Pamukkale Üniversitesi
Mühendislik Bilimleri Dergisi,
Cilt 15, Sayı 2, 2009, Sayfa 187-193

Derivation of Equations for Flexure and Shear Deflections of Simply Supported Beams

Basit Mesnetli Kirişlerde Eğilme ve Kaymadan Dolayı Oluşan Sehim Denklemlerinin Bulunması

Ümran ESENDEMİR*

Süleyman Demirel Üniversitesi, Mühendislik Mimarlık Fakültesi, Makine Mühendisliği Bölümü, 32060, Isparta

Geliş Tarihi/Received : 16.09.2008, Kabul Tarihi/Accepted : 06.02.2009

ABSTRACT

Shear deflection of wood beams generally is exluded in plannning calculations. Ignoring shear deflection could cause significant errors, expecially for short and thick beams. In this study, two deflection functions due to flexure and shear of simply supported composite beam subjected to single force are obtained analytically. Wood being high shear modulus according to other material is selected for sample problem. The deflections the mid point of the beam are calculated to see the effect of shear by using the obtained functions for 0, 15, 30, 45, 60 and 90 orientation angles. Also, bending stresses at the mid point of the short beam are given for 0, 15, 30, 45, 60 and 90 orientation angles. It is shown that the magnitude of shear deflection depends on force, length and height of the beam. The shear effect is the smallest for 45 orientation angle and the biggest for 0 orientation angle.

Keywords : *Shear effect, Shear deflection, Simply supported composite beam, Wood.*

ÖZET

Kiriş uygulamalarının genelinde kaymadan dolayı oluşan sehimler ihmal edilir. Fakat; yüksek kayma modülüne sahip, kısa ve kalın kirişlerde kaymadan dolayı oluşan sehimin ihmal edilmesi çok büyük hatalara neden olmaktadır. Bu çalışmada her iki tarafında mesnetlenmiş orta noktasından tekil yüke maruz kompozit kirişlerdeki eğilme ve kaymadan dolayı ortaya çıkan sehim denklemleri analitik olarak elde edilmiştir. Örnek malzeme olarak kayma modülü diğer malzemelere göre yüksek olan ahşap seçilmiştir. Kaymanın etkisini incelemek için, elde edilen fonksiyonlar kullanılarak, kirişin orta noktasındaki maksimum sehimler 0, 30, 45, 60 and 90 oryantasyon açıları için elde edilmiştir. Aynı zamanda kayma etkisinin en fazla olduğu kısa kirişin orta noktasındaki eğilme gerilmeleri 0, 30, 45, 60 ve 90 oryantasyon açıları için verilmiştir. Kaymadan dolayı oluşan sehimin; kirişe uygulanan yüke, kirişin uzunluğuna ve yüksekliğine göre değiştiği tespit edilmiştir. Kayma etkisi; 45 oryantasyon açısında en küçük, 0 oryantasyon açısında ise en büyük olmaktadır.

Anahtar kelimeler : *Kayma etkisi, Kaymadan dolayı oluşan sehim, Basit desteklenmiş kompozit kiriş, Ahşap.*

1. INTRODUCTION

Wood may be described as an orthotropic material with independent mechanical properties in the directions of three mutually perpendicular axes: longitudinal (L), radial (L), and tangential (T). These are called the principal material axes, and the mechanical properties referred to them are the engineering constants (Liu and Rammer, 2003). In addition to the deflection due to pure bending in a beam, there is shear force in all cases of non-uniform bending and a further deflection, due to shear stresses. This additional shear deflection usually is assumed to be negligible and is not considered in computing the total deflection of a beam. The magnitude of shear deflection depends on both the span to depth ratio of the beam and the elastic properties of the species involved. It increases as the

* Yazışılan yazar/Corresponding author. E-posta adresi /E-mail adress: esen@mmf.sdu.edu.tr (Ü. Esendemir)

effective span to depth ratio of the composite beam decreases and as the core ratio of pure modulus of elasticity to modulus of rigidity increases (Biblis, 1997).

