_{42} & p_{43} \\ 0 & p_{52} & p_{52} \end{bmatrix}
= \begin{array}{c} 4 \\ 5 \end{array}
\begin{bmatrix} 0 & 0 & 0.28 \\ 0 & 0.0219 & 0.0438 \end{bmatrix}
$$

$$
\boldsymbol{Q} = \begin{array}{c} 4 \\ 5 \end{array}
\begin{bmatrix} p_{44} & p_{45} \\ p_{54} & p_{55} \end{bmatrix}
= \begin{array}{c} 4 \\ 5 \end{array}
\begin{bmatrix} 0.6 & 0.12 \\ 0.0511 & 0.8832 \end{bmatrix}
$$

Since the chain starting from state 5 will eventually visit one of the states in ***S*** and will never return state 4 or 5 again, this class of recurrent states ***S*** can be evaluated as an absorbing state.

When the closed class of recurrent states ***S*** represented by ***C*** which consists of states 1, 2 and 3,

$$
\boldsymbol{P} = \begin{array}{c} \boldsymbol{C} \\ 4 \\ 5 \end{array}
\left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0.28 & 0.6 & 0.12 \\ 0.0657 & 0.0511 & 0.8832 \end{array}\right]
= \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \boldsymbol{D_C} & \boldsymbol{Q} \end{bmatrix}
$$

where

$$\boldsymbol{D_C} = \begin{bmatrix} 0.28 \\ 0.0657 \end{bmatrix} \quad \text{and} \quad \boldsymbol{Q} = \begin{bmatrix} 0.6 & 0.12 \\ 0.0511 & 0.8832 \end{bmatrix}$$

Suppose that the chain begins in transient state 5. In order to determine the expected number of times the chain visits a (all) transient state(s) before it is absorbed in a recurrent state, $\boldsymbol{U}$ given in (13) should be calculated.

$$\boldsymbol{U} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} = \begin{matrix} 4 \\ 5 \end{matrix}\begin{bmatrix} 0.4 & -0.12 \\ -0.0511 & 0.1168 \end{bmatrix}^{-1} = \begin{matrix} 4 \\ 5 \end{matrix}\begin{bmatrix} 2.878 & 2.957 \\ 1.259 & 9.855 \end{bmatrix}$$

One of the elements of $\boldsymbol{U}$, $u_{54} = 1.259$ means that the expected number of visits to transient state 4 before the chain enters one of the recurrent states, given that the chain begins in transient state 5 is 1.259 and according to the experimental process mentioned above, this quantity corresponds to 1.259×250 cycles $\cong 315$ cycles. Let u_i denote the expected number of times the chain visits all transient states before the chain enters an absorbing state, given that the chain begins in transient state *i*. For $i = 5$, $u_5 = u_{54} + u_{55} = 1.259 + 9.855 = 11.114$ means that the expected number of visits all transient states before the chain enters an absorbing state, given that the chain begins in transient state 5 is 11.114 and similarly, based on the experimental process, this quantity equals approximately 11.114×250 cycles $\cong 2779$ cycles.

The matrix of eventual passage probabilities from transient states 4 and 5 to the recurrent states 1, 2 and 3 is

$$\boldsymbol{F_C} = \begin{matrix} 4 \\ 5 \end{matrix}\begin{bmatrix} 2.878 & 2.957 \\ 1.259 & 9.855 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0.28 \\ 0 & 0.0219 & 0.0438 \end{bmatrix} \cong \begin{matrix} \\ 4 \\ 5 \end{matrix}\begin{matrix} 1 & 2 & 3 \\ \left[\begin{matrix} 0 \\ 0 \end{matrix}\right. & \begin{matrix} 0.0648 \\ 0.2158 \end{matrix} & \left.\begin{matrix} 0.9354 \\ 0.7842 \end{matrix}\right] \end{matrix} = \begin{matrix} \\ 4 \\ 5 \end{matrix}\begin{matrix} 1 & 2 & 3 \\ \left[\begin{matrix} f_{41} \\ f_{51} \end{matrix}\right. & \begin{matrix} f_{42} \\ f_{52} \end{matrix} & \left.\begin{matrix} f_{43} \\ f_{53} \end{matrix}\right] \end{matrix}$$

