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THE EFFECTS OF ASPECT RATIO AND INITIAL IMPERFECTION SHAPE ON THE UNIAXIAL PLATE STRENGTH

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ABSTRACT

This study is concerned with the imperfection and aspect-ratio sensitivity of the uniaxial strength of rectangular steel plates. To investigate the imperfection and aspect ratio sensitivity of the uniaxial strength of rectangular plates, over 90 different plate problems have been modelled and solved by using the user-element in ANSYS 5.0 which is a commercial finite element package. The user-element which is offered by ANSYS 5.0 as an extension of the element library, is used to implement subroutines written for analysing large deflection elastic-perfectly-plastic material behaviour of plates by using finite element method. The results obtained for each plate problem have been presented in figures. The maximum average strength of plates have been obtained for the different aspect ratios, the initial geometric imperfection modes and the complex initial geometric imperfections, achieved by combination of these modes. The levels of initial geometric imperfections are assumed as found in the literature. The effects of complex initial geometric imperfections and aspect ratios on plate strength for various modes have been determined. The combination of initial imperfection modes creates new geometrical imperfections and magnitudes, generally higher than previous values. These factors are shown to effect the plate by reducing its strength. The results obtained can help to design the rectangular plates and plated structures under uniaxial compression.

Key Words : Plate strength, Collapse loads of plate, Plate imperfections

PLAKLARDAKİ BOYUT ORANININ VE BAŞLANGIÇTAKİ DÜZLEMSİZLİKLERİN PLAK MUKAVEMETİNE ETKİSİ

ÖZET

Bu çalışma dikdörtgen çelik plakların boyut oranlarının ve başlangıçtaki düzlemsizliklerinin plak mukavemetine olan duyarlılığı ile ilgilidir. Bu plak boyut oranlarının ve düzlemsizliklerinin plak mukavemetine duyarlılığını araştırmak için ticari bir sonlu elemenler programı olan ANSYS 5.0'deki kullanıcıya ait olacak şekilde sunulan elemanı (user-element) kullanarak 90 dan fazla plak problemlemlerinin modellemesi yapılıp çözüldü. ANSYS 5.0 tarafından sunulan bu kullanıcı eleman plakların büyük deformasyon ve elasto-plastik malzeme davranışlarının analizinde kullanılan sonlu elemanlar metodu kullanılarak Fortran 77 dili ile yazılmış alt programlardan oluşturulmuştur. Her bir plaka probleminin sonuçları grafiklerle gösterilmiştir. Farklı plak boyutlarının oranı ve ilk düzlemsizlik modları ve bu modların birleşimi ile oluşan kompleks ilk düzlemsizlikleri için ortalama maksimum plak mukavemetleri hesaplanmıştır. Başlangıçtaki düzlemsizliklerin büyüklükleri literatürdeki makalelerden alınmıştır. Farklı modların oluşturduğu kompleks başlangıç düzlemsizliklerin ve plak boyut oranlarının mukavemetine olan etkileri hesaplanmıştır. Farklı modların oluşturduğu başlangıç düzlemsizliklerinin birleşimi yeni fakat farklı büyüklükteki düzlemsizlikler oluşturmaktadır. Bu faktörler plak mukavemetini azaltan değerler olarak oluşmaktadır. Elde edilen sonuçlar baskı altındaki dikdörtgen çelik plakların ve bunlardan oluşan çelik plak yapılarının tasarımında kullanılabilir.

Anahtar Kelimeler : Plak mukavemeti, Plakların deforme yükleri, Plak düzlemsizlikleri

1. INTRODUCTION

Plates occur in steel construction as individual faces; smaller cross section and larger cross section structural members. The smaller cross sections are found as box columns and box beams. Larger sections are found as large box girders in bridge construction and ship hull plating.

Individually plates can be categorised as internal and external panels in structures. Internal and external panels can be remote from cross-frames or bounded by stiff cross-frames. Longitudinal edges of internal panels are assumed to be held straight against out-of-plane behaviour. The external panels are assumed free to take any in-plane shape. The internal and external panels, remote from cross frames, are assumed to expand freely in transverse direction, whereas the other panels, bounded by stiff cross frames, are likely to contain an in-plain shear loading, and do not allow any movement in transverse direction (Little, 1980).

The classical results for the elastic buckling of a rectangular plate under longitudinal compression show that the buckled shape consists of nearly square sinusoidal half-waves the number, m, of buckles along the length taking the value which gives lowest buckling stress. For plates with simply supported edges, this corresponds to a buckle aspect ratio a/mb of 1, and for clamped edges, a shorter buckle aspect ratio a/mb of 0.6 occurs. There is no clear reason to assume the same geometry when buckling occurs above the elastic limit, or for the post-buckled behaviour.