Thomas (2002) studied shear and flexural deflection equations for oriented strand board floor decking with point load by using finite element method. Lee and Reddy (2004) carried out nonlinear deflection control of laminated plates using third-order shear deformation theory by using finite element method. Schramm et al. (1994) carried out shear deformation coefficient in beam theory. Pilkey et al., (1995) carried out new structural matrices for a beam element with shear deformation. He derived a new general beam stiffness matrix which accounts for bending and shear deflection. Wang (1998) carried out calculations for the maximum deflection of steel-concrete composite beams with partial shear interaction. This deflection is related to the strength of shear connector in the composite beam. Altenbach (2000) studied on the deformation of transverse shear stiffness of orthotropic plates. Aydoğan (1995) studied stiffness-matrix formulation of beams with shear effect on elastic foundation. Onu (2000) examined shear effect in beam finite element on two-parameter elastic foundation. Faella et al., (2003) presented a finite element procedure considering nonlinear load slip relationship for shear connectors. A wide parametric analysis is performed with reference to the evaluation of deflections for simply supported beam. Nie and Cai (1998) studied the effects of shear slip on the deformation of steel-concrete composite beams. Nie and Cai (2000) developed an analytical model for deflection of cracked RC beams under sustained loading. Kubojima et al., (2004) examined effect of shear deflection on vibrational properties of compressed wood. Evangelos (1967) studied shear deflection of two species laminated wood beams. Hiroaki and Tohru (1993) examined shear deflection of anisotropic plates using the finite element method. Machado and Cortinez (2005) studied nonlinear model for stability of thin walled composite beams with shear deformation. Kılıç et al., (2001) investigated the effects of shear on the deflection of orthotropic cantilever beam by the use of anisotropic elasticity theory. Esendemir (2005) analyzed the effects of shear on the deflection to linearly loaded composite cantilever beam. Esendemir et al., (2006) studied the effects of shear on the deflection of simply supported composite beam loaded linearly. Usal et al., (2008) carried out static and dynamic analysis of simply supported beams

In this study, deflection functions due to flexure and shear of two deflection functions due to flexure and shear of simply supported composite beams for 0, 15, 30, 45, 60 and 90 orientation angles are obtained using the anisotropic elasticity theory by Lekhnittskii (1968). And, the deflections at the mid point of the beams are calculated for various lengths, heights, loads and orientation angles.

2. GENERAL THEORY

For a plate, stress-strain relations in anisotropic elasticity theory are given as (Lekhnitskii, 1981)

$$\varepsilon_x = a_{11}\sigma_x + a_{12}\sigma_y + a_{16}\tau_{xy} \quad (1)$$

$$\varepsilon_y = a_{12}\sigma_x + a_{22}\sigma_y + a_{26}\tau_{xy} \quad (2)$$

$$\gamma_{xy} = a_{16}\sigma_x + a_{26}\sigma_y + a_{66}\tau_{xy} \quad (3)$$

where a_{ij} are the components of the compliance matrix. The elements of compliance matrix are given as,

$$\begin{aligned}
a_{11} &= S_{11}\cos^4\theta + (2S_{12}+S_{66})\sin^2\theta\cos^2\theta \\
&\quad + S_{22}\sin^4\theta \\
a_{12} &= S_{12}(\sin^4\theta + \cos^4\theta) + \\
&\quad (S_{11}+S_{22}-S_{66})\sin^2\theta\cos^2\theta \\
a_{22} &= S_{11}\sin^4\theta + (2S_{12}+S_{66})\sin^2\theta\cos^2\theta \\
&\quad + S_{22}\cos^4\theta \\
a_{16} &= (2S_{11}-2S_{12}-S_{66})\sin\theta\cos^3\theta \\
&\quad - (2S_{22}-2S_{12}-S_{66})\sin^3\theta\cos\theta \\
a_{26} &= (2S_{11}-2S_{12}-S_{66})\sin^3\theta\cos\theta \\
&\quad - (2S_{22}-2S_{12}-S_{66})\sin\theta\cos^3\theta \\
a_{66} &= 2(2S_{11}+2S_{22}-4S_{12}-S_{66})\sin^2\theta\cos^2\theta \\
&\quad - S_{66}(\sin^4\theta + \cos^4\theta)
\end{aligned} \quad (4)$$

$$S_{11} = \frac{1}{E_1},\; S_{12} = -\frac{\upsilon_{12}}{E_1},\; S_{22} = \frac{1}{E_2},\; S_{66} = \frac{1}{G_{12}}$$

In addition, the strain components are given as (Jones, 1975).