The matrix of eventual passage probabilities from transient states 4 and 5 to the recurrent states 1, 2 and 3 is

$$\boldsymbol{f_C} = \begin{matrix} 4 \\ 5 \end{matrix}\begin{bmatrix} 2.878 & 2.957 \\ 1.259 & 9.855 \end{bmatrix}\begin{bmatrix} 0.28 \\ 0.0657 \end{bmatrix} = \begin{matrix} \\ 4 \\ 5 \end{matrix}\begin{matrix} \mathbf{C} \\ \begin{bmatrix} f_{4C} \\ f_{5C} \end{bmatrix} \end{matrix} \cong \begin{matrix} \\ 4 \\ 5 \end{matrix}\begin{matrix} \mathbf{C} \\ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{matrix} = \begin{bmatrix} f_{41} + f_{42} + f_{43} \\ f_{51} + f_{52} + f_{53} \end{bmatrix}$$

Irrespective of the transient beginning state, chain is absorbed, and this means that the fabric will eventually be unusable.

The limiting probability vector for the recurrent closed class $\boldsymbol{D_C}$ can be obtained using (14).

$$\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix}\begin{bmatrix} 0.9574 & 0.0426 & 0 \\ 0.2097 & 0.7742 & 0.0161 \\ 0 & 0.3036 & 0.6964 \end{bmatrix}$$

$$\begin{aligned} \pi_1 &= 0.9574\,\pi_1 + 0.2097\,\pi_2 \\ \pi_2 &= 0.0426\,\pi_1 + 0.7742\,\pi_2 + 0.3036\,\pi_3 \\ \pi_3 &= 0.0161\,\pi_2 + 0.6964\,\pi_3 \\ \pi_1 &+ \pi_2 + \pi_3 = 1 \end{aligned}$$

After solving the set of equations given above, the following results are obtained:

$$[\pi_1 \quad \pi_2 \quad \pi_3] = [\ 0.8240 \quad 0.1674 \quad 0.0088\ ]$$

As stated previously, initial state is 5, $\boldsymbol{\pi}'$ is defined using (15) as follows:

$$\boldsymbol{\pi}' = \Big[\underbrace{f_{5C}\pi_1 \quad f_{5C}\pi_2 \quad \cdots \quad f_{5C}\pi_N}_{\text{The limiting probabilities of recurrent states}} \quad \underbrace{0 \quad \cdots \quad 0}_{\text{The limiting probabilities of transient states}} \Big]$$

As a result, the limiting distribution of the process is

$$\boldsymbol{\pi}' = [\ 0.8240 \quad 0.1674 \quad 0.0088 \quad 0 \quad 0\ ]$$

This means that the long run proportions of time spend in state 1, 2 and 3 are equal to 0.8240, 0.1674 and 0.0088, respectively.

The conditional mean first passage times for state 4 to enter recurrent state 1 for the first time given that the fabric will eventually be in unpleasant appearance can be determined. To calculate m_{41}, the conditional mean first passage times m_{21} and m_{31} for the recurrent closed class ***C***={1,2,3} must first be computed. These passage times from recurrent states 2 and 3 to recurrent state 1 in ***C*** can be calculated as follows:

$$m_{21} = 1 + p_{22}m_{21} + p_{23}m_{31} = 1 + 0.7742m_{21} + 0.0161m_{31}$$
$$m_{31} = 1 + p_{32}m_{21} + p_{33}m_{31} = 1 + 0.3036m_{21} + 0.6964m_{31}$$