The first simplification proposed for the modelling of the inelastic behaviour of long plates comes from Moxham (1971). He suggested that a long plate can be considered as an assembly of short plates joined by hinges extending across the plate width. All the short plates can go into elastic behaviour above the elastic limit, but as the maximum load is reached only one will fail and follow the descending load-shortening curve, whilst the others will start to unload elastically. His model is approximate since it contains an arbitrary release of the slope continuity condition by the introduction of the hinges, but should be close representation of the real behaviour. It introduces the concept that the analyst must work a choice of the aspect ratio of the short plate.

The maximum load carried is expressed as a mean stress on the loaded end, defined as $\sigma_m.b.t = P_{max}$ where σ_m, b, t and P_{max} are mean stress, width, thickness of plate and maximum load.

Design data has generally been presented as a strength-slenderness curve which may be non-dimensionalised by plotting as non-dimensional strength, $\frac{\sigma_m}{\sigma_0}$, against non-dimensional slenderness, $\frac{b}{t}\sqrt{\frac{\sigma_0}{E}}$. Two conventions exist for expressing the non-dimensional slenderness, the simple slenderness is defined as $\beta = \frac{b}{t}\sqrt{\frac{\sigma_0}{E}}$, and scaled slenderness is written as $\beta_L = \frac{1}{1.9}\frac{b}{t}\sqrt{\frac{\sigma_0}{E}}$. The most important phenomena around the point where elastic critical stress and yield stress are found occurs. The scaling places this point at $\beta_L = 1$.

The problem for the compiler of design standards is to establish the strength-slenderness curve. There are two approaches to solve this problem.

The first approach fits the design curve to test results. Since individual tests are expensive, the number of test results available is limited. From the test results, particularly for more complex geometry, upper and lower bounds corresponding approximately to mean ± 2 standard deviations may be obtained. Strength data obtained in this way may be used either within a traditional lower-bound/load factor design procedure or within a more sophisticated limit state design process employing partial safety factor (Smith et al., 1987).

The second approach fits the design curve to the results of numerical computations. This requires a detailed understanding of the phenomena occurring in order to set up valid models. There is a need to calibrate of this understanding and the numerical modelling against test results, and considerable care is needed in setting up the loading arrangements and boundary conditions for the specimen plates used in the modelling. One such a modelling scheme is established, it is relatively cheep to obtain additional results.

Typically, the use of numerical method depends on establishing, quite carefully, the conditions which will give the lowest strength. This includes assuming that initial imperfection happens to take up the most unfavourable shape. This will often occur when the initial shape has either the same shape as the out-of-plane deformations of the failure mode giving lowest strength or the initial shape includes such a component in addition to other component.

2. INFLUENCE OF ASPECT RATIO

Two approaches are found in earlier work, as to the choice of plate dimensions used for numerical studies.

In the first approach the interest is in long plates. These are modelled by one short plate as the element determining the plate strength and accepting the validity of Moxham's model to give an equal strength for a long plate. Initial studies such as Moxham's nearly flat plates showed that the aspect ratio had an effect on strength, and this was weaker than the effect of major parameters of in-plane dimensions, thickness and yield stress.

For a long plate it is natural to assume the same constraint for strength calculations as for elastic buckling, that integer number of buckle half wavelengths would occupy the whole plate length. This is only a weak constraint on the aspect ratio of the single short plate being modelled. For the purpose of numerical study and to generate conservative data, we choose to allow the aspect ratio to vary freely and seek the aspect ratio giving the lowest strength.

This approach was used by two workers.

Moxham (1971), first performed a small set of numerical studies with small initial imperfections and slenderness ratios $b/t = 40, 55, 80$ in mild steel and suggested that minimum strength is considered at the aspect ratio $a/b = 0.875$. His later results were obtained for large deflections.

Little (1980) has analysed 960 different simply supported rectangular plates primarily under longitudinal compression. He considered six different geometries which might occur in wide stiffened panel and narrow box beam constructions, and classified them A, B, C, D, E, F. The longitudinal edges might be either constrained to remain straight (representing an internal panel) to be free to pull-in and develop a local in-plane wave (representing an external, edge panel of a narrow member). The total in plane force on the nominally unloaded edge might indeed be zero, (representing a location remote from a cross frame or transverse stiffeners) a might be non-zero, with the displacement in-plane held to zero, (representing the constraint adjacent to a cross-frame). His type A (internal panel, edges constrained to remain straight but zero total force) was examined in most detail. Type B corresponds to the boundary conditions considered here.

He stated formally that results for different yield stresses could be non dimensionalised, providing initial out of flatness, expressed as $\frac{\delta_0}{t}$, remained constant. For this type A plates, a full range of out-of-flatness magnitudes was considered $\left(\frac{\delta_0}{t} = 0.005 - 0.05 - 0.15 - 0.35 - 0.75\right)$, whilst other types were restricted to a reduced range $\left(\frac{\delta_0}{t} = 0.05 - 0.15\right)$. Slenderness β_L of 0.5-1.75 in steps of 0.25 were considered. Aspect ratios $\frac{a}{b}$ of 0.4, 0.6, 0.8, 1.0 were investigated.