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad (5)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \quad (6)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad (7)$$

where u and v are displacements in the x and y directions.

2. DERIVATION OF DEFLECTION FUNCTIONS OF SIMPLY SUPPORTED COMPOSITE BEAMS UNDER A FORCE ACTED AT THE MID POINT

An composite beam supported from two ends loaded by a single force at the mid point is shown in Figure 1.

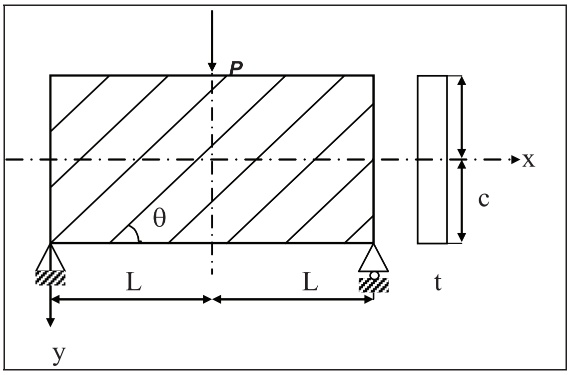

Figure 1. Simply supported composite beam under a single force.

For This beam, stress components are given as (Esendemir, 2004).

$$\sigma_x = \frac{3P}{4tc^3}\left(xy + ry^2 - \frac{rc^2}{3}\right) \tag{8}$$

$$\sigma_y = 0 \tag{9}$$

$$\tau_{xy} = \frac{3P}{8tc^3}\left(c^2 - y^2\right) \tag{10}$$

Where, $r = \dfrac{a_{16}}{a_{11}}$

2. 1. Deflection Due To Flexure

If the equationon obtained from the substitution of Equations (8)-(10) into Eqn. (1) is equalized to Eqn. (5), and if the resulting equality is later integrated with respect to x, then the displacement function; u, in the x direction is found as

$$u = \frac{3P}{4tc^3} a_{11}\left(\frac{x^2 y}{2} + ry^2 x - \frac{rc^2 x}{3}\right) + \frac{3P}{8tc^3} a_{16}\left(c^2 x - y^2 x\right) + f(y) \tag{11}$$

In the same manner, From (8)-(10), (2), and (6) we obtain

$$v_{flexure} = \frac{3P}{4tc^3} a_{12}\left(\frac{xy^2}{2} + \frac{ry^3}{3} - \frac{rc^2 y}{3}\right) + \frac{3P}{8tc^3} a_{26}\left(c^2 y - \frac{y^3}{3}\right) + g(x) \tag{12}$$

g(x) should be known to find $v_{flexure}$,. Therefore, if γ_{xy} in Eqn. (3) in equalised to γ_{xy} in Eqn. (7), we obtain,

$$\frac{3P}{4tc^3} a_{16}\left(xy + ry^2 - \frac{rc^2}{3} + xy\right) + \frac{3P}{8tc^3} a_{66}\left(c^2 - y^2\right) - \frac{3P}{4tc^3} a_{11}\left(\frac{x^2}{2} + 2ryx\right) - \frac{3P}{4tc^3} a_{12}\left(\frac{y^2}{2}\right) - g^{1}(x) + f(y) = 0 \tag{13}$$

Because, of equality of Eqn. (13) to zero, the summation of the terms depending on x, the summation of the terms depending on y and the summation of the terms depending on xy should be equal to seperate constants. Thus,

$$A(x) - g'(x) = b \tag{14}$$

Where, $A(x) = -\dfrac{3P}{4tc^3} a_{11}\left(\dfrac{x^2}{2}\right)$

From Eqn. (14), we obtain,

$$g(x) = \int \left[\left(A(x) - b\right)\right] dx \tag{15}$$

and if this equation is integrated, we obtain

$$g(x) = -\frac{3P}{4tc^3} a_{11}\left(\frac{x^3}{6}\right) - bx + e \tag{16}$$

where, e is a constant of integration. Substitution of $g(x)$ into Eqn. (12) gives the deflection of the beam in tems of b and e as

$$v_{flexure} = \frac{3P}{4tc^3} a_{12}\left(\frac{xy^2}{2} + \frac{ry^3}{3} - \frac{rc^2 y}{3}\right) + \frac{3P}{8tc^3} a_{26}\left(c^2 y - \frac{y^3}{3}\right) - \frac{3P}{24tc^3} a_{11} x^3 - bx + e \tag{17}$$

