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Research Article

An approach for unbalanced fully rough interval transportation problem

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Abstract

In this study, we consider an unbalanced fully rough interval transportation problem, where all the parameters and decision variables are represented by rough interval numbers. A method named as split and separation method has been proposed in the literature to find the optimal solution of balanced fully rough interval transportation problem. As per our knowledge, no method exists in the literature to solve an unbalanced fully rough interval transportation problem. Therefore, a new method is proposed in this study to solve such problem. Using proposed methodology, firstly the unbalanced problem is converted into a balanced one and then the optimal solution of the balanced problem is obtained. To show the applicability of the proposed methodology, a numerical example is solved. Finally, the study's conclusions and future research directions are discussed.

Mathematics Subject Classification (2020). 90B06, 90C27, 90C70

Keywords. Transportation problem, rough interval coefficient, unbalanced fully rough interval transportation problem, optimal solution

1. Introduction

The transportation problem, developed by [17] in 1941, is a special kind of linear programming problem. The classical transportation problem is designed to obtain the optimal resource allocation from supply sites to demand points while reducing the total transportation cost and it is solved as a crisp linear programming problem. Crisp linear programming problem means that all the parameters such as cost of transportation between sources and destinations, available supply at each of the sources and the demand of each destination is completely known. It is assumed that the decision maker has no confusion/ambiguity about the values of the parameters. In the literature, various approaches [2, 4, 19, 20] have been developed to solve the crisp transportation problem such as Korukouglu and Balli [20] solved a transportation problem with crisp parameters using modified Vogel's approximation method. To obtain the better feasible solution for balanced transportation problem, Amilah et al. [4] proposed the supply selection method. Karagul and Sahin [19]

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proposed a new approximation method to solve the crisp transportation problem. But in real-life applications, because of environmental fluctuations, global market instability, rapid price fluctuations etc., the parameters of transportation problem can not be determined crisply. To handle this situation, in 1965, Zadeh [35] developed the fuzzy set (FS) theory which assigns a degree of membership to each element of the set. Number of researchers [1,5,7,8,12,18] have solved the transportation problems with fuzzy parameters. To handle the uncertainty of transportation problem, Baidya et al. [8] proposed generalized credibility measure and CV-Based reduction methods for type-2 triangular fuzzy numbers. Anukokila et al. [5] solved fractional transportation problem with interval-valued fuzzy parameters. Adhami and Ahmad [1] proposed Pythagorean-hesitant fuzzy programming technique to solve the multi-objective transportation problem with fuzzy parameters. Bagheri et al. [7] used triangular fuzzy numbers to tackle the uncertainty of transportation problem and developed fuzzy DEA approach to solve it. Kacher and Singh [18] proposed fuzzy harmonic mean based technique to address the fully fuzzy transportation problem.

Many times a situation arises in the transportation system when total supply of product is not equal to its total demand, such transportation problem is called an unbalanced transportation problem. Many authors have worked on the unbalanced transportation problem such as Kumar and Kaur [21] developed the methods to solve an unbalanced fully fuzzy transportation problems. The fully fuzzy transportation problem in which total availability of product exceeds total demand has been addressed by [29]. Muthuperuma et al. [27] proposed a new approach to identify the basic feasible solution of an unbalanced transportation problem. For the further generlization of FS theory, Atanassov [6] developed intuitionistic fuzzy (IF) set theory. To address the uncertainty of product blending transportation problem, Roy and Midya [31] employed triangular IF numbers. Mahajan and Gupta [23] proposed IF programming approach with non linear membership functions to solve the fully IF transportation problem. Ghosh et al. [15] developed (α, β)-cut approach for the fully IF solid transportation problem.

Apart from fuzziness, the rough set theory [28] is one another tool to handle the uncertainty of optimization problems. The rough set theory has a significant role in managing uncertainty and ambiguity simultaneously in distinctive kinds of fields. But rough set theory is unable to handle continuous variables and is only relevant for tackling problems with discrete data. So, as a specific instance of the rough set, Robolledo [30] introduced the idea of rough interval. In addition to satisfying all the properties of rough set and basic concepts, a rough interval is also capable to describe the continuous variables. To understand the significance of rough interval in real life problems, we consider an example. Let Δ is the "demand for flour" in a town which fluctuate between 35 to 55 tonnes per day. Usually, it fluctuate between 45 to 50 tonnes per day. The other values in the range [35, 55] occurs in specific circumstance such as at certain events, seasons, or holidays, etc. This demonstrates that the demand for flour can be represented by the rough interval $\Delta = (\underline{\Delta}, \overline{\Delta})$ [45, 50][35, 55].

Currently, a number of researchers [9,14,25,26,32,33] have given their consideration on various properties of rough interval to solve the transportation problem such as Bera et al. [9] investigated the model of 4D transportation problem with rough intervals to maximize the profit. Garg and Allah [14] proposed a novel approach to solve the rough interval multi-objective transportation problems. Roy et al. [32] used random rough parameters to handle the impreciseness in transportation problems. Midya and Roy [26] employed fuzzy programming and weighted-sum method to solve the transportation problem with rough interval parameters. A transportation problem in which along with parameters, decision variables are also represented by rough interval numbers is referred to as a fully rough interval transportation problem (FRITP). The literature survey reveals that no approach has been developed to solve the unbalanced FRITP. To address this limitation, a new methodology to solve the unbalanced FRITP is proposed in this paper, which involves converting the unbalanced FRITP into a balanced form. The proposed approach is demonstrated by obtaining the rough interval optimal solution of a numerical example. Some existing research work on transportation problems are presented in Table 1.

Table 1. Existing research work on transportation problem.

References	Nature of transportation problem	Environment	Balanced/ Unbalanced	Number of objective functions
Muthuperuma [27]	TP	Fuzzy	Unbalanced	Single
Bagheri et al. [7]	TP	Fuzzy	Balanced	Multi
Kumar and Kaur [21]	TP	Fully fuzzy	Unbalanced	Single
Rani et al. [29]	TP	Fully fuzzy	Unbalanced	Single
Kacher and Singh [18]	TP	Fully fuzzy	Balanced	Multi
Roy and Midya [31]	STP	IF	Unbalanced	Multi
Mahomoodirad et al. [24]	TP	Fully IF	Balanced	Single
Mahajan and Gupta [23]	TP	Fully IF	Balanced	Multi
Ghosh et al. [15]	STP	Fully IF	Balanced	Multi
Midya and Roy [26]	TP	Rough	Unbalanced	Multi
Akilbasha et al. [3]	TP	Fully Rough	Balanced	Single
Proposed work	TP	Fully Rough	Unbalanced	Single

Contribution of the proposed work:

- The model of an unbalanced FRITP is formulated using the rough set theory.
- A novel method has been proposed to solve the unbalanced FRITP.
- The efficiency of the proposed method is demonstrated by solving a numerical illustration.
- The solutions obtained by the proposed method are compared with the existing methods.

This paper is summarized as: Section 2 depicts the basic definitions related to rough set theory. Section 3 defines the mathematical model of an unbalanced FRITP. The drawbacks of some existing studies are explored in Section 4. A new proposed methodology to solve the unbalanced FRITP is defined in Section 5. Section 6 illustrates the application of proposed methodology to solve a numerical example of an unbalanced FRITP. Results and discussion are given in Section 7. Finally, Section 8 summarizes the conclusions of this study and suggests directions for future research.

2. Preliminaries

This section depicts some fundamental definitions, which are utilized in this work.

Definition 2.1. [22] Let Υ be a non-empty set and S is a σ algebra of its subset Υ. The Ω is an element in S and ω be positive, real-valued, additive set function. Then, $(\Upsilon, \Omega, S, \omega)$ is referred to as a rough space.

Definition 2.2. [34] Let U be a universe and R be an equivalence relation on U, then the pair (U, R) is called as approximation space. Let $X \subseteq U$, then the lower and upper approximation of X is defined as

- The lower approximation of X

$$\underline{R}(X) = \bigcup_{x \in X} \{R(x) : R(x) \subseteq X\}.$$

- The upper approximation of X

$\overline{R}(X) = \bigcup_{x\in X} \{R(x) : R(x) \cap X \neq \phi\}.$

- The boundary region of X

$bn_R(X) = \overline{R}(X) - \underline{R}(X).$

The graphical depiction of the rough set is given in Figure 1.

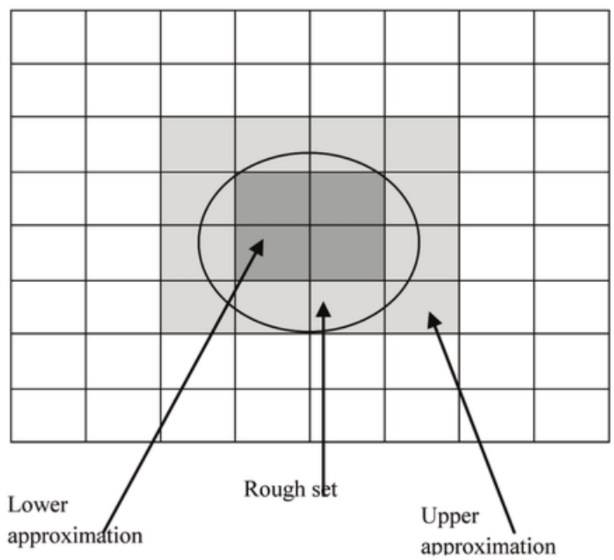

Figure 1. Rough set.

