



# A COMPARATIVE STUDY ON THE STRESS INTENSITY FACTOR OF CARBON EPOXY COMPOSITE PLATES HAVING CENTRAL HOLES

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Geliş Tarihi : 15.01.2000

## ABSTRACT

In this paper, a comparative study regarding the determination of stress intensity factors (SIF) for non-standard thin composite plates is presented. Carbon-Epoxy composite plates are considered for the study. Unnotched specimens and specimens containing either 4, 6, or 8 mm dia. holes are prepared and then tested. Based on the experimental data thus obtained, two analytical approaches and a finite element fracture analysis tool called FRANC have been used to compare the critical SIF values for this composite material.

**Key Words** : Stress intensity factor, Fracture, Crack, Composite, Carbon epoxy

## DELİKLİ KARBON-EPOKSİ PLAKALARIN GERİLME ŞİDDETİ FAKTÖRLERİ ÜZERİNDE KARŞILAŞTIRMALI BİR ÇALIŞMA

### ÖZET

Makalede standart olmayan ince kompozit levhaların gerilme şiddeti faktörlerinin (GŞF) hesaplanmasıyla ilgili karşılaştırmalı bir çalışma sunulmaktadır. Çalışmada Karbon-Epoksi kompozit levhalar kullanılmıştır. Çentiksiz ve üzerinde 4, 6 veya 8 mm çapında delik bulunan deney numuneleri hazırlanıp test edilmiştir. Elde edilen deney sonuçlarına dayanarak, iki analitik yaklaşım ve FRANC-sonlu elemanlar programı kullanılarak kompozitin kritik GŞF'leri hesaplanıp karşılaştırmalar yapılmıştır.

**Anahtar Kelimeler** : Gerilme şiddeti faktörü, Kırılma, Çatlak, Kompozit, Karbon epoksi

### 1. INTRODUCTION

The increasing use of composite materials in many engineering applications has motivated researchers to understand their behaviour. One of the areas of interest has been the study of the resistance of composites to crack formation and propagation. At this stage the determination of stress intensity factor becomes important. Studies on this have yielded various approaches for the calculation of SIF. These include analytical and numerical methods. Purely analytical methods are only applicable for very well defined crack geometries and semi-fictitious boundary conditions. Despite this, various criteria

have been developed the most notable of which are given by Waddoups et al., (1971) and Whitney and Nuismer, (1974).

Waddoups and his group have applied the classical fracture mechanics to predict the static strengths of flawed laminated composites. In their model, intense energy regions characterised by a constant through crack length 'a' and extending symmetrically from each side of the holes on specimens was assumed to define the SIF. Although the data showed reasonable correlation to their model, the characteristic length 'a' is seen to be not constant but varying. In the case of the application of the method to a different

geometry, i.e., slits on each side of the hole in the specimens, the assumption that the diameter of the hole be small compared to the slit size was not well justified.

Whitney and Nuismer have, on the other hand, proposed the Point Stress and the Average Stress criteria. In the former, failure was assumed to occur where the unnotched tensile strength of the material coincided with the notched specimen's stress distribution. The latter criterion assumes failure when the average value of the stress distribution over a fixed distance near a notch reaches the unnotched tensile strength. These criteria were applied to graphite/epoxy and glass/epoxy laminates and reasonably good agreement regarding failure characterised by critical SIF values were obtained.

The finite element technique has successfully been used in many engineering applications. However the stress singularity at the crack tip has presented an obstacle for its successful application to fracture mechanics up until the development of quarter point elements (QPE's) as presented by Murti et al., (1984). However the QPE may not give extremely accurate SIF's as rectangular type QPE elements are less versatile in difficult geometry and the determinant of the Jacobian is not constant when cracks propagate. Triangular type QPE elements present the same problem when one of the sides is curved. In addition to these, the accuracy of the finite element method depends on the size, geometrical shapes and the distribution of the QPE's around the crack tip. Despite these disadvantages and the programming effort to be spent, the FEM with straight-sided triangular QPE's gives satisfactory results. In this study, FRANC2D/L, a fracture analysis code, (Wawrzynak and Ingraffea, 1987) has been used as an analysis tool to present the capabilities of finite element approaches in comparison with the previously mentioned analytical methods.

