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A classification method based on Hamming pseudo-similarity of intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices

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Abstract — In this study, firstly, Hamming pseudo-similarity of intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices) have been defined. Afterwards, a classifier based on Hamming pseudo-similarity of *ifpifs*-matrices (IFPIFS-HC) has been developed. The classifier's simulations have been performed using datasets provided in the UCI Machine Learning Database, and its performance results via the performance metrics accuracy, precision, recall, macro F-score, and micro F-score have been obtained. Thereafter, the results have been compared with those of the well-known methods. Then, the statistical evaluations of the performance results have been conducted using Friedman and Nemenyi post-hoc tests, and the critical diagrams of the Nemenyi post-hoc test are presented. The results and the statistical evaluations show that the proposed classifier has performed better than the others in 12 of 21 datasets in terms of the five performance metrics, in 4 of 21 in terms of the four performance metrics, and 17 of 21 in terms of accuracy performance metric. Moreover, the mean accuracy, precision, recall, precision, macro F-score, and micro F-score results of Fuzzy kNN, FSSC, FussCyier, HDFSSC, and FPFS-EC for the 21 datasets are 84.90, 71.96, 67.95, 71.91, and 75.28; 78.12, 68.01, 68.05, 66.53, and 67.68; 80.76, 68.63, 69.07, 68.36, and 70.65; 81.93, 69.43, 69.95, 70.25, and 72.36; and 89.59, 80.27, 78.40, 81.20, and 83.60, while those of IFPIFS-HC are 90.59, 82.88, 80.75, 82.89, and 85.48, respectively. Finally, the applications of *ifpifs*-matrices to machine learning have been discussed for further research.

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1. Introduction

Fuzzy sets are a mathematical tool put forward by Zadeh [1] to overcome the problems involving uncertainties in which classical sets are insufficient in modelling. Another mathematical tool propounded to model such problems is soft sets [2]. So far, many hybrid versions of these two concepts have been described [3-5]. Fuzzy parameterized fuzzy soft sets (*fpfs*-sets) [6], the general form of these hybrid versions, come to the fore with their ability to model situations where both parameters and alternatives (objects) are fuzzy. The concept of fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices)

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[7] has been defined to avail of the modelling capabilities of *fpfs*-sets and cope with a large number of data in decision-making.

Recently, some soft decision-making (SDM) methods constructed through hybrid versions of fuzzy sets and soft sets in the literature have been configured into the *fpfs*-matrices space [8-12]. In addition, some of these configured SDM methods have been mathematically simplified, providing a great advantage in terms of the running time of the methods [13-18]. Also, Memiş et al. [19] have proposed a classification algorithm employing the Hamming pseudo-similarity of *fpfs*-matrices and successfully applied it to the classification problem related to the medical datasets including "Breast Cancer Wisconsin (Diagnostic)", "Immunotherapy", "Pima Indian Diabetes", and "Statlog Heart". Moreover, Memiş and Enginoğlu [20] developed a classification algorithm by utilising Chebyshev pseudo-similarity of *fpfs*-matrices and successfully applied this algorithm to a classification problem involving the medical data sets "Cryotherapy", "Diabetic Retinopathy", "Hepatitis", and "Immunotherapy".

Despite these successes of *fpfs*-matrices, since fuzzy sets cannot model intuitionistic fuzzy [21] uncertainties, the concepts of intuitionistic fuzzy soft sets [22], intuitionistic fuzzy parameterized soft sets [23], intuitionistic fuzzy parameterized fuzzy soft sets [24], and fuzzy parameterized intuitionistic fuzzy soft sets [25] have been put forward. Then, the concept of intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets (*ifpifs*-sets), which can model problems where both parameters and objects contain intuitionistic fuzzy uncertainties, has been defined and successfully applied to an SDM problem [26]. Recently, the concept of intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices) [27] has been proposed and successfully applied to two SDM problems. Afterwards, Arslan and Enginoğlu [28] have defined the algebraic sum and algebraic product on *ifpifs*-matrices and propounded an SDM method. Moreover, they have applied this SDM method to a performance-based value assignment problem in image processing.

Similar to the studies mentioned above, it is worth studying to construct new classification algorithms by defining similarity measures in the *ifpifs*-matrices space, thereby improving the capabilities of *ifpifs*-matrices in SDM and classification problems. This study is a pioneer study related to similarity measures of *ifpifs*-sets/matrices. It is also substantial because it will lead to further studies on the definition of distance/similarity measures on *ifpifs*-matrices.

In the second part of this study, some basic definitions to be required for the next section are provided. Section 3 defines the concept of pseudo-similarity over *ifpifs*-matrices space and proposes a new classification method using Hamming pseudo-similarity, i.e., IFPIFS-HC. In Section 4, the properties of 21 UCI datasets [29], employed in the comparison of classification methods, are presented. Secondly, mathematical notations of performance metrics commonly used to measure the performance of classification methods are provided. Thirdly, the simulation results of IFPIFS-HC, the well-known and state-of-the-art classifiers such as Fuzzy kNN [30], FSSC [31], FussCyier [32], HDFSSC [33], and FPFS-EC [34] are obtained and presented using the aforesaid datasets and performance metrics. Fourthly, the performance results obtained from the simulation are statistically analysed using the Friedman test [35] and the Nemenyi post-hoc test [36], and their Nemenyi diagrams are presented. The last part discusses the need for further research on *ifpifs*-matrices.