Definition 2.3. [34] A rough interval is defined as $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ where, η^{LL}, η^{UL}, η^{LU}, η^{UU} are all real numbers and $\eta^{LU} \leq \eta^{LL} \leq \eta^{UL} \leq \eta^{UU}$. The interval $[\eta^{LL}, \eta^{UL}]$ is called lower approximation interval and $[\eta^{LU}, \eta^{UU}]$ is called upper approximation interval such that

- If $y \in [\eta^{LL}, \eta^{UL}]$, then $\widetilde{S}^{RI}$ surely takes y.
- If $y \in [\eta^{LU}, \eta^{UU}]$, then $\widetilde{S}^{RI}$ probably takes y.
- If $y \notin [\eta^{LU}, \eta^{UU}]$, then $\widetilde{S}^{RI}$ definitely does not takes y.

Remark 2.4. Let $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ be a rough interval. If $\eta^{LL} = \eta^{LU}$ and $\eta^{UL} = \eta^{UU}$. Then rough interval $\widetilde{S}^{RI}$ converts to a crisp interval.

Remark 2.5. A rough interval $\widetilde{S}^{RI} = [0, 0][0, 0]$ is referred to as zero rough interval number.

Definition 2.6. A rough interval $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ is said to be positive if and only if $\eta^{LU} \geq 0$.

Definition 2.7. Let $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ and $\widetilde{T}^{RI} = [\theta^{LL}, \theta^{UL}][\theta^{LU}, \theta^{UU}]$ are two rough intervals. Then $\widetilde{S}^{RI} = \widetilde{T}^{RI}$ if and only if $\eta^{LL} = \theta^{LL}, \eta^{UL} = \theta^{UL}, \eta^{LU} = \theta^{LU}, \eta^{UU} = \theta^{UU}$.

Definition 2.8. Let $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ and $\widetilde{T}^{RI} = [\theta^{LL}, \theta^{UL}][\theta^{LU}, \theta^{UU}]$ be two rough intervals with $\eta^{LU}, \theta^{LU} \geq 0$. Then the arithmetic operations on these two rough intervals are defined as follows:

- $\widetilde{S}^{RI} \oplus \widetilde{T}^{RI} = [\eta^{LL} + \theta^{LL}, \eta^{UL} + \theta^{UL}][\eta^{LU} + \theta^{LU}, \eta^{UU} + \theta^{UU}]$,
- $\widetilde{S}^{RI} \ominus \widetilde{T}^{RI} = [\eta^{LL} - \theta^{UL}, \eta^{UL} - \theta^{LL}][\eta^{LU} - \theta^{UU}, \eta^{UU} - \theta^{LU}]$,

- $\widetilde{S}^{RI} \otimes \widetilde{T}^{RI} = [\eta^{LL}\theta^{LL}, \eta^{UL}\theta^{UL}],\ [\eta^{LU}\theta^{LU}, \eta^{UU}\theta^{UU}]$,
- $\widetilde{S}^{RI} \oslash \widetilde{T}^{RI} = [\eta^{LL}/\theta^{UL}, \eta^{UL}/\theta^{LL}][\eta^{LU}/\theta^{UU}, \eta^{UU}/\theta^{LU}]$,
- $k\widetilde{S}^{RI} = \begin{cases} [k\eta^{LL}, k\eta^{UL}][k\eta^{LU}, k\eta^{UU}], & \text{if } k \geq 0 \\ [k\eta^{UL}, k\eta^{LL}][k\eta^{UU}, k\eta^{LU}], & \text{if } k < 0. \end{cases}$

Definition 2.9. [13] Let $\widetilde{S}^{RI} = [\underline{S^{RI}}, \overline{S^{RI}}]$ be a rough interval on the rough space $(\Upsilon, \Omega, S, \omega)$. Then the lower trust measure of the rough interval $\widetilde{S}^{RI} \leq r$ is defined as

$$\underline{Tr}\{\widetilde{S}^{RI} \leq r\} = \frac{(y \in \underline{S^{RI}} | y \leq r)}{card(\underline{S^{RI}})},$$

where $card()$ denotes the number of elements in a given set.
Similarly the upper trust measure is defined as

$$\overline{Tr}\{\widetilde{S}^{RI} \leq r\} = \frac{(y \in \overline{S^{RI}} | y \leq r)}{card(\overline{S^{RI}})}.$$

Hence, the trust measure is defined as the convex combination of the lower and the upper trusts measure which is as

$$Tr\{\widetilde{S}^{RI} \leq r\} = \frac{1}{2}\Big(\underline{Tr}\{\widetilde{S}^{RI} \leq r\} + \overline{Tr}\{\widetilde{S}^{RI} \leq r\}\Big).$$

Definition 2.10. [13] Let $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ be a rough interval, then the trust measure of $\widetilde{S}^{RI} \leq r$ is defined as

$$Tr\{\widetilde{S}^{RI} \leq r\} = \begin{cases} 0, & \text{if } r \leq \eta^{LU} \\ \frac{1}{2}\left(\frac{r-\eta^{LU}}{\eta^{UU}-\eta^{LU}}\right), & \text{if } \eta^{LU} \leq r \leq \eta^{LL} \\ \frac{1}{2}\left(\frac{r-\eta^{LU}}{\eta^{UU}-\eta^{LU}} + \frac{r-\eta^{LL}}{\eta^{UL}-\eta^{LL}}\right), & \text{if } \eta^{LL} \leq r \leq \eta^{UL} \\ \frac{1}{2}\left(\frac{r-\eta^{LU}}{\eta^{UU}-\eta^{LU}} + 1\right), & \text{if } \eta^{UL} \leq r \leq \eta^{UU} \\ 1, & \text{if } \eta^{UU} \leq r \end{cases}$$

$$Tr\{\widetilde{S}^{RI} \geq r\} = \begin{cases} 0, & \text{if } \eta^{UU} \leq r \\ \frac{1}{2}\left(\frac{\eta^{UU}-r}{\eta^{UU}-\eta^{LU}}\right), & \text{if } \eta^{UL} \leq r \leq \eta^{UU} \\ \frac{1}{2}\left(\frac{\eta^{UU}-r}{\eta^{UU}-\eta^{LU}} + \frac{\eta^{UL}-r}{\eta^{UL}-\eta^{LL}}\right), & \text{if } \eta^{LL} \leq r \leq \eta^{UL} \\ \frac{1}{2}\left(\frac{\eta^{UU}-r}{\eta^{UU}-\eta^{LU}} + 1\right), & \text{if } \eta^{LU} \leq r \leq \eta^{LL} \\ 1, & \text{if } r \leq \eta^{LU} \end{cases}$$

Definition 2.11. [22] Let $\widetilde{S}^{RI}$ be a rough variable on the rough space $(\Upsilon, \Omega, S, \omega)$. The expected value of $\widetilde{S}^{RI}$ is defined by

$$E[\widetilde{S}^{RI}] = \int_0^\infty Tr\{\widetilde{S}^{RI} \geq r\}dr - \int_{-\infty}^0 Tr\{\widetilde{S}^{RI} \leq r\}dr,$$

where E represents the expected value operator and Tr is the trust measure.

Theorem 2.12. *[34] Let $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ be a rough interval. Then the expected value of $\widetilde{S}^{RI}$ is defined as $E[\widetilde{S}^{RI}] = \frac{1}{2}[\tau(\eta^{LL} + \eta^{UL}) + (1-\tau)(\eta^{LU} + \eta^{UU})]$, where $\tau \in [0, 1]$.*

Remark 2.13. If $\tau = 0.5$, then $E[\widetilde{S}^{RI}] = \frac{1}{4}(\eta^{LL} + \eta^{UL} + \eta^{LU} + \eta^{UU})$.

Theorem 2.14. *[34] Let $\widetilde{S}^{RI}$ and $\widetilde{T}^{RI}$ are two rough intervals with finite expected values. Then for any real number p and q, $E[p\widetilde{S}^{RI} + q\widetilde{T}^{RI}] = pE[\widetilde{S}^{RI}] + qE[\widetilde{T}^{RI}]$.*

Proof. Since $\widetilde{S}^{RI} = [\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}]$ and $\widetilde{T}^{RI} = [\theta^{LL}, \theta^{UL}][\theta^{LU}, \theta^{UU}]$ are two rough intervals and p, q are two positive real numbers. Therefore, using arithmetic operations on rough intervals, we get

$$p\widetilde{S}^{RI} + q\widetilde{T}^{RI} = [p\eta^{LL} + q\theta^{LL}, p\eta^{UL} + q\theta^{UL}][p\eta^{LU} + q\theta^{LU}, p\eta^{UU} + q\theta^{UU}]$$

which is also a rough interval. Then using Theorem 2.12,

$$\begin{aligned} E[p\widetilde{S}^{RI} + q\widetilde{T}^{RI}] &= \frac{1}{4}(p\eta^{LL} + q\theta^{LL} + p\eta^{UL} + q\theta^{UL} + p\eta^{LU} + q\theta^{LU} + p\eta^{UU} + q\theta^{UU}) \\ &= \frac{p}{4}(\eta^{LL} + \eta^{UL} + \eta^{LU} + \eta^{UU}) + \frac{q}{4}(\theta^{LL} + \theta^{UL} + \theta^{LU} + \theta^{UU}) \\ &= pE[\widetilde{S}^{RI}] + qE[\widetilde{T}^{RI}] \end{aligned}$$