But this date it was suggested that, in bridge plating, imperfection magnitude was best represented as being proportional to b with magnitudes of $\delta_0/b = 1/1000$ as average and $\delta_0/b = 1/300$ as maximum value. These correspond to $\left(\delta_0/t\right)$ values of about 0.05 and 0.15 for b/t of 25 to 80.

His results showed that in most cases the minimum plate strength occurs in the range of aspect ratio $\left(a/b\right)$ 0.4 to 0.6. The aspect ratio had a weaker effect on strength than plate slenderness and initial out-of-flatness.

The alternative approach is to consider the long plate of one fixed, but fairly small aspect ratio, and to consider this as representative of any other aspect ratio.

In Frieze's study (Frieze et al., 1977), for long plates, aspect ratio $\left(a/b = 3\right)$ with single half-sine wave initial bows of maximum imperfection magnitudes $\delta_0 = 1.5t$ and $\delta_0 = 0.188t$ were considered. Their results are compared with results of the square plates $\left(a/b = 1\right)$ with the same magnitudes of initial deformation and the same slenderness ($\beta = 2.074$ or $b/t = 60$ for mild steel). From the results of his studies, the peak stiffness and maximum strength of the longer plate are greater in each imperfection cases ($\delta_0 = 1.5t$ and $\delta_0 = 0.188t$) than those of the square plate.

In the case of both plates the imperfection has been taken as a single wave in both directions. This results in a peakier response for the $\left(a/b = 3\right)$ plate because of later surface yielding with a fast unloading characteristic induced by snap-through buckling into a three-wave mode. If the initial imperfection for $\left(a/b = 3\right)$ plate had been in a three half-wave longitudinal mode the response would have approximated to that of the square plate. The results indicated that strength and stiffness calculations based on the square plates would provide conservative estimates of the strength and stiffness of longer plates of more practical proportions.

Applying Little's results for short plates raises a fundamental question about this second approach. If the aspect ration giving lowest strength for a single imperfection mode was should be around $\left(a/b = 0.4 - 0.6\right)$, then in a long plate of aspect ratio $\left(a/b = 3\right)$, the mode number giving the lowest strength might be $m = \frac{3}{0.4}$ to $m = \frac{3}{0.6}$ i.e. $m = 5$ to 7. The choice of a short wavelength mode number $m = 3$ for these studies may not be conservative, but may have overestimated all strength.

The study reported in this chapter addresses this question further.

3. MAGNITUDE OF INITIAL IMPERFECTION ASSUMED

Theoretical and experimental investigations of plate stiffness and strength have presented clearly the important influence of imperfections caused by weld-induced distortions in fabrication, deflects in rolling, accidental damage in service and slamming damage to bow plating in ships structures.

There are significant differences in fabrication methods between different applications. In bridge building, light, often intermittent, welding of stiffeners to plating reduce overall distortions and also reduce weld-induced wrap-up of the plate edges. In ship building, more transverse stiffeners and continuous welding to improve damage stability causes greater wrap-up. In light hull construction of warships the whole bow plating may become deformed inwards between stiffeners as a result of slamming into seas as ship pitches in heavy weather. The measured initial out-of-flatness may be different in these two cases.

Theoretical analysis (Crisfield, 1975; Frieze et al., 1977; Little 1980; Smith et al., 1987) has shown that the most significant form of initial deformation in plates is a periodic or isolated imperfection of amplitude δ_0 with half-wave length equal to or somewhat less then the plate width, b. In square or nearly square plates ($a/b \approx 1.0$) this corresponds to the dominant weld-induced distortion having a single half-wave over the plate length. In longer rectangular plates ($a/b > 1.5$), the overall imperfection δ_0 has relatively little effect on compressive strength and may actually increase plate strength by obstructing formation of the preferred buckling mode. Under loading applied in the shorter direction and in square or nearly square plates, overall imperfection δ_0 is in all cases the most important imperfection (Smith et al., 1987).

Most researches of plating in welded box-girder bridges and ships have referred only to the maximum or central amplitude of imperfection δ_0 measured over the plate length (Ellis, 1977; Somerville et al., 1977; Antoniou et al., 1984). A preferable method of predicting, δ_0, is measurement of maximum imperfection over a short gauge length, b, at any position along the length of a plate. The imperfection amplitude, δ_0, is sometimes non-dimensionalised with respect to b since measurements made on bridges (Bradfield, 1974) conclude that δ_0 is roughly proportional to b. He suggested a mean value for δ_0/b of about $1/1000$ and a maximum value of about $1/300$. This imperfection δ_0 has been employed in some surveys of bridge structures (Ellis, 1977), but unfortunately has not generally been used in surveys of ships' plating (Smith et al., 1987).

Smith explains that "measurements carried out in this way for bridges have referred mainly to intermittently welded structure and are judged to be unrepresentative of continuously welded ships' plating. It appears therefore that no satisfactory definition of mean and coefficient of variation of significant imperfection δ_0 for ships' plating is available at present. Initial deformations have been assumed by some investigators to be proportional to β and by others to be proportional β^2, where $\beta = b/t \sqrt{\sigma_0/E}$ is a plate slenderness parameter. β^2 is suggested as being marginally the better choice and used for assumed imperfection levels" (Smith et al., 1987).