## 2. EXPERIMENTS

A (0°)<sub>3</sub> Carbon-Epoxy composite laminate with a thickness of 0.89 mm, a fibre volume fraction of 32 %, and a weight of 400 gr/m<sup>2</sup> is used in the experiments. The tensile properties of this laminate are given in Table 1.

Table 1. The Tensile Properties of the (0°)<sub>3</sub> Carbon-Epoxy Composite Specimen

E <sub>1</sub> (GPa)	E <sub>2</sub> (GPa)	G <sub>12</sub> (GPa)	ν <sub>12</sub>	t (mm)	V <sub>f</sub> (%)
29	27	2.03	0.25	0.89	32

Out of a total of nine, three groups of notched tensile specimens with 25-mm width and 110-mm gauge

length were prepared. The first group had 4-mm dia. holes drilled at the centre and the remaining two groups had 6 and 8-mm holes located at their centres, respectively. Each group was then tested according to ASTM D1602-60 standard and their respective notched tensile stresses were determined. Examples of the tested specimens from each group are shown in Figure 1. The nominal stress values, which are given in Table 2, were obtained by averaging. All of the tests were performed on a 100 kN Instron 8516+ universal testing machine at a cross-head speed of 1mm/min.

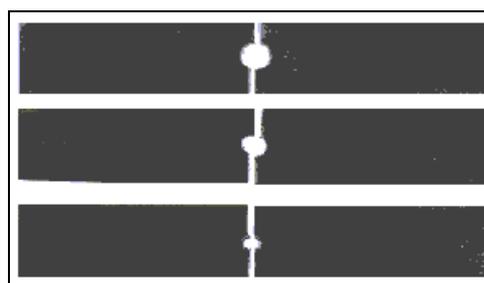


Figure 1. Scanner images of some of the tested specimens

## 3. ANALYTICAL APPROACHES

The linear elastic stress distribution,  $\sigma_y$ , along the net section plane (x axis) for an infinite orthotropic plate containing a circular hole of diameter 2r, is given as (Glyn et al., 1997);

$$\sigma_y = \left\{ 2 + \left(\frac{r}{x}\right)^2 + 3\left(\frac{r}{x}\right)^4 - (K_T^\infty - 3) \left[ 5\left(\frac{r}{x}\right)^6 - 7\left(\frac{r}{x}\right)^8 \right] \right\} \frac{\sigma_N^\infty}{2} \quad (1)$$

In this equation, the stress concentration factor  $K_T^\infty$  can be expressed as;

$$K_T^\infty = 1 + \sqrt{\frac{2}{A_{11}} \left( \sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2(A_{66})} \right)} \quad (2)$$

$A_{ij}$  are the elements of the orthotropic in plane stiffness matrix of the plate.  $\sigma_N^\infty$  is the remote applied tensile stress for a plate of infinite width. From these, the  $\sigma_y$  stress distribution ahead of each hole was obtained as shown in Figure 2. The average Stress criterion by Whitney and Nuismer was then used to find critical crack lengths ( $a_c$ ) for the specimens (see Table 2). This criterion is based on the assumption that failure occurs when the average value of  $\sigma_y$  over some fixed distance,  $a_c$ , from the edge of the hole reaches the unnotched tensile strength of the material,  $\sigma_0$  (see Figure 2). This criterion is given by,

$$\frac{1}{a_c} \int_r^{r+a_c} \sigma_y(x,0) dx = \sigma_0 \quad (3)$$

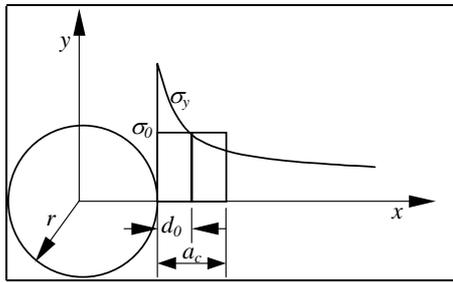


Figure 2. Average stress criterion

By combining Eqs. (1) and (3), we find that the ratio of notched to unnotched strength is,

$$\frac{\sigma_N^\infty}{\sigma_0} = \frac{2(1-\xi)}{2-\xi^2-\xi^4+(K_T^\infty-3)(\xi^6-\xi^8)} \quad (4)$$

where  $\xi = \frac{r}{r+a_c}$  and  $\sigma_N^\infty$  is the notched tensile strength of the infinite width laminate.