□

3. Mathematical model

Here, the model of an unbalanced FRITP is constructed in which product is supplied from u ($l = 1, 2, \ldots, u$) sources to v ($m = 1, 2, \ldots, v$) destinations. The main objective of the decision maker is to calculate the amount of item to be shipped from l^{th} source to m^{th} destination in a way that minimizes the total transportation cost, which is represented by Eq. (3.1). Constraints (3.2), (3.3) and (3.4) represent supply, demand and non-negativity constraints, respectively. To articulate the proposed mathematical model, we make use of the following notations and assumptions:

Notations:
u: the number of sources ($l = 1, 2, \ldots, u$)
v: the number of destinations ($m = 1, 2, \ldots, v$)
$[c_{lm}^{LL}, c_{lm}^{UL}]$ $[c_{lm}^{LU}, c_{lm}^{UU}]$: unit transportation cost from l^{th} source to m^{th} destination
$[x_{lm}^{LL}, x_{lm}^{UL}]$ $[x_{lm}^{LU}, x_{lm}^{UU}]$: amount of item shipped from l^{th} source to m^{th} destination
$[a_l^{LL}, a_l^{UL}]$ $[a_l^{LU}, a_l^{UU}]$: availability of the item at l^{th} source
$[b_m^{LL}, b_m^{UL}]$ $[b_m^{LU}, b_m^{UU}]$: demand for the item at m^{th} destination

Assumptions :

- All the parameters (transportation cost, availability, demand) and decision variables (transported amount) are rough intervals.
- $[a_l^{LL}, a_l^{UL}]$ $[a_l^{LU}, a_l^{UU}] \geq 0$, $[b_m^{LL}, b_m^{UL}]$ $[b_m^{LU}, b_m^{UU}] \geq 0 \quad \forall \ l, \ m.$
- The problem is unbalanced, i.e.,

$$\sum_{l=1}^{u} [a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}] \neq \sum_{m=1}^{v} [b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}].$$

Mathematically the model of an unbalanced FRITP is formulated as:

Model 1:

$$\text{Minimize} \quad \widetilde{Z}^{RI} = \sum_{l=1}^{u} \sum_{m=1}^{v} [c_{lm}^{LL}, c_{lm}^{UL}][c_{lm}^{LU}, c_{lm}^{UU}] \otimes [x_{lm}^{LL}, x_{lm}^{UL}][x_{lm}^{LU}, x_{lm}^{UU}] \tag{3.1}$$

subject to

$$\sum_{m=1}^{v} [x_{lm}^{LL}, x_{lm}^{UL}][x_{lm}^{LU}, x_{lm}^{UU}] \leq [a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}]; \quad l = 1, 2, 3, \ldots, u \tag{3.2}$$

$$\sum_{l=1}^{u} [x_{lm}^{LL}, x_{lm}^{UL}][x_{lm}^{LU}, x_{lm}^{UU}] \geq [b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}]; \quad m = 1, 2, 3, \ldots, v \tag{3.3}$$

$$[x_{lm}^{LL}, x_{lm}^{UL}][x_{lm}^{LU}, x_{lm}^{UU}] \geq 0 \quad \forall \ l, m \tag{3.4}$$

4. Drawbacks of some of the existing methods

It can be observed from the literature that various researchers [3,13,14,26] have studied the transportation problem in rough interval environment. But there are some drawbacks of the existing methods which are presented below:

- In order to solve the fully rough interval integer transportation problem, Akilbasha et al. [3] proposed split and separation method based on the zero point method. However, the problem solved by the authors is a balanced problem, and the proposed solution approach in [3] is not applicable on an unbalanced problem.
- Das et al. [13] introduced rough interval approach to solve the profit maximizing solid transportation problem in which the original problem is transformed into four different transportation problems. But the method proposed by the author obtained crisp solution to the problem with rough interval data.
- Methods proposed in [14, 26] are suitable for solving transportation problems in which all parameters except decision variables are represented by rough intervals. However, with these methods we can not find the solution of FRITP.
- Numerous researchers [16, 21, 24, 27, 29] have studied unbalanced transportation problem in fully fuzzy or intuitionistic fuzzy environment. But no research paper has been published on unbalanced transportation problem in the context of fully rough interval environment.

5. Proposed method

A new method to solve an unbalanced FRITP (Model 1) has been proposed in this section. The stepwise methodology to obtain the solution of Model 1 is as follows:

Step 1: Determine whether the problem is balanced or not, i.e., either

$$\sum_{l=1}^{u} [a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}] = \sum_{m=1}^{v} [b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}]$$

or

$$\sum_{l=1}^{u} [a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}] \neq \sum_{m=1}^{v} [b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}].$$

Case 1: If FRITP is balanced, i.e., $\sum_{l=1}^{u} [a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}] = \sum_{m=1}^{v} [b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}]$, then go to Step 2.

Case 2: If FRITP is an unbalanced, i.e.,

$$\sum_{l=1}^{u} [a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}] \neq \sum_{m=1}^{v} [b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}],$$

then proceed according to the following subcases to make it balanced.

Subcase 2a: If

$$\sum_{l=1}^{u} a_l^{LU} \leq \sum_{m=1}^{v} b_m^{LU},$$

$$\sum_{l=1}^{u} a_l^{LL} - \sum_{l=1}^{u} a_l^{LU} \le \sum_{m=1}^{v} b_m^{LL} - \sum_{m=1}^{v} b_m^{LU},$$

$$\sum_{l=1}^{u} a_l^{UL} - \sum_{l=1}^{u} a_l^{LL} \le \sum_{m=1}^{v} b_m^{UL} - \sum_{m=1}^{v} b_m^{LL},$$

and

$$\sum_{l=1}^{u} a_l^{UU} - \sum_{l=1}^{u} a_l^{UL} \le \sum_{m=1}^{v} b_m^{UU} - \sum_{m=1}^{v} b_m^{UL},$$

then add dummy source with availability $[a^{LL}, a^{UL}][a^{LU}, a^{UU}]$, where

$$a^{LL} = \sum_{m=1}^{v} b_m^{LL} - \sum_{l=1}^{u} a_l^{LL},\ a^{UL} = \sum_{m=1}^{v} b_m^{UL} - \sum_{l=1}^{u} a_l^{UL},$$
$$a^{LU} = \sum_{m=1}^{v} b_m^{LU} - \sum_{l=1}^{u} a_l^{LU},\ a^{UU} = \sum_{m=1}^{v} b_m^{UU} - \sum_{l=1}^{u} a_l^{UU}.$$

Subcase 2b: If

$$\sum_{l=1}^{u} a_l^{LU} \ge \sum_{m=1}^{v} b_m^{LU},$$

$$\sum_{l=1}^{u} a_l^{LL} - \sum_{l=1}^{u} a_l^{LU} \ge \sum_{m=1}^{v} b_m^{LL} - \sum_{m=1}^{v} b_m^{LU},$$

$$\sum_{l=1}^{u} a_l^{UL} - \sum_{l=1}^{u} a_l^{LL} \ge \sum_{m=1}^{v} b_m^{UL} - \sum_{m=1}^{v} b_v^{LL}$$

and

$$\sum_{l=1}^{u} a_l^{UU} - \sum_{l=1}^{u} a_l^{UL} \ge \sum_{m=1}^{v} b_m^{UU} - \sum_{m=1}^{v} b_m^{UL},$$

then add dummy destination with demand $[b^{LL}, b^{UL}][b^{LU}, b^{UU}]$, where

$$b^{LL} = \sum_{l=1}^{u} a_l^{LL} - \sum_{m=1}^{v} b_m^{LL},\ b^{UL} = \sum_{l=1}^{u} a_l^{UL} - \sum_{m=1}^{v} b_m^{UL},$$
$$b^{LU} = \sum_{l=1}^{u} a_l^{LU} - \sum_{m=1}^{v} b_m^{LU},\ b^{UU} = \sum_{l=1}^{u} a_l^{UU} - \sum_{m=1}^{v} b_m^{UU}.$$