He categorised initial imperfection values, δ_0/t, as $=0.025\beta^2$ slight, $=0.1\beta^2$ average and $=0.3\beta^2$ severe levels. Those imperfection values are the function of slenderness, β, since the numerical results can determined by setting the imperfection magnitudes depends on the slenderness to fit the test results on the strength-slenderness curves.

Horne and Narayanan (1975) recommend that imperfection magnitude, δ_0, should be obtained by the following empirical expression

$$\delta_0 = \left(\overline{\delta_0} + \frac{\overline{\delta_{0m}}}{2}\right)\frac{b}{30t}\sqrt{\frac{\sigma_0}{245}}$$

where $\overline{\delta_0}$ is the measured imperfection and $\overline{\delta_{0m}}$ is the plate tolerance recommended by the Merrison rules, that is,

$$\frac{b}{30t}\left(1+\frac{b}{5000}\right)\text{mm} \quad \text{for} \quad t < 25\text{mm}$$

$$\frac{b}{750t}\left(1+\frac{b}{5000}\right)\text{mm} \quad \text{for} \quad t \geq 25\text{mm}.$$

The Bridge code seems to use one strength-slenderness curve for any yield stress, implying a constant $\frac{\delta_0}{t}$. The curves were obtained from work on mild steel $\sigma_0 \cong 250\text{N/mm}^2$ and correspond to a sinusoidal shape magnitude $\frac{\delta_0}{b} = 0.005$ (but then expressed as a lower value since the standard places more emphasis on high yield steel of about $\sigma_0 = 350\text{N/mm}^2$). The fabrication section of the standard uses a similar magnitude of initial imperfection, but gives a measurement procedure based on a gauge length g = 2b. For an idealised sinusoidal shape, the inspection would measure the peak-to-peak magnitude i. e. double the amplitude. So this implies a tighten fabrication standard. To evaluate the imperfection magnitudes, δ_0, chosen and recommended by different authors and also used in British standard, a mild steel plate can be chosen. The slenderness, the yield stress, the elastic modules and the thickness of this plate are chosen as $\frac{b}{t} = 55$, $\sigma_0 = 250\text{N/mm}^2$, $E = 205000\text{N/mm}^2$, and $t = 6\text{mm}$.

For this plate;
Moxham's imperfection magnitudes are as $\delta_0 = 0$, and $\delta_0 = 1.65\,\text{mm}$.
Frieze's imperfection magnitudes are as $\delta_0 = 1.128\,\text{mm}$, and $\delta_0 = 9\,\text{mm}$.
Little's imperfection magnitudes are as $\delta_0 = 0.03$ mm, 0.3 mm, 0.9 mm, 2.1mm and 4.5 mm.
Smith's imperfection magnitude are as $\delta_0 = 0.55\,\text{mm}$ (slight), $\delta_0 = 2.211\text{mm}$ (average) and $\delta_0 = 6.633\,\text{mm}$ (severe).
Horne and Narayanan's recommended value is as follows $\delta_0 = 1.809\text{mm}$ if measured imperfection is chosen as $\overline{\delta_0} = 0$.

In British standard BS5400 (Bridge Code) initial imperfection magnitude is as follow $\delta_0 = 1.67\text{mm}$. From the above imperfection magnitudes for general application Smith's and Little's values can be considered as representative since average value of Smith's imperfection magnitude have some similarities with Little's value. This is the Little's imperfection value is same with Simith's Average imperfection value at $\beta = 2$. Smith's initial imperfection levels as Severe, Average and Slight are for the ships' plating while Little's values are for the bridges' plating. In Figure 1, initial imperfection values which is assumed by Smith (1987) are plotted against the slenderness ratio, β. Therefore, in this study, without any specific application Smith's Average imperfection levels are selected as being representative.

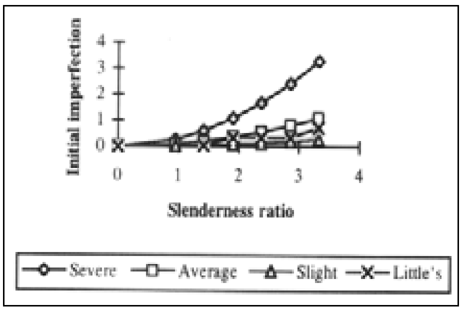

Figure 1. Initial imperfections (Severe, Average and Slight levels are assumed by Smith et all., 1987)

4. GEOMETRIES STUDIED

The geometry of plates, shown in Figure 2, have an initial out-of-flatness, δ_o, shaped as a single half sine wave in both X and Y direction (m = 1, n = 1), and two (m = 2, n = 1), three (m = 3, n = 1) and four (m = 4, n = 1) half wave mode in X directions. An initial out-of-flatness shape having a single half sine wave in Y direction (n = 1) is chosen for all modes since significantly higher strengths would be obtained for any higher value of shape.