2. Preliminaries

This section, firstly, presents the concept of *ifpifs*-matrices [27] and its some of basic properties. Across the present paper, let E be a parameter set and U be an alternative (object) set.

Definition 2.1. [21] Let E be a universal set and $\mu, \nu: E \to [0,1]$ such that $\mu(x) + \nu(x) \leq 1$, for all $x \in E$. Then, the set $\{(x, \mu(x), \nu(x)) : x \in E\}$ is called intuitionistic fuzzy set (*if*-set) over E.

Here, for all $x \in E$, $\mu(x)$ and $\nu(x)$ are called the membership and non-membership degrees of x, respectively, and $\pi(x) = 1 - \big(\mu(x) + \nu(x)\big) = 1 - \mu(x) - \nu(x)$ is the indeterminacy degree of x. Moreover, for all $x \in E$, $0 \leq \pi(x) \leq 1$ is straightforward.

In the present paper, the set of all *if*-sets over E is denoted by $IF(E)$ and $f \in IF(E)$. For brevity, the notation ${}^{\mu(x)}_{\nu(x)}x$ is used instead of $\big(x, \mu(x), \nu(x)\big)$. In other words, an *if*-set over E is denoted by $f = \left\{{}^{\mu(x)}_{\nu(x)}x : x \in E\right\}$. Furthermore, we do not display the element ${}^{0}_{1}x$ in an *if*-set.

Definition 2.2. [26] Let U be a universal set, $f \in IF(E)$, and α be a function from f to $IF(U)$. Then, the set $\left\{\left({}^{\mu(x)}_{\nu(x)}x, \alpha\left({}^{\mu(x)}_{\nu(x)}x\right)\right) : x \in E\right\}$ being the graphic of α is called an intuitionistic fuzzy parameterized intuitionistic fuzzy soft set (*ifpifs*-set) parameterized via E over U (or briefly over U).

Hereinafter, the set of all the *ifpifs*-sets over U is denoted by $IFPIFS_E(U)$. In $IFPIFS_E(U)$, since the graph(α) and α generate each other uniquely, the notations are interchangeable. Therefore, if it causes no confusion, we denote an *ifpifs*-set graph(α) by α. In addition, for convenience, we do not display the elements $({}^{0}_{1}x, 0_U)$ in an *ifpifs*-set. Here, 0_U is the empty *if*-set over U.

Example 2.3. Let $E = \{x_1, x_2, x_3, x_4\}$ and $U = \{u_1, u_2, u_3\}$. Then,

$$\alpha = \left\{\left({}^{1}_{0}x_1, \{{}^{0.7}_{0.2}u_1, {}^{0.3}_{0.4}u_2\}\right), \left({}^{0.5}_{0.5}x_2, \{{}^{0.1}_{0.7}u_2, {}^{1}_{0}u_3\}\right), \left({}^{0}_{1}x_4, \{{}^{0.5}_{0.5}u_1, {}^{0.9}_{0}u_2, {}^{0}_{0.4}u_3\}\right)\right\}$$

and

$$\beta = \left\{\left({}^{0.6}_{0.2}x_2, \{{}^{0.3}_{0.3}u_1, {}^{0.7}_{0.1}u_2, {}^{0.2}_{0.6}u_3\}\right), \left({}^{0}_{0.5}x_3, \{{}^{0.8}_{0.1}u_1, {}^{0.1}_{0.8}u_3\}\right), \left({}^{0.7}_{0}x_4, \{{}^{1}_{0}u_2\}\right)\right\}$$

are two *ifpifs*-sets over U.

Definition 2.4. [27] Let $\alpha \in IFPIFS_E(U)$. Then, $[a_{ij}]$ is called *ifpifs*-matrix of α and is defined by

$$[a_{ij}] \coloneqq \begin{bmatrix} a_{01} & a_{02} & a_{03} & \cdots & a_{0n} & \cdots \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{0, 1, 2, \dots\}$ and $j \in \{1, 2, \dots\}$,

$$a_{ij} \coloneqq \begin{cases} \dfrac{\mu(x_j)}{\nu(x_j)}, & i = 0 \\ \alpha\left({}^{\mu(x_j)}_{\nu(x_j)}x_j\right)(u_i), & i \neq 0 \end{cases}$$

or briefly $a_{ij} \coloneqq {}^{\mu_{ij}}_{\nu_{ij}}$. Here, if $|U| = m - 1$ and $|E| = n$, then $[a_{ij}]$ has order $m \times n$.

In this paper, as long as it causes no confusion, the membership and non-membership functions of $[a_{ij}]$, i.e., μ_{ij} and ν_{ij}, will be represented by μ^a_{ij} and ν^a_{ij}, respectively. Moreover, the set of all the *ifpifs*-matrices parameterized via E over U is denoted by $IFPIFS_E[U]$ and $[a_{ij}], [b_{ij}], [c_{ij}] \in IFPIFSS_E[U]$.