Subcase 2c: If none of Subcase 2a or Subcase 2b is satisfied, then add dummy source with availability $[a^{LL}, a^{UL}][a^{LU}, a^{UU}]$, where

$$a^{LL} = \max\left\{0, \sum_{m=1}^{v} b_m^{LU} - \sum_{l=1}^{u} a_l^{LU}\right\} + \max\left\{0, \left(\sum_{m=1}^{v} b_m^{LL} - \sum_{m=1}^{v} b_m^{LU}\right) - \left(\sum_{l=1}^{u} a_l^{LL} - \sum_{l=1}^{u} a_l^{LU}\right)\right\},$$
$$a^{UL} = \max\left\{0, \sum_{m=1}^{v} b_m^{LU} - \sum_{l=1}^{u} a_l^{LU}\right\} + \max\left\{0, \left(\sum_{m=1}^{v} b_m^{LL} - \sum_{m=1}^{v} b_m^{LU}\right) - \left(\sum_{l=1}^{u} a_l^{LL} - \sum_{l=1}^{u} a_l^{LU}\right)\right\}$$
$$+ \max\left\{0, \left(\sum_{m=1}^{v} b_m^{UL} - \sum_{m=1}^{v} b_m^{LL}\right) - \left(\sum_{l=1}^{u} a_l^{UL} - \sum_{l=1}^{u} a_l^{LL}\right)\right\},$$
$$a^{LU} = \max\left\{0, \sum_{m=1}^{v} b_m^{LU} - \sum_{l=1}^{u} a_l^{LU}\right\},$$

$$a^{UU}=\max\Big\{0,\sum_{m=1}^{v}b_m^{LU}-\sum_{l=1}^{u}a_l^{LU}\Big\}+\max\Big\{0,\Big(\sum_{m=1}^{v}b_m^{LL}-\sum_{m=1}^{v}b_m^{LU}\Big)-\Big(\sum_{l=1}^{u}a_l^{LL}-\sum_{l=1}^{u}a_l^{LU}\Big)\Big\}$$
$$+\max\Big\{0,\Big(\sum_{m=1}^{v}b_m^{UL}-\sum_{m=1}^{v}b_m^{LL}\Big)-\Big(\sum_{l=1}^{u}a_l^{UL}-\sum_{l=1}^{u}a_l^{LL}\Big)\Big\}$$
$$+\max\Big\{0,\Big(\sum_{m=1}^{v}b_m^{UU}-\sum_{m=1}^{v}b_m^{UL}\Big)-\Big(\sum_{l=1}^{u}a_l^{UU}-\sum_{l=1}^{u}a_l^{UL}\Big)\Big\},$$

and add dummy destination with demand $[b^{LL},b^{UL}][b^{LU},b^{UU}]$ where

$$b^{LL}=\max\Big\{0,\sum_{l=1}^{u}a_l^{LU}-\sum_{m=1}^{v}b_m^{LU}\Big\}+\max\Big\{0,\Big(\sum_{l=1}^{u}a_l^{LL}-\sum_{l=1}^{u}a_l^{LU}\Big)-\Big(\sum_{m=1}^{v}b_m^{LL}-\sum_{m=1}^{v}b_m^{LU}\Big)\Big\},$$
$$b^{UL}=\max\Big\{0,\sum_{l=1}^{u}a_l^{LU}-\sum_{m=1}^{v}b_m^{LU}\Big\}+\max\Big\{0,\Big(\sum_{l=1}^{u}a_l^{LL}-\sum_{l=1}^{u}a_l^{LU}\Big)-\Big(\sum_{m=1}^{v}b_m^{LL}-\sum_{m=1}^{v}b_m^{LU}\Big)\Big\}$$
$$+\max\Big\{0,\Big(\sum_{l=1}^{u}a_l^{UL}-\sum_{l=1}^{u}a_l^{LL}\Big)-\Big(\sum_{m=1}^{v}b_m^{UL}-\sum_{m=1}^{v}b_m^{LL}\Big)\Big\},$$
$$b^{LU}=\max\Big\{0,\sum_{l=1}^{u}a_l^{LU}-\sum_{m=1}^{v}b_m^{LU}\Big\},$$
$$b^{UU}=\max\Big\{0,\sum_{l=1}^{u}a_l^{LU}-\sum_{m=1}^{v}b_m^{LU}\Big\}+\max\Big\{0,\Big(\sum_{l=1}^{u}a_l^{LL}-\sum_{l=1}^{u}a_l^{LU}\Big)-\Big(\sum_{m=1}^{v}b_m^{LL}-\sum_{m=1}^{v}b_m^{LU}\Big)\Big\}$$
$$+\max\Big\{0,\Big(\sum_{l=1}^{u}a_l^{UL}-\sum_{l=1}^{u}a_l^{LL}\Big)-\Big(\sum_{m=1}^{v}b_m^{UL}-\sum_{m=1}^{v}b_m^{LL}\Big)\Big\}$$
$$+\max\Big\{0,\Big(\sum_{l=1}^{u}a_l^{UU}-\sum_{l=1}^{u}a_l^{UL}\Big)-\Big(\sum_{m=1}^{v}b_m^{UU}-\sum_{m=1}^{v}b_m^{UL}\Big)\Big\}.$$

Let us suppose that the unit transportation cost corresponding to dummy source or destination be zero rough interval number. Now the unbalanced problem (Model 1) turns into balanced form as follows:

Model 2:

$$\text{Minimize } \widetilde{Z}^{RI}=\sum_{l=1}^{w}\sum_{m=1}^{z}[c_{lm}^{LL},c_{lm}^{UL}][c_{lm}^{LU},c_{lm}^{UU}]\otimes[x_{lm}^{LL},x_{lm}^{UL}][x_{lm}^{LU},x_{lm}^{UU}] \tag{5.1}$$

subject to

$$\sum_{m=1}^{z}[x_{lm}^{LL},x_{lm}^{UL}][x_{lm}^{LU},x_{lm}^{UU}]=[a_l^{LL},a_l^{UL}][a_l^{LU},a_l^{UU}];\;\; l=1,2,3,\dots,w \tag{5.2}$$

$$\sum_{l=1}^{w}[x_{lm}^{LL},x_{lm}^{UL}][x_{lm}^{LU},x_{lm}^{UU}]=[b_m^{LL},b_m^{UL}][b_m^{LU},b_m^{UU}];\;\; m=1,2,3,\dots,z \tag{5.3}$$

$$[x_{lm}^{LL},x_{lm}^{UL}][x_{lm}^{LU},x_{lm}^{UU}]\geq 0,\;\;\forall\, l,m, \tag{5.4}$$

where $w=u$ or $u+1$, $\;z=v$ or $v+1$.

Step 2: By applying the equality and non-negativity conditions on the constraints (5.2) to (5.4), i.e.,

$$[\eta^{LL},\eta^{UL}][\eta^{LU},\eta^{UU}]=[\theta^{LL},\theta^{UL}][\theta^{LU},\theta^{UU}]\Rightarrow\eta^{LL}=\theta^{LL},\eta^{UL}=\theta^{UL},\eta^{LU}=\theta^{LU},\eta^{UU}=\theta^{UU}$$

and

$$[\eta^{LL},\;\eta^{UL}][\eta^{LU},\;\eta^{UU}]\geq 0\Rightarrow\eta^{LU}\geq 0,\;\;\eta^{LL}-\eta^{LU}\geq 0,\;\;\eta^{UL}-\eta^{LL}\geq 0,\;\;\eta^{UU}-\eta^{UL}\geq 0.$$

Model 2 is becomes Model 3 as:

Model 3:

$$\text{Minimize } \widetilde{Z}^{RI} = \sum_{l=1}^{w}\sum_{m=1}^{z}[c_{lm}^{LL}, c_{lm}^{UL}][c_{lm}^{LU}, c_{lm}^{UU}] \otimes [x_{lm}^{LL}, x_{lm}^{UL}][x_{lm}^{LU}, x_{lm}^{UU}] \tag{5.5}$$

subject to

$$\sum_{m=1}^{z} x_{lm}^{LL} = a_l^{LL}, \quad \forall \ l = 1, 2, \ldots, w \tag{5.6}$$

$$\sum_{m=1}^{z} x_{lm}^{UL} = a_l^{UL}, \quad \forall \ l = 1, 2, \ldots, w \tag{5.7}$$

$$\sum_{m=1}^{z} x_{lm}^{LU} = a_l^{LU}, \quad \forall \ l = 1, 2, \ldots, w \tag{5.8}$$

$$\sum_{m=1}^{z} x_{lm}^{UU} = a_l^{UU}, \quad \forall \ l = 1, 2, \ldots, w \tag{5.9}$$

$$\sum_{l=1}^{w} x_{lm}^{LL} = b_m^{LL}, \quad \forall \ m = 1, 2, \ldots, z \tag{5.10}$$

$$\sum_{i=1}^{w} x_{lm}^{UL} = b_m^{UL}, \quad \forall \ m = 1, 2, \ldots, z \tag{5.11}$$

$$\sum_{l=1}^{w} x_{lm}^{LU} = b_m^{LU}, \quad \forall \ m = 1, 2, \ldots, z \tag{5.12}$$

$$\sum_{l=1}^{w} x_{lm}^{UU} = b_m^{UU}, \quad \forall \ m = 1, 2, \ldots, z \tag{5.13}$$

$$x_{lm}^{LU} \geq 0, \quad \forall \ l = 1, 2, \ldots, w, \ \ m = 1, 2, \ldots, z \tag{5.14}$$

$$x_{lm}^{LL} - x_{lm}^{LU} \geq 0, \quad \forall \ l = 1, 2, \ldots, w, \ \ m = 1, 2, \ldots, z \tag{5.15}$$

$$x_{lm}^{UL} - x_{lm}^{LL} \geq 0, \quad \forall \ l = 1, 2, \ldots, w, \ \ m = 1, 2, \ldots, z \tag{5.16}$$

$$x_{lm}^{UU} - x_{lm}^{UL} \geq 0, \quad \forall \ l = 1, 2, \ldots, w, \ \ m = 1, 2, \ldots, z \tag{5.17}$$