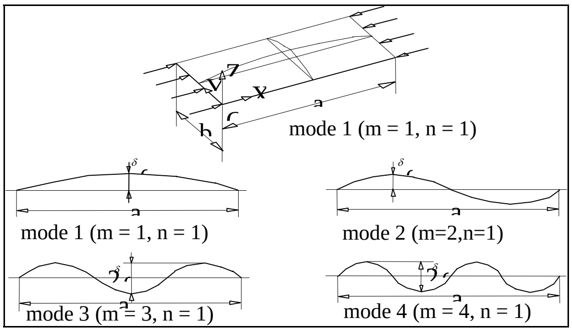

Figure 2. Plate geometry and assumed mode shapes of initial out-of-flatness

All four edges are considered to be simply supported against out-of-plane deformation in Z direction. The loaded edges are loaded by applying increments in the in-plane displacement whilst remaining straight. The unloaded edges are free to pull-in in X and Y directions.

The boundary conditions on long centre line $y = b/2$ are symmetric, see Figure 3. For $m = 1$ and 3 boundary conditions on transverse centre line $x = a/2$ are symmetric while for $m = 2$ and 4 they are anti-symmetric.

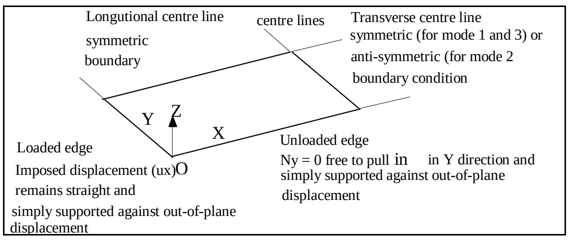

Figure 3 Boundary conditions of a quarter of a plate

In the first study, the slenderness has been chosen as $\beta = 1.92$ ($\frac{b}{t} = 55$ for mild steel) where the yield stress and critical buckling stress are similar, and the sensitivity to imperfection is maximum. The initial imperfection magnitudes are taken as the "average" value $\delta_0 = 0.1t\beta^2$ assumed by Smith. Aspect ratios, a/b, vary between 0.2 to 4.0. This study investigates the variation of strength with aspect ratio and transition between the modes ($m = 1, 2, 3$, and 4).

The second study repeats this investigation with $\beta = 3$. These plates are more slender and the critical buckling stress of the plate is significantly less than the yield stress, so that very large out-of-plane deflections are needed before yield occurs. The aim of this study to investigate the plate behaviour at high slenderness.

The third study takes an intermediate slenderness, $\beta = 2.3$, and varies the imperfection magnitude as $\delta_0 = 1.83$ mm, 2.213 mm, 3.174 mm, 4 mm for the two half wave mode (m = 2, n = 1). The aim of this study is to consider the effect of imperfection magnitude.

A quadrant of the plate is modelled and meshed, with symmetric and anti-symmetric boundary conditions. All plates have $t = 6\text{mm}$, $E = 205000\text{N/mm}^2$ and $\sigma_0 = 250\text{N/mm}^2$ but results are non-dimensionalised and valid for other plate sizes and properties.

The finite element mesh is graded to avoid any drastic changes in the element size when the types of initial imperfection varies from first mode, m = 1, to fourth mode, m = 4. For mode 1 a (6 x 6) mesh is found to give adequate results in minimum computation time whilst fine meshes (6 x 9) and (6 x 12) produce only as 0.5 % and 0.8 % increase in plate strength.

Figure 4 shows curves for fine meshes for the longer mode members. Meshes of 6 x 6, 6 x 9 and 6 x 12 were used. Generally the fine meshes gave higher strengths. All later results use a 6 x 9 mesh for $m = 2$ and (6 x 12) mesh for $m = 3$ and 4. These were considered sufficient to fine.

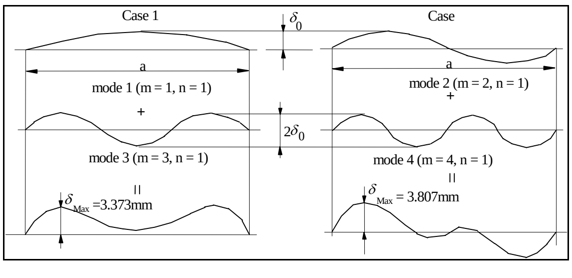

Figure 4. Variation of plate strenght against aspect raio for plates with β = 1.92 and $\delta_0 = 0.1t\beta^2$

Solution control parameters followed those used for the program verification test results presented by Bektaş (1997). The force convergence criterion was taken as 0.001, and a displacement increment was applied corresponding to $\Delta\varepsilon_x / \varepsilon_0 = 0.0432$. From the ratio of in-plane strain increment $\Delta\varepsilon_x$ in the loaded direction to yield strain ε_0, displacement increment Δu is obtained as;

Selection $\frac{\Delta\varepsilon_x}{\varepsilon_0} = 0.0432$ and substituting for $\Delta\varepsilon_x$ and ε_0 gives $\Delta u = 0.0432 \frac{\sigma_0}{E} a \frac{1}{2}$.