Example 2.5. The *ifpifs*-matrices of α and β provided in Example 2.3 are as follows:

$$[a_{ij}] = \begin{bmatrix} \substack{1 \\ 0} & \substack{0.5 \\ 0.5} & \substack{0 \\ 1} & \substack{0 \\ 1} \\ \substack{0.7 \\ 0.2} & \substack{0 \\ 1} & \substack{0 \\ 1} & \substack{0.5 \\ 0.5} \\ \substack{0.3 \\ 0.4} & \substack{0.1 \\ 0.7} & \substack{0 \\ 1} & \substack{0.9 \\ 0} \\ \substack{0 \\ 1} & \substack{1 \\ 0} & \substack{0 \\ 1} & \substack{0 \\ 0.4} \end{bmatrix} \text{ and } [b_{ij}] = \begin{bmatrix} \substack{0 \\ 1} & \substack{0.6 \\ 0.2} & \substack{0 \\ 0.5} & \substack{0.7 \\ 0} \\ \substack{0 \\ 1} & \substack{0.3 \\ 0.3} & \substack{0.8 \\ 0.1} & \substack{0 \\ 1} \\ \substack{0 \\ 1} & \substack{0.7 \\ 0.1} & \substack{0 \\ 1} & \substack{1 \\ 0} \\ \substack{0 \\ 1} & \substack{0.2 \\ 0.6} & \substack{0.1 \\ 0.8} & \substack{0 \\ 1} \end{bmatrix}$$

3. Proposed Classification Method: IFPIFS-HC

This section, firstly, provides the basic notations to be needed for proposed classification method based on *ifpifs*-matrices. Throughout the study, let $D = [d_{ij}]_{m\times(n+1)}$ denotes a data matrix whose last column contains class labels of the data, where m and n represent the number of samples and the number of attributes in data matrix, respectively. $(D_{train})_{m_1\times n}$, $C_{m_1\times 1}$, and $(D_{test})_{m_2\times n}$ stand for a training matrix, class matrix of the training matrix, and the testing matrix obtained from the data matrix D, respectively, such that $m_1 + m_2 = m$. Let $U_{k\times 1}$ be a matrix consisting of unique class labels of $C_{m_1\times 1}$. $D_{i-train}$ and D_{i-test} denote i^{th} rows of D_{train} and D_{test}, respectively. Similarly, $D_{train-j}$ and D_{test-j} denote j^{th} rows of D_{train} and D_{test}, respectively. Moreover, $T_{m_2\times 1}$ represents predicted class labels of the testing samples, $I_n := \{1,2,3,\dots,n\}$, and $I_n^* := \{0,1,2,\dots,n\}$.

Definition 3.1. Let $u, v \in \mathbb{R}^n$. Then, the function $P: \mathbb{R}^n \times \mathbb{R}^n \to [-1,1]$ defined by

$$P(u,v) := \frac{n\sum_{i=1}^{n} u_i v_i - \left(\sum_{i=1}^{n} u_i\right)\left(\sum_{i=1}^{n} v_i\right)}{\sqrt{\left[n\sum_{i=1}^{n} u_i^2 - \left(\sum_{i=1}^{n} u_i\right)^2\right]\left[n\sum_{i=1}^{n} v_i^2 - \left(\sum_{i=1}^{n} v_i\right)^2\right]}}$$

is called the Pearson correlation coefficient between u and v.

Definition 3.2. Let $u \in \mathbb{R}^n$. Then, the vector $\hat{u} \in \mathbb{R}^n$ defined by

$$\hat{u}_i := \begin{cases} \dfrac{u_i - \min\limits_{k\in I_n}\{u_k\}}{\max\limits_{k\in I_n}\{u_k\} - \min\limits_{k\in I_n}\{u_k\}}, & \max\limits_{k\in I_n}\{u_k\} \neq \min\limits_{k\in I_n}\{u_k\} \\ 1, & \max\limits_{k\in I_n}\{u_k\} = \min\limits_{k\in I_n}\{u_k\} \end{cases}$$

is called normalizing vector of u.

Definition 3.3. Let $D = [d_{ij}]_{m\times(n+1)}$ be a data matrix, $i \in I_m$, and $j \in I_n$. Then, the matrix $\tilde{D} = [\tilde{d}_{ij}]_{m\times n}$ defined by

$$\tilde{d}_{ij} := \begin{cases} \dfrac{d_{ij} - \min\limits_{k\in I_m}\{d_{kj}\}}{\max\limits_{k\in I_m}\{d_{kj}\} - \min\limits_{k\in I_m}\{d_{kj}\}}, & \max\limits_{k\in I_m}\{d_{kj}\} \neq \min\limits_{k\in I_m}\{d_{kj}\} \\ 1, & \max\limits_{k\in I_m}\{d_{kj}\} = \min\limits_{k\in I_m}\{d_{kj}\} \end{cases}$$

is called column normalized matrix (feature-fuzzification matrix) of D.

Definition 3.4. Let $(D_{train})_{m_1\times n}$ be a training matrix obtained from $D = [d_{ij}]_{m\times(n+1)}$. Then, the matrix $\tilde{D}_{train} = [\tilde{d}_{ij-train}]_{m_1\times n}$ defined by

$$\tilde{d}_{ij-train} := \begin{cases} \dfrac{d_{ij-train} - \min\limits_{k\in I_m}\{d_{kj}\}}{\max\limits_{k\in I_m}\{d_{kj}\} - \min\limits_{k\in I_m}\{d_{kj}\}}, & \max\limits_{k\in I_m}\{d_{kj}\} \neq \min\limits_{k\in I_m}\{d_{kj}\} \\ 1, & \max\limits_{k\in I_m}\{d_{kj}\} = \min\limits_{k\in I_m}\{d_{kj}\} \end{cases}, \qquad i \in I_{m_1} \text{ and } j \in I_n$$

is called column normalized matrix (feature-fuzzification matrix) of D_{train}.