Step 3: Use the multiplication operation on rough interval which is given in Definition 2.8, i.e.,

$$[\eta^{LL}, \eta^{UL}][\eta^{LU}, \eta^{UU}] \otimes [\theta^{LL}, \theta^{UL}][\theta^{LU}, \theta^{UU}] = [\eta^{LL}\theta^{LL}, \eta^{UL}\theta^{UL}][\eta^{LU}\theta^{LU}, \eta^{UU}\theta^{UU}],$$

to transform the Model 3 into its equivalent model, i.e., Model 4 as

Model 4:

$$\text{Minimize } \widetilde{Z}^{RI} = \Big[\sum_{l=1}^{w}\sum_{m=1}^{z}\Big([c_{lm}^{LL}x_{lm}^{LL}, c_{lm}^{UL}x_{lm}^{UL}][c_{lm}^{LU}x_{lm}^{LU}, c_{lm}^{UU}x_{lm}^{UU}]\Big)\Big] \tag{5.18}$$

subject to constraints (5.6) to (5.17)

Step 4: The Model 4 can not be directly solve due to the existence of rough parameters. Therefore, by using the expected value of rough interval numbers discussed in Theorem 2.12, the Model 4 is converted into a crisp model, i.e. Model 5:

Model 5:

$$\text{Minimize } E(\widetilde{Z}^{RI}) = \frac{1}{4}\Big[\sum_{l=1}^{w}\sum_{m=1}^{z}\Big(c_{lm}^{LL}x_{lm}^{LL} + c_{lm}^{UL}x_{lm}^{UL} + c_{lm}^{LU}x_{lm}^{LU} + c_{lm}^{UU}x_{lm}^{UU}\Big)\Big] \quad (5.19)$$

subject to constraints (5.6) to (5.17)

Step 5: Determine the optimal solution $[x_{lm}^{LL*}, x_{lm}^{UL*}][x_{lm}^{LU*}, x_{lm}^{UU*}]$ of the crisp model (Model 5) by using any of the optimization solver.

Step 6: Use the solution determine in Step 5 to calculate the optimal value of objective function as follows:

$$\widetilde{Z}^{RI} = \sum_{l=1}^{w}\sum_{m=1}^{z}[c_{lm}^{LL}x_{lm}^{LL*}, c_{lm}^{UL}x_{lm}^{UL*}][c_{lm}^{LU}x_{lm}^{LU*}, c_{lm}^{UU}x_{lm}^{UU*}].$$

5.1. Advantages of the proposed method:

(i) The rough set theory is advantageous over the fuzzy set theory and intuitionistic fuzzy set theory as it manages the ambiguity and imprecision in a better way. Therefore, the mathematical model of the real world problem in terms of rough intervals reflects the actual situation and the solutions obtained by using the proposed approach are better than those obtained by using the other theories.

(ii) Most of the real-world transportation problems are unbalanced and proposed methodology can handle such problems effectively. Hence, the proposed approach is best suited for practical applications.

(iii) The proposed method obtains the rough interval solution to a problem with rough parameters, which gives the decision makers a broader range of solutions and a better understanding of them.

(iv) It provides the non-negative solution for the considered problem, which enhances its feasibility and applicability.

6. Numerical example

A fruit supply company has three supply points at Delhi, Chandigarh and Lucknow with product availability a_l $(l = 1, 2, 3)$ and four destination points at Ahmedabad, Jaipur, Ludhiana and Jodhpur having demand b_m $(m = 1, 2, 3, 4)$. Due to imprecise information, the transportation cost between source and destination, demand of destination points and supply of origins are taken as rough interval, which are presented in Table 2. The main objective of the decision maker is to find the amount of product $\widetilde{x}_{lm}^{RI}$ shipped from l^{th} source to m^{th} destination so that we get the minimum total transportation cost. The transportation cost is considered in dollar ($) per ton and supply, demand are measured in tons.

Table 2. Unbalanced fully rough interval transportation problem (FRITP).

	D_1	D_2	D_3	D_4	Supply
S_1	[36, 37][35, 38]	[40, 42][38, 44]	[38, 40][36, 42]	[37, 39][35, 40]	[7, 8][6, 10]
S_2	[41, 42][40, 44]	[40, 42][39, 45]	[42, 45][40, 46]	[30, 32][28, 33]	[9, 10][7, 12]
S_3	[30, 32][29, 33]	[28, 33][26, 35]	[35, 37][34, 39]	[42, 43][40, 44]	[16, 20][15, 22]
Demand	[4, 5][3, 7]	[6, 10][5, 13]	[10, 12][9, 14]	[8, 11][7, 12]	

From Table 2, it is seen that $\sum_{l=1}^{3}[a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}] \neq \sum_{m=1}^{4}[b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}]$, hence the problem is unbalanced. We find from Section 5 that neither Subcase 2a nor Subcase 2b of Step 1 hold to make it balanced. So as per Subcase 2c, we introduce a dummy source with capacity $[a_4^{LL}, a_4^{LU}][a_4^{LU}, a_4^{UU}] = [0,4][0,6]$ and dummy destinations with demand $[b_5^{LL}, b_5^{LU}][b_5^{LU}, b_5^{UU}] = [4,4][4,4]$ so that the total availabilities and total demand becomes equal, i.e., $\sum_{l=1}^{4}[a_l^{LL}, a_l^{UL}][a_l^{LU}, a_l^{UU}] = \sum_{m=1}^{5}[b_m^{LL}, b_m^{UL}][b_m^{LU}, b_m^{UU}] = [32,42][28,50]$. The transportation cost from dummy source to dummy destination is considered as zero rough interval, i.e., $[c_{4m}^{LL}, c_{4m}^{UL}][c_{4m}^{LU}, c_{4m}^{UU}] = [0,0][0,0] \quad \forall\, m = 1,2,3,4,5$ and $[c_{l5}^{LL}, c_{l5}^{UL}][c_{l5}^{LU}, c_{l5}^{UU}] = [0,0][0,0] \quad \forall\, l = 1,2,3,4$. After this procedure the unbalanced FRITP presented in Table 2 is converted into a balanced transportation problem presented in Table 3.

Table 3. Balanced fully rough interval transportation problem (FRITP).

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	[36, 37][35, 38]	[40, 42][38, 44]	[38, 40][36, 42]	[37, 39][35, 40]	[0, 0][0, 0]	[7, 8][6, 10]
S_2	[41, 42][40, 44]	[40, 42][39, 45]	[42, 45][40, 46]	[30, 32][28, 33]	[0, 0][0, 0]	[9, 10][7, 12]
S_3	[30, 32][29, 33]	[28, 33][26, 35]	[35, 37][34, 39]	[42, 43][40, 44]	[0, 0][0, 0]	[16, 20][15, 22]
S_4	[0, 0][0, 0]	[0, 0][0, 0]	[0, 0][0, 0]	[0, 0][0, 0]	[0, 0][0, 0]	[0, 4][0, 6]
Demand	[4, 5][3, 7]	[6, 10][5, 13]	[10, 12][9, 14]	[8, 11][7, 12]	[4, 4][4, 4]	[32, 42][28, 50]

The model of obtained balanced problem is formulated as:

$$
\begin{aligned}
(P)\quad \text{Minimize } \widetilde{Z}^{RI} = {} & [36,37][35,38] \otimes [x_{11}^{LL}, x_{11}^{UL}][x_{11}^{LU}, x_{11}^{UU}] \oplus [40,42][38,44] \otimes [x_{12}^{LL}, x_{12}^{UL}][x_{12}^{LU}, x_{12}^{UU}] \\
& \oplus [38,40][36,42] \otimes [x_{13}^{LL}, x_{13}^{UL}][x_{13}^{LU}, x_{13}^{UU}] \oplus [37,39][35,40] \otimes [x_{14}^{LL}, x_{14}^{UL}][x_{14}^{LU}, x_{14}^{UU}] \\
& \oplus [0,0][0,0] \otimes [x_{15}^{LL}, x_{15}^{UL}][x_{15}^{LU}, x_{15}^{UU}] \oplus [41,42][40,44] \otimes [x_{21}^{LL}, x_{21}^{UL}][x_{21}^{LU}, x_{21}^{UU}] \\
& \oplus [40,42][39,45] \otimes [x_{22}^{LL}, x_{22}^{UL}][x_{22}^{LU}, x_{22}^{UU}] \oplus [42,45][40,46] \otimes [x_{23}^{LL}, x_{23}^{UL}][x_{23}^{LU}, x_{23}^{UU}] \\
& \oplus [30,32][28,33] \otimes [x_{24}^{LL}, x_{24}^{UL}][x_{24}^{LU}, x_{24}^{UU}] \oplus [0,0][0,0] \otimes [x_{25}^{LL}, x_{25}^{UL}][x_{25}^{LU}, x_{25}^{UU}] \\
& \oplus [30,32][29,33] \otimes [x_{31}^{LL}, x_{31}^{UL}][x_{31}^{LU}, x_{31}^{UU}] \oplus [28,33][26,35] \otimes [x_{32}^{LL}, x_{32}^{UL}][x_{32}^{LU}, x_{32}^{UU}] \\
& \oplus [35,37][34,39] \otimes [x_{33}^{LL}, x_{33}^{UL}][x_{33}^{LU}, x_{33}^{UU}] \oplus [42,43][40,44] \otimes [x_{34}^{LL}, x_{34}^{UL}][x_{34}^{LU}, x_{34}^{UU}] \\
& \oplus [0,0][0,0] \otimes [x_{35}^{LL}, x_{35}^{UL}][x_{35}^{LU}, x_{35}^{UU}] \oplus [0,0][0,0] \otimes [x_{41}^{LL}, x_{41}^{UL}][x_{41}^{LU}, x_{41}^{UU}] \\
& \oplus [0,0][0,0] \otimes [x_{42}^{LL}, x_{42}^{UL}][x_{42}^{LU}, x_{42}^{UU}] \oplus [0,0][0,0] \otimes [x_{43}^{LL}, x_{43}^{UL}][x_{43}^{LU}, x_{43}^{UU}] \\
& \oplus [0,0][0,0] \otimes [x_{44}^{LL}, x_{44}^{UL}][x_{44}^{LU}, x_{44}^{UU}] \oplus [0,0][0,0] \otimes [x_{45}^{LL}, x_{45}^{UL}][x_{45}^{LU}, x_{45}^{UU}]
\end{aligned}
$$