This displacement increment, Δu, is applied to all problems in a load step or a sub step to obtain the accurate results in a short computation time.

5. THE RESULTS OBTAINED

Figure 4 shows the computed results for $m = 1$, together with a strength curve computed from Little's results by interpolating to $\delta_0 / t = 0.1\beta^2$. They show a similar shape of curve, with a minimum around $a/b = 0.6$, but Little's results are 6 % higher.

It is worth noting that the curves, in Figure 5, have some similarities with the elastic buckling curves in Figure 5 taken from Bulson (1970), these are as follows;

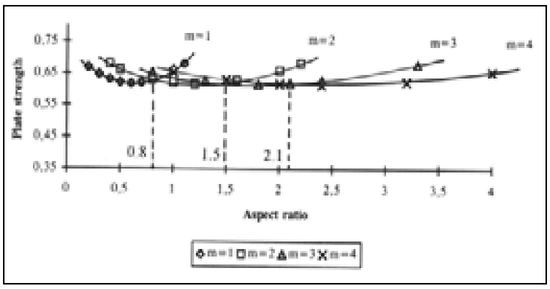

Figure 5. Variation of elastic buckling coefficient $K = \frac{m^2}{\Phi^2} + \frac{\Phi^2}{m^2} + 2$ against the aspect ratio $\Phi = \frac{a}{b}$ (Bulson, 1970), σ_{cr} is proportional to K for any given $\frac{b}{t}$ ratio

- Both curves have transition points where curves cut each other.
- The lowest elastic buckling stresses of short (mode 1) and long (mode 2, 3 ,4) plates are same, and the lowest plate strengths of short (mode 1) and long (mode 2, 3, 4) plates are same as well.
- Both figures suggest that the lowest elastic buckling stress and the lowest plate strength can be obtained below the transition points as selecting modes depending on the aspect ratio or selecting aspect ratio depending on the modes.
- Elastic buckling curves have minima at multiplies of $a/b = 1.0$, the strength curves have minima at multiplies of $a/b = 0.6$.

Minimum strength is considered at the point of aspect ratio, $a/b = 0.6$ for mode $m = 1$, $a/b = 1.2$ for mode $m = 2$, $a/b = 1.8$ for mode $m = 3$ and $a/b = 2.4$ for mode $m = 4$. This shows that minima occur at multiples of $a/b = 0.6$ (i.e. 1.2, 1.8, 2.4) for $m = 1,2,3,4$ for this case $\beta = 1.92$, $\delta_0 / t = 0.1\beta^2$. This confirms use of short plate to estimate strength of a long plate.

For design of long plates which have the imperfection modes, $m = 2$, $m = 3$ and $m = 4$, reference can be made to the design of the single plate about obtaining minimum strength. It is worth noting that the sensitivity of aspect ratio to the strength of plates can be seen clearly in Figure 5. It can be noted that the aspect ratio sensitivity to the uniaxial strength of plates decreases for long plates which have mode 2, 3 and 4. So that short plates are more sensitive to the aspect ratio than the long plates. Above the first minimum at $a/b = 0.6$, the strength variation with aspect ratio is small. The maxima, at transitions between modes, lie at less than 1.6 % of σ_0 and 2.5 % of σ_m / σ_0 above the minimum.

The investigation is repeated for $\beta = 3$, retaining $\frac{\delta_0}{t} = 0.1\beta^2$ showing behaviour in the region where the critical buckling stress of the plate is significantly less than the yield stress and very large out-of-plane deflections are needed before yield occurs for real plates. The results are presented in Figure 6, and strength compared with the earlier results for $\beta = 1.92$, the strength has reduced significantly, as expected, from $\sigma_m / \sigma_0 = 0.616$ to 0.402, whilst the minima now occur at multiples of

$a/b = 0.4$. The four curves have minima at slightly different strengths. This is probably due to an unbalanced fine mesh for the high mode number, and would be useful to repeat these analyses using meshes of 6 x 12 for mode 2, 6 x 18 for mode 3 and 6 x 24 for mode 4.

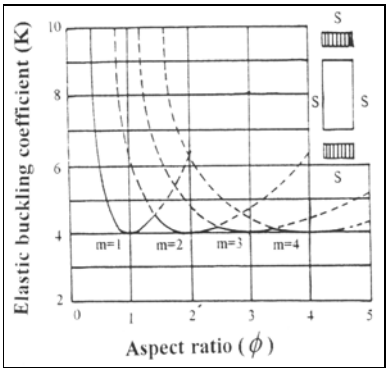

Figure 6. Variation of plate strength against aspect ratio for plates with $\beta = 3$ and $\delta_0 = 0.1t\beta^2$

The effect of varying imperfection magnitude is shown in Figure 8 for an intermediate slenderness $\beta = 2.3$, $m = 2$ and varying initial out-of-flatness show where the Average imperfection fits against this new scale. Smith's initial imperfection values are in relation with the slenderness ratio as $\left(\delta_0 = 0.1t\beta^2\right)$ to fit the numerical results to the test results. From the results of this study as seen in Figure 8, the increasing imperfection magnitude causes the strength to fall and shifts the lowest strength from approximately $a/b = 1.5$ to $a/b = 1.2$. The curves in Figure 7 became flat when the deflections are largest. This shows that the increment of deflection reduces the effects of aspect ratio on the plate strength. Therefore, variation of lowest plate strength on the aspect ratio depends considerably on the slenderness β and imperfection magnitudes δ_0. It is worth noting that the strength degrease in the higher region of the $\frac{a}{b} = 2$ for the last curve which is evaluated for highest imperfection, $\delta_0 = 4\text{mm}$, shown in Figure 7.