Definition 3.5. Let $(D_{test})_{m_2 \times n}$ be a training matrix obtained from $D = [d_{ij}]_{m\times(n+1)}$. Then, the matrix $\widetilde{D}_{test} = [\tilde{d}_{ij-test}]_{m_2\times n}$ defined by

$$\tilde{d}_{ij-test} \coloneqq \begin{cases} \dfrac{d_{ij-test} - \min\limits_{k\in I_m}\{d_{kj}\}}{\max\limits_{k\in I_m}\{d_{kj}\} - \min\limits_{k\in I_m}\{d_{kj}\}}, & \max\limits_{k\in I_m}\{d_{kj}\} \neq \min\limits_{k\in I_m}\{d_{kj}\} \\ 1, & \max\limits_{k\in I_m}\{d_{kj}\} = \min\limits_{k\in I_m}\{d_{kj}\} \end{cases}, \qquad i \in I_{m_2} \text{ and } j \in I_n$$

is called column normalized matrix (feature-fuzzification matrix) of D_{test}.

Definition 3.6. Let $D_{train} = [d_{ij-train}]_{m_1\times n}$ and $C_{m_1\times 1}$ be a training matrix and its class matrix obtained from a data matrix D, respectively. Then, the matrix $ifw^{\lambda P}_{D_{train}} = \begin{bmatrix} \mu^{\lambda P}_{1j} \\ \nu^{\lambda P}_{1j} \end{bmatrix}$ is called intuitionistic fuzzification weight matrix based on Pearson correlation coefficient of D_{train} and is defined by

$$\mu^{\lambda P}_{1j} \coloneqq 1 - \left(1 - \left|P(D_{train-j}, C)\right|\right)^{\lambda}$$

and

$$\nu^{\lambda P}_{1j} \coloneqq \left(1 - \left|P(D_{train-j}, C)\right|\right)^{\lambda(\lambda+1)}$$

such that $j \in I_n$ and $\lambda \in [0, \infty)$.

Definition 3.7. Let $\widetilde{D}_{train} = [\tilde{d}_{ij-train}]_{m_1\times n}$ be a training matrix obtained from a data matrix D. Then, the matrix $\widetilde{\widetilde{D}}^{\lambda}_{train} = \left[\tilde{\tilde{d}}^{\lambda}_{train-ij}\right] = \begin{bmatrix} \mu^{\widetilde{\widetilde{D}}^{\lambda}}_{ij-train} \\ \nu^{\widetilde{\widetilde{D}}^{\lambda}}_{ij-train} \end{bmatrix}$ is called intuitionistic fuzzification of $\widetilde{D}_{train}$ and is defined by

$$\mu^{\widetilde{\widetilde{D}}^{\lambda}}_{ij-train} \coloneqq 1 - \left(1 - \tilde{d}_{ij-train}\right)^{\lambda}$$

and

$$\nu^{\widetilde{\widetilde{D}}^{\lambda}}_{ij-train} \coloneqq \left(1 - \tilde{d}_{ij-train}\right)^{\lambda(\lambda+1)}$$

such that $i \in I_{m_1}$, $j \in I_n$, and $\lambda \in [0, \infty)$.

Definition 3.8. Let $\widetilde{D}_{test} = [\tilde{d}_{ij-test}]_{m_2\times n}$ be a testing matrix obtained from a data matrix D. Then, the matrix $\widetilde{\widetilde{D}}^{\lambda}_{test} = \left[\tilde{\tilde{d}}^{\lambda}_{test-ij}\right] = \begin{bmatrix} \mu^{\lambda\widetilde{\widetilde{D}}}_{ij-test} \\ \nu^{\lambda\widetilde{\widetilde{D}}}_{ij-test} \end{bmatrix}$ is called intuitionistic fuzzification of $\widetilde{D}_{test}$ and is defined by

$$\mu^{\lambda\widetilde{\widetilde{D}}}_{ij-test} \coloneqq 1 - \left(1 - \tilde{d}_{ij-test}\right)^{\lambda}$$

and

$$\nu^{\lambda\widetilde{\widetilde{D}}}_{ij-test} \coloneqq \left(1 - \tilde{d}_{ij-test}\right)^{\lambda(\lambda+1)}$$

such that $i \in I_{m_2}$, $j \in I_n$, and $\lambda \in [0, \infty)$.

Definition 3.9. Let $(\tilde{D}_{test})_{m_2\times n}$ be the column normalized matrix of a matrix $(D_{test})_{m_2\times n}$ and $\tilde{\tilde{D}}^{\lambda}_{test} = \left[\tilde{\tilde{d}}^{\lambda}_{test-ij}\right] = \begin{bmatrix}\mu^{\lambda\tilde{\tilde{D}}}_{ij-test} \\ \nu^{\lambda\tilde{\tilde{D}}}_{ij-test}\end{bmatrix}$ be the intuitionistic fuzzification of $\tilde{D}_{test}$. Then, the *ifpifs*-matrix $\left[a^{\tilde{\tilde{D}}^{\lambda}_{k-test}}_{ij}\right]_{2\times n}$ is called the *ifpifs*-matrix obtained by k^{th} row of $\tilde{\tilde{D}}^{\lambda}_{test}$ and $ifw^{\lambda P}_{D_{train}}$ and is defined by

$$a^{\tilde{\tilde{D}}^{\lambda}_{k-test}}_{0j} := \begin{matrix}\mu^{\lambda P}_{1j} \\ \nu^{\lambda P}_{1j}\end{matrix} \quad \text{and} \quad a^{\tilde{\tilde{D}}^{\lambda}_{k-test}}_{1j} := \begin{matrix}\mu^{\lambda\tilde{\tilde{D}}}_{kj-test} \\ \nu^{\lambda\tilde{\tilde{D}}}_{kj-test}\end{matrix}$$

such that $k \in I_{m_1}$ and $j \in I_n$.