subject to

$$
\begin{aligned}
&[x_{11}^{LL}, x_{11}^{UL}][x_{11}^{LU}, x_{11}^{UU}] \oplus [x_{12}^{LL}, x_{12}^{UL}][x_{12}^{LU}, x_{12}^{UU}] \oplus [x_{13}^{LL}, x_{13}^{UL}][x_{13}^{LU}, x_{13}^{UU}] \oplus [x_{14}^{LL}, x_{14}^{UL}][x_{14}^{LU}, x_{14}^{UU}]\\
&\oplus[x_{15}^{LL}, x_{15}^{UL}][x_{15}^{LU}, x_{15}^{UU}] = [7,8][6,10],\\
&[x_{21}^{LL}, x_{21}^{UL}][x_{21}^{LU}, x_{21}^{UU}] \oplus [x_{22}^{LL}, x_{22}^{UL}][x_{22}^{LU}, x_{22}^{UU}] \oplus [x_{23}^{LL}, x_{23}^{UL}][x_{23}^{LU}, x_{23}^{UU}] \oplus [x_{24}^{LL}, x_{24}^{UL}][x_{24}^{LU}, x_{24}^{UU}]\\
&\oplus[x_{25}^{LL}, x_{25}^{UL}][x_{25}^{LU}, x_{25}^{UU}] = [9,10][7,12],\\
&[x_{31}^{LL}, x_{31}^{UL}][x_{31}^{LU}, x_{31}^{UU}] \oplus [x_{32}^{LL}, x_{32}^{UL}][x_{32}^{LU}, x_{32}^{UU}] \oplus [x_{33}^{LL}, x_{33}^{UL}][x_{33}^{LU}, x_{33}^{UU}] \oplus [x_{34}^{LL}, x_{34}^{UL}][x_{34}^{LU}, x_{34}^{UU}]\\
&\oplus[x_{35}^{LL}, x_{35}^{UL}][x_{35}^{LU}, x_{35}^{UU}] = [16,20][15,22],\\
&[x_{41}^{LL}, x_{41}^{UL}][x_{41}^{LU}, x_{41}^{UU}] \oplus [x_{42}^{LL}, x_{42}^{UL}][x_{42}^{LU}, x_{42}^{UU}] \oplus [x_{43}^{LL}, x_{43}^{UL}][x_{43}^{LU}, x_{43}^{UU}] \oplus [x_{44}^{LL}, x_{44}^{UL}][x_{44}^{LU}, x_{44}^{UU}]\\
&\oplus[x_{45}^{LL}, x_{45}^{UL}][x_{45}^{LU}, x_{45}^{UU}] = [0,4][0,6],\\
&[x_{11}^{LL}, x_{11}^{UL}][x_{11}^{LU}, x_{11}^{UU}] \oplus [x_{21}^{LL}, x_{21}^{UL}][x_{21}^{LU}, x_{21}^{UU}] \oplus [x_{31}^{LL}, x_{31}^{UL}][x_{31}^{LU}, x_{31}^{UU}]\\
&\oplus[x_{41}^{LL}, x_{41}^{UL}][x_{41}^{LU}, x_{41}^{UU}] = [4,5][3,7],\\
&[x_{12}^{LL}, x_{12}^{UL}][x_{12}^{LU}, x_{12}^{UU}] \oplus [x_{22}^{LL}, x_{22}^{UL}][x_{22}^{LU}, x_{22}^{UU}] \oplus [x_{32}^{LL}, x_{32}^{UL}][x_{32}^{LU}, x_{32}^{UU}]\\
&\oplus[x_{42}^{LL}, x_{42}^{UL}][x_{42}^{LU}, x_{42}^{UU}] = [6,10][5,13],\\
&[x_{13}^{LL}, x_{13}^{UL}][x_{13}^{LU}, x_{13}^{UU}] \oplus [x_{23}^{LL}, x_{23}^{UL}][x_{23}^{LU}, x_{23}^{UU}] \oplus [x_{33}^{LL}, x_{33}^{UL}][x_{33}^{LU}, x_{33}^{UU}]\\
&\oplus[x_{43}^{LL}, x_{43}^{UL}][x_{43}^{LU}, x_{43}^{UU}] = [10,12][9,14],\\
&[x_{14}^{LL}, x_{14}^{UL}][x_{14}^{LU}, x_{14}^{UU}] \oplus [x_{24}^{LL}, x_{24}^{UL}][x_{24}^{LU}, x_{24}^{UU}] \oplus [x_{34}^{LL}, x_{34}^{UL}][x_{34}^{LU}, x_{34}^{UU}]\\
&\oplus[x_{44}^{LL}, x_{44}^{UL}][x_{44}^{LU}, x_{44}^{UU}] = [8,11][7,12],\\
&[x_{15}^{LL}, x_{15}^{UL}][x_{15}^{LU}, x_{15}^{UU}] \oplus [x_{25}^{LL}, x_{25}^{UL}][x_{25}^{LU}, x_{25}^{UU}] \oplus [x_{35}^{LL}, x_{35}^{UL}][x_{35}^{LU}, x_{35}^{UU}]\\
&\oplus[x_{45}^{LL}, x_{45}^{UL}][x_{45}^{LU}, x_{45}^{UU}] = [4,4][4,4],\\
&[x_{11}^{LL}, x_{11}^{UL}][x_{11}^{LU}, x_{11}^{UU}] \geq 0, [x_{12}^{LL}, x_{12}^{UL}][x_{12}^{LU}, x_{12}^{UU}] \geq 0, [x_{13}^{LL}, x_{13}^{UL}][x_{13}^{LU}, x_{13}^{UU}] \geq 0,\\
&[x_{14}^{LL}, x_{14}^{UL}][x_{14}^{LU}, x_{14}^{UU}] \geq 0, [x_{15}^{LL}, x_{15}^{UL}][x_{15}^{LU}, x_{15}^{UU}] \geq 0,\\
&[x_{21}^{LL}, x_{21}^{UL}][x_{21}^{LU}, x_{21}^{UU}] \geq 0, [x_{22}^{LL}, x_{22}^{UL}][x_{22}^{LU}, x_{22}^{UU}] \geq 0, [x_{23}^{LL}, x_{23}^{UL}][x_{23}^{LU}, x_{23}^{UU}] \geq 0,\\
&[x_{24}^{LL}, x_{24}^{UL}][x_{24}^{LU}, x_{24}^{UU}] \geq 0, [x_{25}^{LL}, x_{25}^{UL}][x_{25}^{LU}, x_{25}^{UU}] \geq 0,\\
&[x_{31}^{LL}, x_{31}^{UL}][x_{31}^{LU}, x_{31}^{UU}] \geq 0, [x_{32}^{LL}, x_{32}^{UL}][x_{32}^{LU}, x_{32}^{UU}] \geq 0, [x_{33}^{LL}, x_{33}^{UL}][x_{33}^{LU}, x_{33}^{UU}] \geq 0,\\
&[x_{34}^{LL}, x_{34}^{UL}][x_{34}^{LU}, x_{34}^{UU}] \geq 0, [x_{35}^{LL}, x_{35}^{UL}][x_{35}^{LU}, x_{35}^{UU}] \geq 0,
\end{aligned}
$$

$$
\begin{aligned}
&[x_{41}^{LL}, x_{41}^{UL}][x_{41}^{LU}, x_{41}^{UU}] \geq 0, [x_{42}^{LL}, x_{42}^{UL}][x_{42}^{LU}, x_{42}^{UU}] \geq 0, [x_{43}^{LL}, x_{43}^{UL}][x_{43}^{LU}, x_{43}^{UU}] \geq 0,\\
&[x_{44}^{LL}, x_{44}^{UL}][x_{44}^{LU}, x_{44}^{UU}] \geq 0, [x_{45}^{LL}, x_{45}^{UL}][x_{45}^{LU}, x_{45}^{UU}] \geq 0
\end{aligned}
$$

Applying Step 2 to 4 of the proposed approach discussed in Section 5, the above balanced problem (P) transformed into an equivalent crisp problem $(P^{'})$.