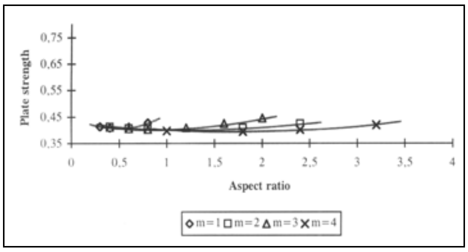

Figure 7. Variation of plate strength against aspect ratio for plates with $\beta = 3$ and w $(\delta_0) = 1.38\,\text{mm}, 3.174\,\text{mm}, 4.0\,\text{mm}$

Possible reasons of this unbalanced strengths obtained against aspect ratio are may be;

- Effects of coarse mesh in modes which chosen as 6x9 mesh , and
- At high imperfections the nature of plate behaviour changes, the instability behaviour becoming less important, and the plate bending behaviour of initially bent plate dominates, and the magnitude of the curvature along a longitudinal line of the plate becomes the dominant parameter.

6. INITIAL OUT - OF-FLATNESS CONTAINING COMPONENTS OF TWO MODES

Initial out-of-flatness as a double sine series $w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_0 \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$ is a simplification of a real initial shape, which is not maintained consistently, and for design, we normally seek to evaluate strength using an upper bound to the adverse components of the initial out-of-flatness. Any real shape can, in principle, be modelled as the sum of the Furrier series of sinusoidal initial out-of-flatness components; one might expect very few of the possible terms to be significant. We might expect the greatest adverse effects on the strength to be produced by a component of the initial out-of-flatness, having a shape similar to the short plate which gives lowest strength. Previous work has considered a representative long plate of $\frac{a}{b} = 3$ and two components of initial out-of-flatness $m = 1$, $\frac{a}{mb} = 3$ modelling an overall bow, due to ends being pulled flat at a diaphragm, cross frame or transverse stiffeners and $m = 3$, $\frac{a}{mb} = 1$ intended to represent the weakening ripple imperfection. As seen earlier, this $\frac{a}{mb} = 1$ component will not have the most adverse effect, and should be adjusted to give $\frac{a}{mb}$ nearer to a value such as 0.6.

Therefore, two imperfection shapes, presented as Case 1 and Case 2 in Figure 8, are occurred by combination of mode 1 to 3 and mode 2 to 4. For

Case 1, symmetric boundary conditions are imposed about the XY plane throughout to X and Y directions, respectively. For Case 2, anti-symmetric boundary conditions are imposed throughout to Y direction, and symmetric boundary conditions are imposed throughout to X direction. The other cases, for example; case 3 include mode 1 and 2, and case 4 consist of mode 1 and 4, can not be modelled since their boundary conditions are not convenient to apply. Symmetry boundary conditions can not be combined with anti-symmetry boundary conditions for modelling of a quarter of the plate. Therefore, only two cases (Case1 and Case 2) are considered (see Figure 8).

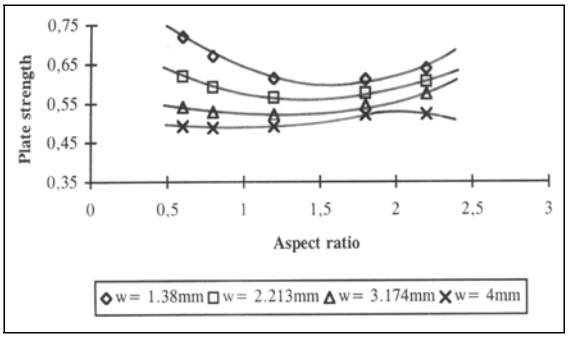

Figure 8 Initial imperfections for Case 1 and Case 2

The slenderness ratio $\beta = 1.92$, is chosen, since it is a point where the yield stress and critical buckling stress of the plate coincide, and the sensitivity to imperfections is maximum. The magnitude of initial imperfections are derived from Smith's assumed average values $\left(\delta_0 = 0.1t\beta^2\right)$. Arbitrarily, each component is chosen to have an equal amplitude. This leads to maximum magnitude of the resultant imperfection shape of Case 1, $\delta_{max} \cong 3.373\text{mm}$ and Case2, $\delta_{max} \cong 3.807\text{mm}$.