Definition 3.10. Let $(\tilde{D}_{train})_{m_1\times n}$ be the column normalized matrix of a matrix $(D_{train})_{m_1\times n}$ and $\tilde{\tilde{D}}^{\lambda}_{train} = \left[\tilde{\tilde{d}}^{\lambda}_{train-ij}\right] = \begin{bmatrix}\mu^{\lambda\tilde{\tilde{D}}}_{ij-train} \\ \nu^{\lambda\tilde{\tilde{D}}}_{ij-train}\end{bmatrix}$ be the intuitionistic fuzzification of $\tilde{D}_{train}$. Then, the *ifpifs*-matrix $\left[b^{\tilde{\tilde{D}}^{\lambda}_{k-train}}_{ij}\right]_{2\times n}$ is called the *ifpifs*-matrix obtained by k^{th} row of $\tilde{\tilde{D}}^{\lambda}_{train}$ and $ifw^{\lambda P}_{D_{train}}$ and is defined by

$$b^{\tilde{\tilde{D}}^{\lambda}_{k-train}}_{0j} := \begin{matrix}\mu^{\lambda P}_{1j} \\ \nu^{\lambda P}_{1j}\end{matrix} \quad \text{and} \quad b^{\tilde{\tilde{D}}^{\lambda}_{k-train}}_{1j} := \begin{matrix}\mu^{\lambda\tilde{\tilde{D}}}_{kj-train} \\ \nu^{\lambda\tilde{\tilde{D}}}_{kj-train}\end{matrix}$$

such that $k \in I_{m_2}$ and $j \in I_n$.

Secondly, we define the concept of pseudo-similarity of *ifpifs*-matrices and propose Hamming pseudo-similarity of *ifpifs*-matrices.

Definition 3.11. Let $s: IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ be a mapping. Then, for all $[a_{ij}], [b_{ij}], [c_{ij}] \in IFPIFS_E[U]$, s is a pseudo-similarity over $IFPIFS_E[U]$ if and only if s satisfies the following properties:

i. $s([a_{ij}],[a_{ij}]) = 1$

ii. $s([a_{ij}],[b_{ij}]) = s([b_{ij}],[a_{ij}])$

iii. $0 \leq s([a_{ij}],[b_{ij}]) \leq 1$

Proposition 3.12. The mapping $s_H: IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ defined by

$$s_H([a_{ij}],[b_{ij}]) := 1 - \frac{1}{2(m-1)n}\sum_{i=1}^{m-1}\sum_{j=1}^{n}\left(\left|\mu^a_{0j}\mu^a_{ij} - \mu^b_{0j}\mu^b_{ij}\right| + \left|\nu^a_{0j}\nu^a_{ij} - \nu^b_{0j}\nu^b_{ij}\right| + \left|\pi^a_{0j}\pi^a_{ij} - \pi^b_{0j}\pi^b_{ij}\right|\right)$$

is a pseudo-similarity over $IFPIFS_E[U]$ and is called Hamming pseudo-similarity.

Thirdly, we propose a new classification method referred to as Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Hamming Classifier (IFPIFS-HC). This method employs Definition 3.6 to obtain feature weight based on the effect of parameters on classification. It then constructs the training *ifpifs*-matrix and the testing *ifpifs*-matrix using Definitions 3.4, 3.5, 3.7, 3.8, 3.9, and 3.10. Then, utilising the Hamming pseudo-similarity, a matrix of similarity values of the testing *ifpifs*-matrix to each training *ifpifs*-matrix is obtained. The class of the training sample with the highest similarity is assigned as the probable class of the test sample. This process proceeds similarly for all test samples. Finally, the predicted class matrix is generated for the test data. IFPIFS-HC's flowchart (Figure 1) and algorithm steps (Algorithm 1) are as follows:

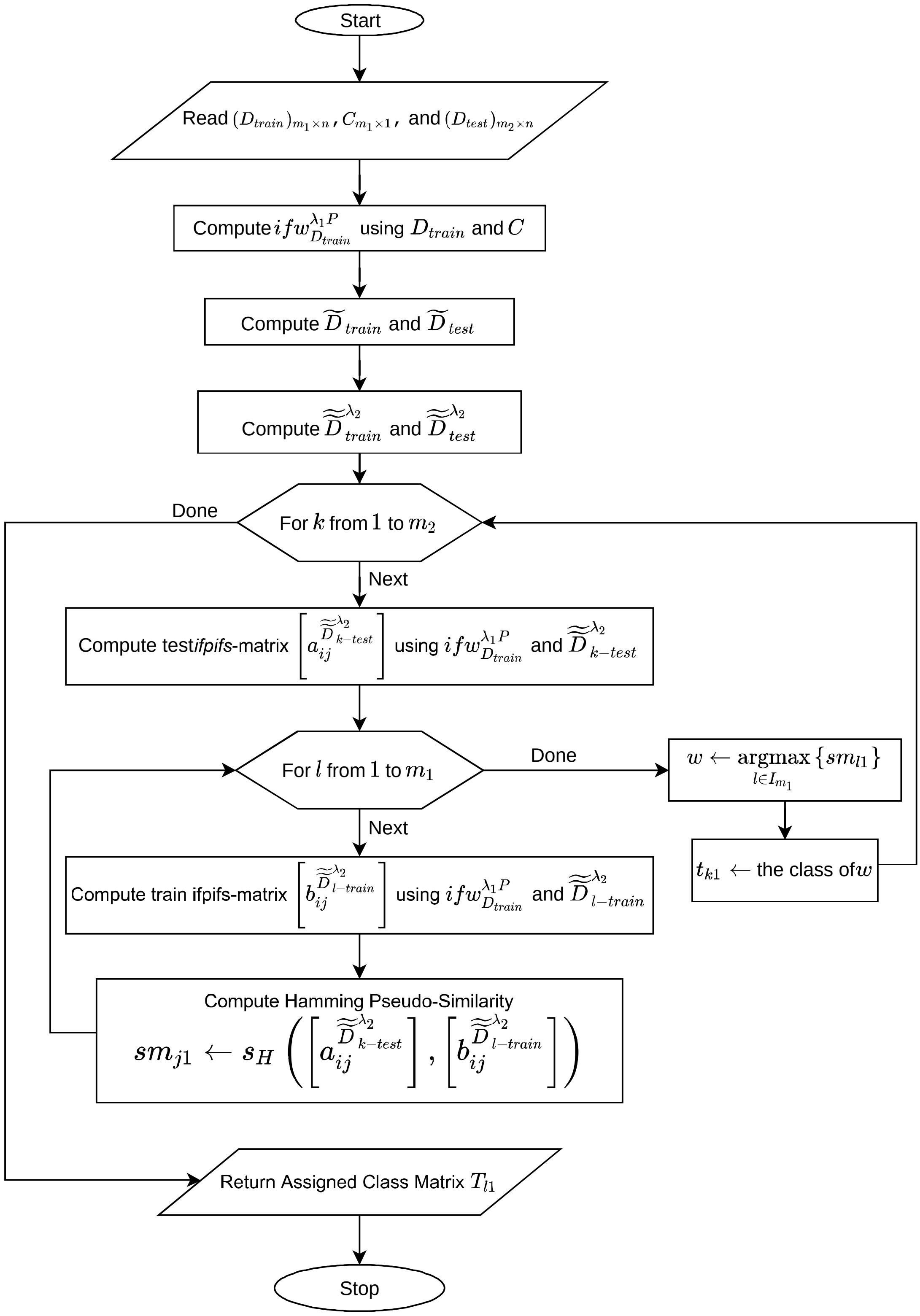

Figure 1. The flowchart of the IFPIFS-HC

Algorithm 1. Pseudocode of IFPIFS-HC Algorithm

Input: $(D_{train})_{m_1 \times n}$, $C_{m_1 \times 1}$, $(D_{test})_{m_2 \times n}$, λ_1, and λ_2

Output: $T_{m_2 \times 1}$

1: **procedure** IFPIFS-HC$(D_{train}, C, D_{test}, \lambda_1, \lambda_2)$

2: Compute $ifw_{D_{train}}^{\lambda_1 P}$ using D_{train} and C

3: Compute feature fuzzification of D_{train} and D_{test}, namely $\widetilde{D}_{train}$ and $\widetilde{D}_{test}$

4: Compute feature intuitionistic fuzzification of $\widetilde{D}_{train}$ and $\widetilde{D}_{test}$, namely $\widetilde{\widetilde{D}}_{train}^{\lambda_2}$ and $\widetilde{\widetilde{D}}_{test}^{\lambda_2}$

5: **for** k from 1 to m_2 **do**

6: Compute the test *ifpifs*-matrix $\left[a_{ij}^{\widetilde{\widetilde{D}}_{k-test}^{\lambda_2}}\right]$ using $ifw_{D_{train}}^{\lambda_1 P}$ and $\widetilde{\widetilde{D}}_{k-test}^{\lambda_2}$

7: **for** l from 1 to m_1 **do**

8: Compute the train *ifpifs*-matrix $\left[b_{ij}^{\widetilde{\widetilde{D}}_{l-train}^{\lambda_2}}\right]$ using $ifw_{D_{train}}^{\lambda_1 P}$ and $\widetilde{\widetilde{D}}_{l-train}^{\lambda_2}$

9: $sm_{l1} \leftarrow s_H\left(\left[a_{ij}^{\widetilde{\widetilde{D}}_{k-test}^{\lambda_2}}\right], \left[b_{ij}^{\widetilde{\widetilde{D}}_{l-train}^{\lambda_2}}\right]\right)$ ▻ $[sm_{l1}]$ stands for similarity matrix

10: **end for**

11: $w \leftarrow \underset{l \in I_{m_1}}{\operatorname{argmax}}\{sm_{l1}\}$

12: $t_{k1} \leftarrow$ the class of w

13: **end for**

14: **return** $T_{m_2 \times 1}$

15: **end procedure**

4. Experimental Study

This section provides the details of the 21 classification datasets in the UCI Machine Learning Repository [29]. It then presents five performance metrics for classification in machine learning. Afterwards, it performs a simulation to manifest that IFPIFS-HC is more efficient than Fuzzy kNN [30], FSSC [31], FussCyier [32], HDFSSC [33], and FPFS-EC [34]. Finally, it carries out statistical analyses of the simulation results utilising the Friedman test [35] and the Nemenyi post-hoc test [36].