$$
\begin{aligned}
(P^{'})\quad \text{Minimize}\quad E(\widetilde{Z}^{RI}) = \frac{1}{4}\big[&36x_{11}^{LL} + 37x_{11}^{UL} + 35x_{11}^{LU} + 38x_{11}^{UU} + 40x_{12}^{LL} + 42x_{12}^{UL} + 38x_{12}^{LU} + 44x_{12}^{UU}\\
&+38x_{13}^{LL} + 40x_{13}^{UL} + 36x_{13}^{LU} + 42x_{13}^{UU} + 37x_{14}^{LL} + 39x_{14}^{UL} + 35x_{14}^{LU} + 40x_{14}^{UU} + 0x_{15}^{LL} + 0x_{15}^{UL} + 0x_{15}^{LU}\\
&+0x_{15}^{UU} + 41x_{21}^{LL} + 42x_{21}^{UL} + 40x_{21}^{LU} + 44x_{21}^{UU} + 40x_{22}^{LL} + 42x_{22}^{UL} + 39x_{22}^{LU} + 45x_{22}^{UU} + 42x_{23}^{LL} + 45x_{23}^{UL}\\
&+40x_{23}^{LU} + 46x_{23}^{UU} + 30x_{24}^{LL} + 32x_{24}^{UL} + 28x_{24}^{LU} + 33x_{24}^{UU} + 0x_{25}^{LL} + 0x_{25}^{UL} + 0x_{25}^{LU} + 0x_{25}^{UU} + 30x_{31}^{LL}\\
&+32x_{31}^{UL} + 29x_{31}^{LU} + 33x_{31}^{UU} + 28x_{32}^{LL} + 33x_{32}^{UL} + 26x_{32}^{LU} + 35x_{32}^{UU} + 35x_{33}^{LL} + 37x_{33}^{UL} + 34x_{33}^{LU} + 39x_{33}^{UU}\\
&+42x_{34}^{LL} + 43x_{34}^{UL} + 40x_{34}^{LU} + 44x_{34}^{UU} + 0x_{35}^{LL} + 0x_{35}^{UL} + 0x_{35}^{LU} + 0x_{35}^{UU} + 0x_{41}^{LL} + 0x_{41}^{UL} + 0x_{41}^{LU} + 0x_{41}^{UU}\\
&+0x_{42}^{LL} + 0x_{42}^{UL} + 0x_{42}^{LU} + 0x_{42}^{UU} + 0x_{43}^{LL} + 0x_{43}^{UL} + 0x_{43}^{LU} + 0x_{43}^{UU} + 0x_{44}^{LL} + 0x_{44}^{UL} + 0x_{44}^{LU} + 0x_{44}^{UU}\\
&+0x_{45}^{LL} + 0x_{45}^{UL} + 0x_{45}^{LU} + 0x_{45}^{UU}\big]
\end{aligned}
$$

subject to

$$
\begin{aligned}
&x_{11}^{LL}+x_{12}^{LL}+x_{13}^{LL}+x_{14}^{LL}+x_{15}^{LL}=7,\ x_{11}^{UL}+x_{12}^{UL}+x_{13}^{UL}+x_{14}^{UL}+x_{15}^{UL}=8,\\
&x_{11}^{LU}+x_{12}^{LU}+x_{13}^{LU}+x_{14}^{LU}+x_{15}^{LU}=6,\ x_{11}^{UU}+x_{12}^{UU}+x_{13}^{UU}+x_{14}^{UU}+x_{15}^{UU}=10,\\
&x_{21}^{LL}+x_{22}^{LL}+x_{23}^{LL}+x_{24}^{LL}+x_{25}^{LL}=9,\ x_{21}^{UL}+x_{22}^{UL}+x_{23}^{UL}+x_{24}^{UL}+x_{25}^{UL}=10,\\
&x_{21}^{LU}+x_{22}^{LU}+x_{23}^{LU}+x_{24}^{LU}+x_{25}^{LU}=7,\ x_{21}^{UU}+x_{22}^{UU}+x_{23}^{UU}+x_{24}^{UU}+x_{25}^{UU}=12,\\
&x_{31}^{LL}+x_{32}^{LL}+x_{33}^{LL}+x_{34}^{LL}+x_{35}^{LL}=16, x_{31}^{UL}+x_{32}^{UL}+x_{33}^{UL}+x_{34}^{UL}+x_{35}^{UL}=20,\\
&x_{31}^{LU}+x_{32}^{LU}+x_{33}^{LU}+x_{34}^{LU}+x_{35}^{LU}=15, x_{31}^{UU}+x_{32}^{UU}+x_{33}^{UU}+x_{34}^{UU}+x_{35}^{UU}=22,\\
&x_{41}^{LL}+x_{42}^{LL}+x_{43}^{LL}+x_{44}^{LL}+x_{45}^{LL}=0, x_{41}^{UL}+x_{42}^{UL}+x_{43}^{UL}+x_{44}^{UL}+x_{45}^{UL}=4,\\
&x_{41}^{LU}+x_{42}^{LU}+x_{43}^{LU}+x_{44}^{LU}+x_{45}^{LU}=0,\ x_{41}^{UU}+x_{42}^{UU}+x_{43}^{UU}+x_{44}^{UU}+x_{45}^{UU}=6,\\
&x_{11}^{LL}+x_{21}^{LL}+x_{31}^{LL}+x_{41}^{LL}=4, x_{11}^{UL}+x_{21}^{UL}+x_{31}^{UL}+x_{41}^{UL}=5, x_{11}^{LU}+x_{21}^{LU}+x_{31}^{LU}+x_{41}^{LU}=3,\\
&x_{11}^{UU}+x_{21}^{UU}+x_{31}^{UU}+x_{41}^{UU}=7,\\
&x_{12}^{LL}+x_{22}^{LL}+x_{32}^{LL}+x_{42}^{LL}=6, x_{12}^{UL}+x_{22}^{UL}+x_{32}^{UL}+x_{42}^{UL}=10, x_{12}^{LU}+x_{22}^{LU}+x_{32}^{LU}+x_{42}^{LU}=5,\\
&x_{12}^{UU}+x_{22}^{UU}+x_{32}^{UU}+x_{42}^{UU}=13,\\
&x_{13}^{LL}+x_{23}^{LL}+x_{33}^{LL}+x_{43}^{LL}=10, x_{13}^{UL}+x_{23}^{UL}+x_{33}^{UL}+x_{43}^{UL}=12, x_{13}^{LU}+x_{23}^{LU}+x_{33}^{LU}+x_{43}^{LU}=9,\\
&x_{13}^{UU}+x_{23}^{UU}+x_{33}^{UU}+x_{43}^{UU}=14,\\
&x_{14}^{LL}+x_{24}^{LL}+x_{34}^{LL}+x_{44}^{LL}=8, x_{14}^{UL}+x_{24}^{UL}+x_{34}^{UL}+x_{44}^{UL}=11, x_{14}^{LU}+x_{24}^{LU}+x_{34}^{LU}+x_{44}^{LU}=7,\\
&x_{14}^{UU}+x_{24}^{UU}+x_{34}^{UU}+x_{44}^{UU}=12,\\
&x_{15}^{LL}+x_{25}^{LL}+x_{35}^{LL}+x_{45}^{LL}=4, x_{15}^{UL}+x_{25}^{UL}+x_{35}^{UL}+x_{45}^{UL}=4, x_{15}^{LU}+x_{25}^{LU}+x_{35}^{LU}+x_{45}^{LU}=4,\\
&x_{15}^{UU}+x_{25}^{UU}+x_{35}^{UU}+x_{45}^{UU}=4\\
&x_{11}^{LU}\geq 0, x_{11}^{LL}-x_{11}^{LU}\geq 0, x_{11}^{UL}-x_{11}^{LL}\geq 0, x_{11}^{UU}-x_{11}^{UL}\geq 0, x_{12}^{LU}\geq 0, x_{12}^{LL}-x_{12}^{LU}\geq 0, x_{12}^{UL}-x_{12}^{LL}\geq 0,\\
&x_{12}^{UU}-x_{12}^{UL}\geq 0, x_{13}^{LU}\geq 0, x_{13}^{LL}-x_{13}^{LU}\geq 0, x_{13}^{UL}-x_{13}^{LL}\geq 0, x_{13}^{UU}-x_{13}^{UL}\geq 0, x_{14}^{LU}\geq 0, x_{14}^{LL}-x_{14}^{LU}\geq 0,\\
&x_{14}^{UL}-x_{14}^{LL}\geq 0, x_{14}^{UU}-x_{14}^{UL}\geq 0, x_{15}^{LU}\geq 0, x_{15}^{LL}-x_{15}^{LU}\geq 0, x_{15}^{UL}-x_{15}^{LL}\geq 0, x_{15}^{UU}-x_{15}^{UL}\geq 0,\\
&x_{21}^{LU}\geq 0, x_{21}^{LL}-x_{21}^{LU}\geq 0, x_{21}^{UL}-x_{21}^{LL}\geq 0, x_{21}^{UU}-x_{21}^{UL}\geq 0, x_{22}^{LU}\geq 0, x_{22}^{LL}-x_{22}^{LU}\geq 0, x_{22}^{UL}-x_{22}^{LL}\geq 0,\\
&x_{22}^{UU}-x_{22}^{UL}\geq 0, x_{23}^{LU}\geq 0, x_{23}^{LL}-x_{23}^{LU}\geq 0, x_{23}^{UL}-x_{23}^{LL}\geq 0, x_{23}^{UU}-x_{23}^{UL}\geq 0, x_{24}^{LU}\geq 0, x_{24}^{LL}-x_{24}^{LU}\geq 0,\\
&x_{24}^{UL}-x_{24}^{LL}\geq 0, x_{24}^{UU}-x_{24}^{UL}\geq 0 x_{25}^{LU}\geq 0, x_{25}^{LL}-x_{25}^{LU}\geq 0, x_{25}^{UL}-x_{25}^{LL}\geq 0, x_{25}^{UU}-x_{25}^{UL}\geq 0,\\
&x_{31}^{LU}\geq 0, x_{31}^{LL}-x_{31}^{LU}\geq 0, x_{31}^{UL}-x_{31}^{LL}\geq 0, x_{31}^{UU}-x_{31}^{UL}\geq 0, x_{32}^{LU}\geq 0, x_{32}^{LL}-x_{32}^{LU}\geq 0, x_{32}^{UL}-x_{32}^{LL}\geq 0,\\
&x_{32}^{UU}-x_{32}^{UL}\geq 0, x_{33}^{LU}\geq 0, x_{33}^{LL}-x_{33}^{LU}\geq 0, x_{33}^{UL}-x_{33}^{LL}\geq 0, x_{33}^{UU}-x_{33}^{UL}\geq 0, x_{34}^{LU}\geq 0, x_{34}^{LL}-x_{34}^{LU}\geq 0,\\
&x_{34}^{UL}-x_{34}^{LL}\geq 0, x_{34}^{UU}-x_{34}^{UL}\geq 0, x_{35}^{LU}\geq 0, x_{35}^{LL}-x_{35}^{LU}\geq 0, x_{35}^{UL}-x_{35}^{LL}\geq 0, x_{35}^{UU}-x_{35}^{UL}\geq 0,\\
&x_{41}^{LU}\geq 0, x_{41}^{LL}-x_{41}^{LU}\geq 0, x_{41}^{UL}-x_{41}^{LL}\geq 0, x_{41}^{UU}-x_{41}^{UL}\geq 0, x_{42}^{LU}\geq 0, x_{42}^{LL}-x_{42}^{LU}\geq 0, x_{42}^{UL}-x_{42}^{LL}\geq 0,\\
&x_{42}^{UU}-x_{42}^{UL}\geq 0, x_{43}^{LU}\geq 0, x_{43}^{LL}-x_{43}^{LU}\geq 0, x_{43}^{UL}-x_{43}^{LL}\geq 0, x_{43}^{UU}-x_{43}^{UL}\geq 0, x_{44}^{LU}\geq 0, x_{44}^{LL}-x_{44}^{LU}\geq 0,\\
&x_{44}^{UL}-x_{44}^{LL}\geq 0, x_{44}^{UU}-x_{44}^{UL}\geq 0, x_{45}^{LU}\geq 0, x_{45}^{LL}-x_{45}^{LU}\geq 0, x_{45}^{UL}-x_{45}^{LL}\geq 0, x_{45}^{UU}-x_{45}^{UL}\geq 0.
\end{aligned}
$$