Resources have not allowed a full study of this problem. A few results are presented for combined imperfections in a different regime. For the first case, the components $m = 1$ and 3 are presented in the initial shape and $\frac{a}{b} = 0.4 - 1.8$ giving, for $m = 1$, $\frac{a}{mb} = 0.4 - 1.8$ and for $m = 3$, $\frac{a}{mb} = 0.13 - 0.6$. In the second case, the components $m = 2$ and 4 are presented in the initial shape, and $\frac{a}{b} = 0.4 - 3.2$, giving for $m = 2$, $\frac{a}{mb} = 0.2 - 1.6$ and for $m = 4$, $\frac{a}{mb} = 0.1 - 0.8$. In each case, the longer half-wavelength component has a geometry which includes the expected minimum strength near $\frac{a}{mb} = 0.6$, see Figure 9. The shorter half-wavelength component may just condition, but is otherwise concentrated of very short half-wavelength.

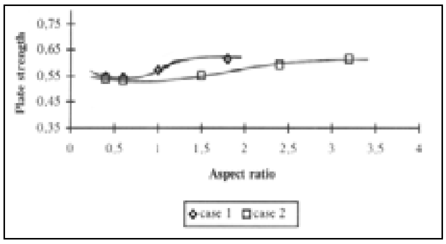

Figure 9. Variation of plate strength against aspect ratio for $\beta = \frac{b}{t}\sqrt{\frac{\sigma_0}{E}} = 1.92$, $\delta_0 = 0.1t\beta^2$ ase 1 (mode 1 + 3 and Case 2 (mode 2 + 4) Plates

Results confirm that a minimum strength is still seen, where the additional short half-wavelength is present. If it plotted together with the single half-wavelength component, it is seen that the additional short half-wave component has reduced the strength, presumably due to the significant additional and secondary bending stresses induced by the membrane stress Nx and the curvature in the longitudinal criterion $\chi_x = -\frac{\partial^2 w}{\partial x^2}$.

For Case 1, the effect of a second component of the initial out-of-flatness is to reduce the strength further, from $\sigma_m/\sigma_0 = 0.618$ for $m = 1$ to 0.545 when $m = 1$ and $m = 3$ components are presented. Estimating the strength for $m = 1$ only and a magnitude of initial out-of-flatness of 3.37mm, as shown in Figure 4, would give a strength of $\sigma_m/\sigma_0 = 0.551$, very similar to that computed.

Similarly for Case 2, the effect of the second component is to reduce the strength again from $\sigma_m/\sigma_0 = 0.618$ for $m = 2$ to 0.535 when $m = 2$ and $m = 4$ components are present. Estimating the strength for $m = 2$ only and a magnitude of initial out-of-flatness of 3.807mm, as shown in Figure 4, would give a strength of $\sigma_m/\sigma_0 = 0.532$, very similar to that computed. If the curves were extended to the highest ratios a/b, they would enter the region where the short half-wavelength component will have the greatest adverse strength. The rise in the curves, as a/b increases, is therefore expected to halt, a maximum strength to be found, before reaching a second minimum, which might be expected near $\frac{a}{b}=1.8-2.0$ for first case and $\frac{a}{b}=2.4-2.8$ for second case. Insufficient computations are available for these geometric for any such fine structure of the variation to be scan.

7. CONCLUSION

Work has been carried out to investigate the strength of initially-imperfect rectangular plates under uniaxial loading, examining the effect of varying plate aspect ratio, and the shape of the assumed initial imperfection. The main investigation considered on initial out-of-flatness comprising a varying number of sinusoidal half-wavelengths within the plate length. It showed how the mode number of the initial shape giving lowest plate strength switches as the plate aspect ratio is varied.

For the main case studied the preferred buckle shape was based on an aspect ratio of each buckle half wavelength, $a/(mb)$ of 0.6. This case had the critical plate slenderness ratio, and a substantial magnitude of initial out-of-flatness, based on the proposed "Average" out-of-flatness for ship plating. Within this limitation, the results at higher slenderness, and for large initial out-of-flatness show that the preferred buckle aspect ratio may vary down to $a/(mb)=0.4$. More work on different geometries, and with the refined mesh, would be needed to explore the variation between values of 0.6 and 0.4 fully.

A small investigation considered a plate having a more complex initial shape, modelled by superposing two sinusoidal shapes. The shapes considered were restricted to either both of odd mode number, or both even, to maintain anti-symmetric and symmetric boundary conditions at the centreline and allow only one quadrant to be modelled. The results confirmed that the addition of a second component to the initial out-of-flatness could reduce the strength, as compressed with the plate having only one component of equal amplitude. The results suggested a need for some caution in accepting the practice used in some earlier plate strength studies. In these, a plate of overall aspect ratio $a/b=3$ was taken as representative of other lengths, and the assumed initial shape contained a third mode component ($a/(mb)=1.0$) either alone or in addition to a first mode component. Within the limitations expressed above, the present results suggest that the choice of this third mode, rather than a higher fifth and seventh mode, may have increased the computed strength and lead to non-conservative results.

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