Boundary condition for this beam is given as,

v=0 at x=0 and y=0

$\dfrac{dv}{dx}$=0 at x=L and y=0

From boundary condition,

$$e = 0,\ b = -\frac{3P}{8tc^3} a_{11} L^2$$

are obtained. In the end, the deflection in y direction is obtained as,

$$v_{flexure} = \frac{3P}{4tc^3} a_{12}\left(\frac{xy^2}{2}+\frac{ry^3}{3}-\frac{rc^2y}{3}\right) + \frac{3P}{8tc^3} a_{26}\left(c^2y-\frac{y^3}{3}\right) - \frac{3P}{24tc^3} a_{11}x^3 + \frac{3P}{8tc^3} a_{11} L^2 x \qquad (18)$$

In order to determine the deflection function of the symmetry axis at the mid point, it is necessary to insert x = L and y = 0 into Eqn. (18) to find

$$v_{flexure} = \frac{P}{4tc^3} a_{11} L^3 \qquad (19)$$

2. 2. Deflection due to Shear

When we substitute Equations (8)-(10) into Eqn. (3) and multiply by an incremental length of the

Table 2. Deflections of simply supported composite beam subjected to single force with respect to half length of beam.

θ(°)	L(mm)	$v_{flexure}$	v_{shear}	v_{shear}	%Error
0	80	0.2034	0.2471	0.4505	54.85
	100	0.3973	0.3088	0.7061	43.74
	120	0.6865	0.3706	1.0570	35.06
	140	1.0900	0.4323	1.5220	28.40
30	80	0.6478	0.1232	0.7710	15.99
	100	1.2650	0.1541	1.4190	10.86
	120	2.1860	0.1849	2.3710	7.79
	140	3.4720	0.2157	3.6870	5.85
45	80	0.9427	0.1219	1.0650	11.45
	100	1.8410	0.1524	1.9940	7.64
	120	3.1820	0.1828	3.3650	5.44
	140	5.0520	0.2133	5.2660	4.05
60	80	1.088	0.1619	1.2500	12.96
	100	2.124	0.2024	2.3270	8.70
	120	3.671	0.2429	3.9140	6.21
	140	5.829	0.2834	6.1130	4.64
90	80	1.0830	0.2471	1.3300	18.57
	100	2.1150	0.3088	2.4240	12.74
	120	3.6550	0.3706	4.0260	9.20
	140	5.8040	0.4323	6.2370	6.93

Table 3. Deflections of simply supported composite beam subjected to single force with respect to half height of beam.

θ(°)	c(mm)	$v_{flexure}$	v_{shear}	v_{shear}	% Error
0	10	3.1780	0.6176	3.7960	16.27
	15	0.9417	0.4118	1.3530	30.42
	20	0.3973	0.3088	0.7061	43.74
	25	0.2034	0.2471	0.4505	54.85
30	10	10.1200	0.3081	10.4300	2.95
	15	2.9990	0.2054	3.2040	6.41
	20	1.2650	0.1541	1.4190	10.86
	25	0.6478	0.1232	0.7710	15.99
45	10	14.7300	0.3047	15.0300	2.02
	15	4.3640	0.2032	4.5680	4.45
	20	1.8410	0.1524	1.9940	7.64
	25	0.9427	0.1219	1.0650	11.45
60	10	17.0000	0.4049	17.4000	2.32
	15	5.0360	0.2699	5.3050	5.08
	20	2.1240	0.2024	2.3270	8.70
	25	1.0880	0.1619	1.2500	12.96
90	10	16.9200	0.6176	17.5400	3.52
	15	5.0140	0.4118	5.4250	7.59
	20	2.1150	0.3088	2.4240	12.74
	25	1.0830	0.2471	1.3300	18.57

Table 4. Deflections of simply supported composite beam subjected to single force with respect to load of beam.