7. Results and discussion

In this section, the optimal solution ($\widetilde{x}^{RI}$) of the crisp linear programming problem (P') is obtained by using LINGO 18.0 optimization solver and is given as

$$
\widetilde{x}^{RI}=\begin{cases}
\widetilde{x}_{11}^{RI}=[1,2][0,4],\ \widetilde{x}_{12}^{RI}=[0,0][0,0],\ \widetilde{x}_{13}^{RI}=[2,2][2,2],\ \widetilde{x}_{14}^{RI}=[0,0][0,0],\ \widetilde{x}_{15}^{RI}=[4,4][4,4],\\
\widetilde{x}_{21}^{RI}=[0,0][0,0],\ \widetilde{x}_{22}^{RI}=[0,0][0,1],\ \widetilde{x}_{23}^{RI}=[1,1][0,1],\ \widetilde{x}_{24}^{RI}=[8,9][7,10],\ \widetilde{x}_{25}^{RI}=[0,0][0,0],\\
\widetilde{x}_{31}^{RI}=[3,3][3,3],\ \widetilde{x}_{32}^{RI}=[6,10][5,12],\ \widetilde{x}_{33}^{RI}=[7,7][7,7],\ \widetilde{x}_{34}^{RI}=[0,0][0,0],\ \widetilde{x}_{35}^{RI}=[0,0][0,0],\\
\widetilde{x}_{41}^{RI}=[0,0][0,0],\ \ \widetilde{x}_{42}^{RI}=[0,0][0,0],\ \ \widetilde{x}_{43}^{RI}=[0,2][0,4],\ \ \widetilde{x}_{44}^{RI}=[0,2][0,2],\ \ \widetilde{x}_{45}^{RI}=[0,0][0,0].
\end{cases}
$$

The graphical depiction of this solution set is given in Figure 2.

The optimal transportation cost is found to be $\widetilde{Z}^{RI}=[897,1172][723,1449]$, which takes into consideration the opinion of all involved experts (intersection) and respects their knowledge (union) by the surely and possibly optimal ranges, respectively. In the obtained optimal transportation cost, the range [897, 1172] is called the surely optimal range and [723, 1449] is called the possibly optimal range. From Figure 2, the solution set corresponding to the lower bound of surely optimal range 897 is obtained

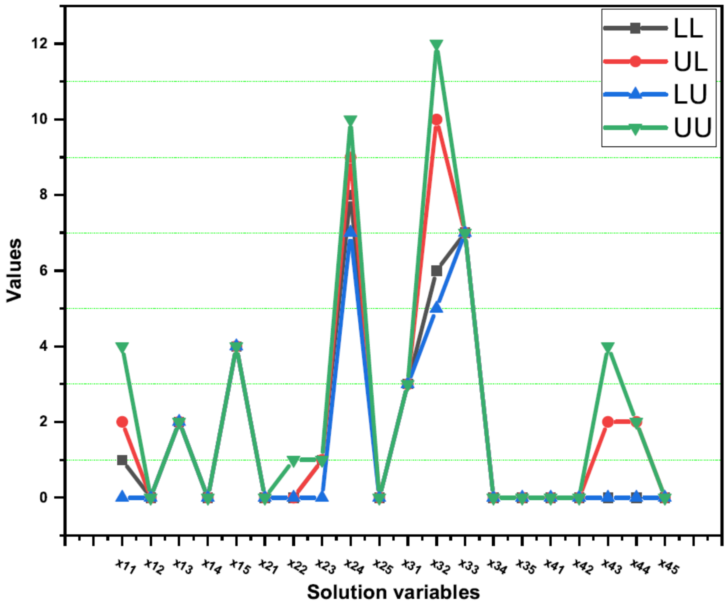

Figure 2. Optimal transported amount.

$x_{ij}^{LL} = \{1, 0, 2, 0, 4; 0, 0, 1, 8, 0; 3, 6, 7, 0, 0; 0, 0, 0, 0, 0\}$ and for the upper bound 1172 is obtained as $x_{ij}^{UL} = \{2, 0, 2, 0, 4; 0, 0, 1, 9, 0; 3, 10, 7, 0, 0; 0, 0, 2, 2, 0\}$. Similarly, for the possibly optimal range the solution set corresponding to the lower bound 723 is obtained as $x_{ij}^{LU} = \{0, 0, 2, 0, 4; 0, 0, 0, 7, 0; 3, 5, 7, 0, 0; 0, 0, 0, 0, 0\}$ and corresponding to the upper bound 1449 is obtained as $x_{ij}^{UU} = \{4, 0, 2, 0, 4; 0, 1, 1, 10, 0; 3, 12, 7, 0, 0; 0, 0, 4, 2, 0\}$. This visualization of the solution (Figure 2) can assist decision maker in determining the appropriate amount of transportation for a given scenario.

Also, to prove the efficiency of proposed methodology, the numerical example considered in [3] is solved and the obtained solutions are presented in Table 4. Table 4 shows that our proposed method is suitable for solving the problem considered in [3] and produces the same solution as [3]. However, the solution approach proposed by [3] is not applicable for the FRITP discussed in Section 6 because this problem is an unbalanced problem.

Table 4. Comparison of solution with existing method and proposed method.

Example	Existing Method [3]	Proposed Method
Akilbasha et al. [3]	[125, 218][60, 325]	[125, 218][60, 325]
Numerical example in Section 6	Not applicable	[897, 1172][723, 1449]

8. Conclusions and future research scope

Rough set theory is practically more applicable to handle the uncertainty because no extra data related information is required while using it. This advantage of rough set theory has inspired us to formulate the model of an unbalanced transportation problem with rough interval parameters called the unbalanced FRITP. To solve the proposed model, a new methodology is developed that demonstrates its robustness in dealing with imprecise mathematical modeling. It has also been empirically proven to be user-friendly for the decision maker dealing with cost, availability, and demand as rough interval parameters. However, the proposed approach cannot be used to solve an unbalanced non-linear or fractional FRITP. The suggested solution approach is also not applicable to the transportation problem with multiple objectives. Therefore, in future, the intrigued researchers may extend the proposed technique to solve the unbalanced non-linear or fractional FRITP. Also, solving an unbalanced fully rough multi-objective transportation problem is an interesting area for future research. It would also be highly beneficial to solve the unbalanced NP-hard FRITP using nature-inspired algorithms [10, 11].

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