The solutions are $m_{21} = 5.0216$ and $m_{31} = 8.315$.
Using the formula given in (17) to compute m_{41}, the following is obtained:

$$m_{41} = m_{4C} + \left(\frac{f_{42}}{f_{4C}}\right)m_{21} + \left(\frac{f_{43}}{f_{4C}}\right)m_{31} = 13.93825 \cong 14$$
$$m_{51} = m_{5C} + \left(\frac{f_{52}}{f_{5C}}\right)m_{21} + \left(\frac{f_{53}}{f_{5C}}\right)m_{31} = 18.7183 \cong 19$$

On the average, a fabric that will eventually be in unpleasant appearance will enter recurrent state 1 which is the worst situation with respect to appearance for the first time after $14 \times 250 = 3500$ and $19 \times 250 = 4750$ cycles when it is known that the pilling degree of the fabric is 4 and 5, respectively. While the conditional mean first passage time from state 2 which is one of the unpleasant appearances to state 1 which is the worst unpleasant appearance is the shortest, as expected the conditional mean first passage time from state 5 which is the best appearance to state 1 is the longest.

The number of cycles calculated above for the transitions between pilling degrees gives an idea about the durability of the fabrics and can also be used to estimate the useful life of the fabrics. Considering that 2000 cycles are used as the standard test time to simulate the pilling behaviour of the fabrics during usage and this test is equivalent to 40 minutes, 3500 and 4750 cycles will be approximately 70 and 95 minutes, respectively. It is clear that under real usage conditions, this period represents a very long time, and therefore this means that the fabrics with similar structure will have long life cycle.

4. CONCLUSION

In this study, the pilling degree changing of wool fabrics during pilling process was evaluated by using stochastic modelling based on Markov chains. Accordingly, 58 woven 100% wool fabrics were tested in similar structure, and the model was applied using experimental data on pilling degree results of these fabrics. Pilling degree evaluation was composed of 5 degrees from 1 (severe pilling) to 5 (non-pilling).

In the first part of the study, an appropriate state transition diagram was designed, and probability transition matrix of corresponding Markov chain was obtained using MLE method and used to model the pilling process of these woollen woven fabrics. Afterwards, limiting distribution of the process was derived and since obtained Markov chain was reducible, conditional mean first passage times were calculated.

The initial condition of pilling degree of the fabrics was considered as 5. Due to the pilling formed by the usage of the fabrics, the pilling degrees were expected to be 1, 2 or 3 that were generally evaluated as low-quality fabrics in textile industry. The results of the limit probability distribution calculated at the end of the study also supported the expected situation in real life. For the woven wool fabrics with 150-300 g/m^2 weight per unit area, the probability of that the fabrics remain in the pilling degree 4 or pilling degree 5 after long-term use was 0, whereas these probabilities were calculated as 0.0088, 0.1674 and 0.8240 for pilling degrees 3, 2 and 1, respectively.

Conditional mean first passage times were also calculated. Accordingly, when the pilling degree of the fabric was 4, the first time that the fabric would eventually be in the worst situation (state 1) is approximately 3500 cycles. This conditional mean first passage times can be used for simulation and optimization of the durability of fabrics regarding the pilling formation. The calculated pilling cycles number for the transitions between pilling degrees can be useful for the estimation of the life cycle of the fabrics.

The results obtained from the study such as probabilities of transitions among the pilling formation states on wool fabrics, limiting probability distribution and conditional mean first passage times can be beneficial for the future works especially intended to be carried out on the computational methods of pilling mechanism. The number of cycles calculated for the transitions between pilling degrees gives an idea about the durability of the fabrics and can also be used to estimate the useful life of the fabrics.

The findings obtained based on the result of the calculations made with the stochastic structure is in line with the real situation, and therefore the stochastic structure constructed can be used for different fabric types, as well. By using the data obtained from the experiments, different results can be obtained for the fabrics using the same stochastic structure.

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