4.1. UCI Datasets

Table 1 presents the details of UCI classification datasets employed in the experiment herein: "Zoo", "Coimbra", "Teaching Assistant Evaluation", "Wine", "Sonar", "Glass", "Vertebral Column 3C", "Leaf", "Ionosphere", "Dermatology", "Wholesale Customers", "Breast Cancer Wisconsin", "HCV Data", "Parkinson's Disease", "Vehicle", "German Credit Data", "Mice Protein Expression", "Semeion Handwritten Digit", "Car Evaluation", "Wireless Indoor Localization", and "Image Segmentation".

Table 1. Details of UCI datasets (# represents "the number of ")

No.	Dataset	# Sample	# Attribute	# Class	Balanced/Imbalanced
1.	Zoo	101	16	7	Imbalanced
2.	Coimbra	116	9	2	Imbalanced
3.	Teaching Assistant Evaluation	151	5	3	Imbalanced
4.	Wine	178	13	3	Imbalanced
5.	Sonar	208	60	2	Imbalanced
6.	Glass	214	9	6	Imbalanced
7.	Vertebral Column 3C	310	6	3	Imbalanced
8.	Leaf	340	14	36	Imbalanced
9.	Ionosphere	351	34	2	Imbalanced
10.	Dermatology	366	34	6	Imbalanced
11.	Wholesale Customers	440	6	3	Imbalanced
12.	Breast Cancer Wisconsin	569	30	2	Imbalanced
13.	HCV Data	589	12	5	Imbalanced
14.	Parkinson's Disease	756	754	2	Imbalanced
15.	Vehicle	846	17	4	Imbalanced
16.	German Credit Data	1000	20	2	Imbalanced
17.	Mice Protein Expression	1077	72	8	Imbalanced
18.	Semeion Handwritten Digit	1593	265	2	Imbalanced
19.	Car Evaluation	1728	6	4	Imbalanced
20.	Wireless Indoor Localization	2000	7	4	Balanced
21.	Image Segmentation (Segment)	2310	19	7	Balanced

4.2. Performance Criteria

This subsection provides the mathematical notations of the performance criteria, namely accuracy (Acc), precision (Pre), recall (Rec), macro F-score (MacF), and micro F-score (MicF) [37, 38] to compare the aforesaid classifiers. Let $X = \{x_1, x_2, \cdots, x_n\}$, $\mathbb{Y} = \{\mathbb{Y}_1, \mathbb{Y}_2, \cdots, \mathbb{Y}_n\}$, $\widehat{\mathbb{Y}} = \{\widehat{\mathbb{Y}}_1, \widehat{\mathbb{Y}}_2, \cdots, \widehat{\mathbb{Y}}_n\}$, and k be the set of n samples to be classified, the set of ground truth class of the samples, the set of prediction class of the samples, and the number of the class of the samples, respectively. Then,

$$\mathrm{Acc}(\mathbb{Y}, \widehat{\mathbb{Y}}) := \frac{1}{k}\sum_{i=1}^{k} \frac{TP_i + TN_i}{TP_i + TN_i + FP_i + FN_i}$$

$$\mathrm{Pre}(\mathbb{Y}, \widehat{\mathbb{Y}}) := \frac{1}{k}\sum_{i=1}^{k} \frac{TP_i}{TP_i + FP_i}$$

$$\mathrm{Rec}(\mathbb{Y}, \widehat{\mathbb{Y}}) := \frac{1}{k}\sum_{i=1}^{k} \frac{TP_i}{TP_i + FN_i}$$

$$\mathrm{MacF}(\mathbb{Y}, \widehat{\mathbb{Y}}) := \frac{1}{k}\sum_{i=1}^{k} \frac{2TP_i}{2TP_i + FP_i + FN_i}$$

$$\mathrm{MicF}(\mathbb{Y}, \widehat{\mathbb{Y}}) := \frac{2\sum_{i=1}^{k} TP_i}{2\sum_{i=1}^{k} TP_i + \sum_{i=1}^{k} FP_i + \sum_{i=1}^{k} FN_i}$$

where TP_i, TN_i, FP_i, and FN_i are the number of true positive, true negative, false positive, and false negative for the class i, respectively, and their mathematical notations are as follows:

$$TP_i := \left|\left\{x_t : i \in \mathbb{Y}_t \wedge i \in \widehat{\mathbb{Y}}_t, t \in I_n\right\}\right|$$

$$TN_i := \left|\left\{x_t : i \notin \mathbb{Y}_t \wedge i \notin \widehat{\mathbb{Y}}_t, t \in I_n\right\}\right|$$

$$FP_i := \left|\left\{x_t : i \notin \mathbb{Y}_t \wedge i \in \widehat{\mathbb{Y}}_t, t \in I_n\right\}\right|$$

$$FN_i := \left|\left\{x_t : i \in \mathbb{Y}_t \wedge i \notin \widehat{\mathbb{Y}}_t, t \in I_n\right\}\right|$$

4.3. Simulation Results

In this subsection, we concentrate mainly on comparing IFPIFS-HC with the well-known and state-of-the-art classifiers based on fuzzy sets and soft sets, i.e., Fuzzy-kNN [30], FSSC [31], FussCyier [32], HDFSSC [33], and FPFS-EC [34]. We carry out a simulation utilising MATLAB R2021a software and a laptop with I(R) Core(TM) I5-3230M CPU @ 2.60GHz and 16 GB RAM. We use 5-fold cross-validation herein to split the data sets as training and testing. In 5-fold cross-validation, the datasets are split into 5 parts of equal size. This process occurs randomly. While a part of these 5 parts is used as validation data (test data), the remaining 4 parts are used as training data to train the classifier. Because the cross-validation process is repeated 5 times, each part is used as test data only once. Therefore, all samples in the dataset are used as both training and test data. In this phase, we record the mean results for 5 iterations. In each iteration in 5-cross-validation, the training and testing phases are carried out independently from other stages (for more details about k-fold cross-validation, see [39,40]). We finally repeat this process 10 times and obtain the mean Acc, Pre, Rec, MacF, MicF, and running time results.