θ(°)	P(N)	$v_{flexure}$	v_{shear}	v_{shear}	% Error
0	100	0.2648	0.2059	0.4707	43.74
	150	0.3973	0.3088	0.7061	43.74
	200	0.5297	0.4118	0.9414	43.74
	250	0.6621	0.5147	1.1770	43.74
30	100	0.8434	0.1027	0.9461	10.86
	150	1.2650	0.1541	1.4190	10.86
	200	1.6870	0.2054	1.8920	10.86
	250	2.1090	0.2568	2.3650	10.86
45	100	1.2280	0.1016	1.3290	7.64
	150	1.8410	0.1524	1.9940	7.64
	200	2.4550	0.2032	2.6580	7.64
	250	3.0690	0.2540	3.3230	7.64
60	100	1.4160	0.1350	1.5510	8.70
	150	2.1240	0.2024	2.3270	8.70
	200	2.8330	0.2699	3.1020	8.70
	250	3.5410	0.3374	3.8780	8.70
90	100	1.4100	0.2059	1.6160	12.74
	150	2.1150	0.3088	2.4240	12.74
	200	2.8200	0.4118	3.2320	12.74
	250	3.5250	0.5147	4.0400	12.74

beam, an increment of shear deflection is obtained. An additional integration between x=x, x=2L over x provides the deflection due to shear for the beam as fallows

$$v_{shear} = \frac{3P}{4tc^3} a_{16} \left(\begin{array}{c} 2L^2 y + 2ry^2 L - \dfrac{2rc^2 L}{3} \\ -\dfrac{x^2 y}{2} - ry^2 x + \dfrac{rc^2 x}{3} \end{array} \right) + \frac{3P}{8tc^3} a_{66} \left(2c^2 L - 2y^2 L - c^2 x + y^2 x \right) \quad (20)$$

In order to determine the deflection function of the symmetry axis at the mid point, it is necessary to insert x=L and y=0 into Eqn. (20) to find

$$v_{shear} = \frac{3P}{4tc^3} a_{16} \left(-\frac{rc^2 L}{3} \right) + \frac{3P}{8tc^3} a_{66} \left(c^2 L \right) \quad (21)$$

Thus, the total deflection is

$$v_{total} = v_{flexure} + v_{shear} \quad (22)$$

To find the total deflection of beam at the mid point, x=L, y=0 values are inserted into Eqn. (22). Finally, we obtain

$$v_{total} = \frac{P}{4tc^3} a_{11} L^3 + \frac{3P}{4tc^3} a_{16} \left(-\frac{rc^2 L}{3} \right) + \frac{3P}{8tc^3} a_{66} \left(c^2 L \right) \quad (23)$$

3. SAMPLE PROBLEM

In this paper, a Sitka spruce is used for numerical calculations. Mechanical properties for Sitka spruce in the longitudinal-radial plane are given in Table 1 (Liu and Rammer, 2003).

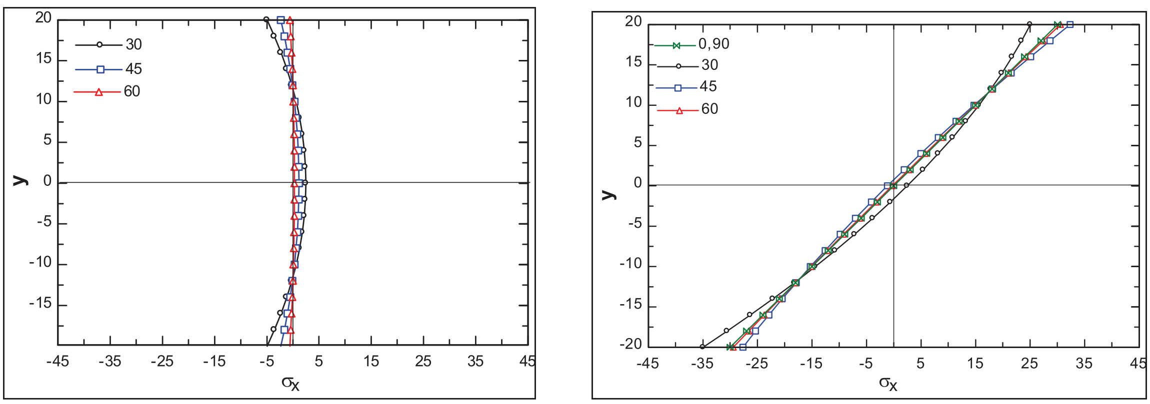

Figure 2. Normal stresses (σ_x) with respect to different orientation angles ((a) x=80 mm, (b) x=0 mm).