The linear elastic fracture mechanics (LEFM) was then used to find the SIF values. An example calculation can be given as follows:

For the specimen with 8 mm dia. hole, using Eq. (4) with  $\sigma_N^\infty/\sigma_0 = 152/405 = 0.375$ ,  $a_c$  was obtained as 0.59 and the corresponding  $f(a_c/r)$  from reference (Fracture Toughness Testing, 1964) was interpolated as 2.57. Using the relation (Usama, 1996);

$$K_I = \sigma_N^\infty \sqrt{\pi a_c} f\left(\frac{a_c}{r}\right) \quad (5)$$

the SIF was found as  $16.80 \text{ MPa}\sqrt{\text{m}}$ . Similar calculations for the 4 mm dia. hole yields  $19.08 \text{ MPa}\sqrt{\text{m}}$  (Figure 3).

From the approach developed by Waddoups, on the other hand, for the 4 mm dia. hole, the following analysis was performed. Waddoups assumes the same critical crack lengths for all specimens.  $a_c$  for

the specimen under consideration is found as:  $f(a/r) = \sigma_0/\sigma_c = 405/191 = 2.12$ , and from reference [6],  $a_c/r = 0.315$  and  $a_c = 0.632 \text{ mm}$ . Using Eq. 3, the SIF is obtained as  $18.042 \text{ MPa}\sqrt{\text{m}}$ . In a similar manner, the remaining SIFs for other specimens from both approaches were calculated and are presented in Table 2.

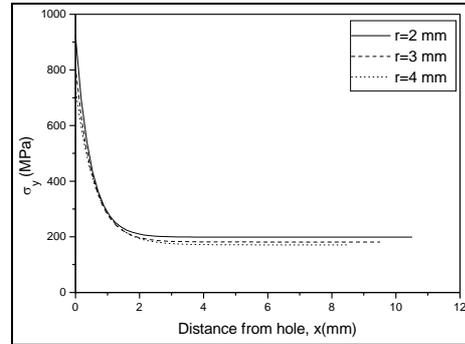


Figure 3. The  $\sigma_y$  stress distribution of each hole

#### 4. NUMERICAL OR FINITE ELEMENT APPROACH

The FE analysis was performed using the material properties for the Carbon Epoxy composite listed in Table 1 and the  $a_c$  values (assumed to be those obtained from Whitney and Nuismer analysis) in Table 2. The mesh used in the analysis consists of 6-node triangular QPEs for crack region and 8-node Quadrilateral elements for the remaining regions. 192 elements and 673 nodes were used. Due to symmetry, only one half of the specimens were modelled. The mesh, constraints and loads are shown in Figure 4. The SIF values are given in Table 2.

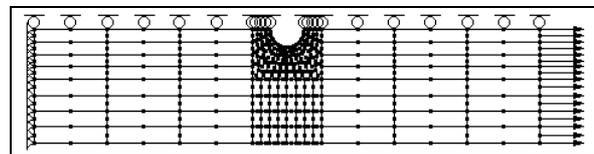


Figure 4. The mesh, constraints and loads of the FE model

Table 2. Experimental, Analytical and Numerical SIF Results

r (mm)	$\sigma_c$ (MPa)	$a_c$ (mm)	$f(a/r)$	SIF, $K_I$ ( $\text{MPa}\sqrt{\text{m}}$ )		
				Waddoups' Approach	Average Stress Criterion	Numerical FRANC
2	191	0.89	1.89		19.08	23.04
3	168	0.78	2.27	18.04	18.87	21.52
4	152	0.59	2.57		16.80	19.12
			Averages	18.04	18.25	21.23

## 5. DISCUSSIONS AND CONCLUSIONS

Analytical and numerical analyses have been performed to determine the stress intensity factor of a Carbon-Epoxy composite plate. As the radius of the hole in the specimens increases, by referring to Table 2, the Average Stress criterion and FE analysis predict decreasing SIF values. The average SIF values are also shown in the Table. The values are close to each other and from a practical point of view they can be considered to be satisfactory for design purposes. Despite the shortcomings of his method as mentioned previously, it is seen that Waddoups' approach should not be overlooked due to its simplicity. The capability to determine the SIF by the Finite Element Method is also demonstrated in this work. Compared to the others, the advantage of the FEM in complicated load and geometries should again be stated.

## 6. ACKNOWLEDGEMENT

We are grateful to Turkish Aerospace Industries (TAI) for technical help and for providing us with the composite material from which the specimens were prepared.

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