Table 1. Mechanical properties of the composite beam (Liu and Rammer, 2003).

E_1(Gpa)	E_2(GPa)	G_{12}(GPa)	ν_{12}
11.800	2.216	0.910	0.37

Eqn. (23) for single force is used to obtained the deflections at the mid point of beams analytically. Firstly; The constant values t,c and P were taken as 1mm, 20 mm and 150 N respectively. Calculations were obtained for 80, 100, 120 and 140 mm beam half lengths for 0, 30, 45, 60 and 90 orientation angles. It was calculated flexure and shear deflections at the mid point of beams, and then obtained total deflections. Also, İt was obtained to shear effects as seen Table 2. As seen from this table, the longer the beam is the smaller shear effect. Shear deflection is the smallest for 45 orientation angle. Secondly; the constant values t; L and P were taken as 1mm, 100 mm and 150 N respectively. Calculations were obtained for 10, 15, 20 and 25 beam half heights lengths for 0, 30, 45, 60 and 90 orientation angles (Table 3).

As seen this table , while cross-sectional height of beam increases the shear effect increases too. But, the shear deflection decreases. Finally; the constant values t, c, L were taken as 1mm, 20 mm and 100 mm respectively. Calculations for single force were obtained for 100, 150, 200 and 250 N in Table 4 for 0, 30, 45, 60 and 90 orientation angles. As seen Table 4, while the values of single forces increase, shear deflections decrease. Tables show the variations of the percentage beam-end deflection error level due to shear effects.

Figure 2 presents the distribution of the normal stress σ_x for L=80 mm having maximum error. Figure 2a shows the distribution of the normal stresses for x=80 mm. The stresses are zero for 0 and 90 orientation angles. Orientation angle increases from zero, the normal stresses distribute parabolically. At the mid point with x=0 mm, the normal stres σ_x distributions are shown in Figure 2b. For 0 and 90 orientation angles, The stresses fall on a straight line. But, for the other orientation angles the stresses distribute parabolically.

Also, as it can be seen in Figure 2, the intensity of normal stress σ_x is maximum at the upper and lower surfaces for 0 and 90 orientation angles. But, the intensity of normal stres σ_x at the upper surface becomes greater than lower surface for 30, 45 and 60 orientation angles.

4. DISCUSSIONS AND CONCLUSIONS

For simply supported composite beams loaded single forces, flexure and shear deflection functions are obtained in this study. Deflection functions are derived for calculations. The results given below are concluded in this investigation.

The error level is the smallest for 45 orientation angle and the biggest for 0 orientation angle.

The smaller the beam is the bigger the shear effect,

The longer the beam is the bigger the shear deflection,

Shear deflection is the smallest for 45 orientation angle and is the biggest for 0 orientation angle.

The heighter the beam is the bigger the shear effect, but shear deflection is the less,

While the values of single force increase, shear and flexure deflections decrease for the same orientation angles. But, the error levels are the same.

While the orientation angles increase shear deflection and the error levels decrease. But, flexure and total deflections increase.

When the orientation angle theta is 0 and 90 the bending stress curve at the mid point is linear. When the orientation angle theta is 30, 45 and 60, the bending stress curves at the mid point is nonlinear.

Bending stress is maximum for 30 orientation angle.

REFERENCES

Altenbach, H. 2000. On the determination of transverse shear stiffnesses of orthotropic plates, *Zeitschrift für Angewandte Mathematik and Physik,* 51 (4), 629-649.

Aydoğan, M. 1995. Stiffness-matrix formulation of beams with shear effect on elastic foundation, *Journal of Structural Engineering,* 121 (9), 1265-1270.

Biblis, E. J. 1997. Shear deflection of two species laminated wood beams, *Wood Science and Technology*. 1 (3), 231-238.

Esendemir, Ü. 2004. An elastic-plastic stress analysis in a polymer-matrix composite beam of arbitrary orientation supported from two ends acted upon with a force at the mid point. *Journal of Reinforced Plastics and Composites,* 23 (6), 613-623.

Esendemir, Ü. 2005. The effects of shear on the deflection of linearly loaded composite cantilever beam, *Journal of Reinforced Plastics and Composites*. 24 (11), 1159-1168.

Esendemir, Ü., Usal, M.R., Usal, M. 2006. The effects of shear on the deflection of simply supported composite beam loaded linearly, *Journal of Reinforced Plastics and Composites*. 25 (8), 835-846.

Evangelas, J. Biblis, 1967. Shear deflection of two species laminated wood beams, *Wood Science and Technology*. 1 (3), 231-238.

Faella, C., Martinelli, E., and Nigro, E. 2003. Shear connection nonlinearity and deflections of steel concrete composite beams: a simplified method, *Journal of Structural Engineering*. 129 (1), 12-20.

Hiroaki, K., Tohru, N. 1993. Shear deflection of anisotropic plates. *JSME Internatianal Journal Series A: Mechanics and Material Engineering*. 36 (1). 73-79.

Jones, R. M. 1975. Mechanics of composite materials. *Mcgraw-Hill,* Kogakusha, Tokyo.

Kılıç, O., Aktaş, A. and Dirikolu, M.H. 2001. An investigating of the effects of shear on the deflection of an orthotropic cantilever beam by use of anisotropic elasticity theory. *Composites Science and Technology*. (61), 2055-2061.

Kubojima, Y., Ohtani, T., and Yoshihara, H. 2004. Effect of shear deflection on vibrational properties of compressed wood, *Wood Science and Technology.* 38 (3), 237-244.

Lee, S.J, Reddy, J.N. 2004. Nonlinear deflection control of laminated plates using third-order shear deformation theory, *International journal of Mechanics and Materials Design,* 1 (1), 33-61.

Lekhnitskii, S.G. 1968. Anisotropic Plates, *Gordon and Breach Science*, New York.

Lekhnitskii, S.G. 1981. Theory of Elasticity of an Anisotropic Body. *Mir Publishers,* Moscow.

Liu, J.Y and Rammer, D.R. 2003. Analysis of wood cantilever loaded at free end, *Wood and Fiber Science*. 35 (3), 334-340.

Machado, S. P., Cortinez, V.H. 2005. Non-linear model for stability of thin-walled composite beams with shear deformation, *Thin-Walled Structures*. (43), 1615-1645.

Nie, J. and Cai, C.S. 1998. Steel-concrete composite beams considering shear slip effects, *Journal of Structural Engineering.* 129 (4), 495-506.

Nie, J., Cai, C.S. 2000. Deflection of cracked rc beams under sustained loading, Journal of Structural Engineering. 126 (6), 708-716.

Onu, G. 2000. Shear effect in beam finite element on two-parameter elastic foundation, *Journal of Structural Engineering.* 126 (9), 1104-1107.

Pilkey, W. D., Kang, W., Schramm, U. 1995. New structural matrices for a beam element with shear deformation, *Finite Elements Journal of Structural Engineering,* 121 (9). *In Analysis and design,* 19 (1), 25-44.

Schramm, U., Kitis, L., Kang, W., Pilkey, W.D. 1994. On the shear deformation coefficient in beam theory, *Finite Elements in Analysis and Design*. 16 (2), 141-162.

Thomas, W. H. 2002. Shear and flexural deflection equations for OSB floor decking with point load, H*olz als Roh-Und Werstoff*. 60 (3), 175-180.

Usal, M.R., Usal, M., Esendemir, Ü. 2008. Static and dynamic analysis of simply supported beams, *Journal of Reinforced Plastics and Composites,* 27 (3): 263-276.

Wang, Y.C. 1998. Deflection of steel-concrete composite beams with partial shear interaction, *Journal of Structural Engineering.* 124 (10), 1159-1165.