Table 2 shows the average Acc, Pre, Rec, MacF, MicF, and running time results of the classifiers for the datasets. In Table 2, it is observed that IFPIFS-HC has classified “Mice Protein” dataset in a maximum classification performance as in HDFSSC and FPFS-EC. Moreover, according to all performance metrics, the performance results of IFPIFS-HC for “Breast Cancer Wisconsin”, “Ionosphere”, “Parkinson’s Disease”, “Wine”, and “Zoo” datasets are over 95%, 90%, 93%, 97%, and 94%, respectively. On the other hand, although IFPIFS-HC does not produce the best results in all performance metrics in “Segment”, “Sonar”, and “Wireless” datasets, it produces the closest results to the best ones for these datasets. For “Segment” dataset, the approximate differences between the results are 0.06, 0.2, 0.2, 0.2, and 0.2, for “Sonar” datasets, 1.5, 1.74, 1.4, 1.46, and 1.5, and for “Wireless” datasets, 0.07, 0.14, 0.13, 0.14, and 0.13. Also, IFPIFS-HC has better performance values than the others in all performance metrics for “Car Evaluation”, “Coimbra”, “Glass”, “Leaf”, “Teaching Assistant Evaluation”, and “Wholesale Customers” datasets. Although IFPIFS-HC does not have the best performance according to some performance metrics in the other datasets, it produces the closest results to the best performances, except for Rec values in “HCV” and “German Credit” datasets.

IFPIFS-HC achieves remarkable classification success by using the Hamming pseudo-similarity of *ifpifs*-matrices based on the Pearson correlation coefficient and evaluating all training samples separately. On the other hand, evaluating all training samples separately leads to an increase in the running time of IFPIFS-HC. Therefore, IFPIFS-HC is generally a bit slower than the other classifiers. On the other hand, it takes approximately 0.02349 to 8.55762 seconds to classify all test samples in a considered dataset. The detailed numerical results mentioned above are presented in Table 2.

Table 2. Performance comparison of the classifiers for the 21 UCI datasets

Datasets	Classifiers	Acc±SD	Pre±SD	Rec±SD	MacF±SD	MicF±SD	Running Time±SD
Breast Cancer Wisconsin	Fuzzy kNN	92.17±2.37	92.19±2.53	91.11±2.74	91.52±2.59	92.17±2.37	0.00972±0.00203
	FSSC	93.61±2.05	93.41±2.25	92.99±2.29	93.14±2.20	93.61±2.05	0.00141±0.00191
	FussCyier	93.56±2.04	94.32±1.93	92.03±2.55	92.93±2.29	93.56±2.04	0.00074±0.00088
	HDFSSC	92.88±2.09	93.07±2.20	91.71±2.49	92.26±2.30	92.88±2.09	0.00105±0.00166
	FPFS-EC	95.18±1.66	94.99±1.77	94.78±1.93	94.84±1.80	95.18±1.66	0.11592±0.02538
	IFPIFS-HC(10,0.5)	**95.65±1.55**	**95.53±1.67**	**95.22±1.77**	**95.33±1.67**	**95.65±1.55**	0.45639±0.08281
Car Evaluation	Fuzzy kNN	94.26±0.71	79.00±3.20	62.29±4.63	66.97±4.85	88.52±1.42	0.04783±0.00367
	FSSC	72.11±1.37	37.98±1.64	56.78±4.10	34.18±2.25	44.21±2.74	0.00467±0.00077
	FussCyier	80.20±1.31	44.12±2.13	64.17±4.61	44.87±2.73	60.40±2.61	0.00142±0.00036
	HDFSSC	86.68±1.08	55.59±2.51	76.35±4.38	60.44±3.14	73.37±2.17	0.00288±0.00069
	FPFS-EC	97.48±0.55	90.17±3.25	89.11±3.64	89.30±3.25	94.95±1.09	0.99152±0.12451
	IFPIFS-HC(0.5,5)	**98.02±0.51**	**90.88±2.88**	**90.47±3.58**	**90.40±2.74**	**96.05±1.03**	4.10788±0.39287
Coimbra	Fuzzy kNN	55.11±9.51	54.61±10.31	54.28±9.52	53.60±9.79	55.11±9.51	0.00068±0.00033
	FSSC	62.93±8.09	67.69±8.92	64.89±7.89	61.85±8.73	62.93±8.09	0.00037±0.00019
	FussCyier	61.66±8.26	68.37±9.46	64.13±7.87	59.91±9.39	61.66±8.26	0.00021±0.0001
	HDFSSC	60.02±8.47	63.28±9.67	61.60±8.32	59.03±9.03	60.02±8.47	0.00027±0.00015
	FPFS-EC	68.00±9.42	68.66±9.79	67.83±9.38	67.33±9.60	68.00±9.42	0.0069±0.00206
	IFPIFS-HC(5,0.5)	**69.81±8.93**	**70.28±9.20**	**69.82±9.09**	**69.32±9.19**	**69.81±8.93**	0.02349±0.00692
Dermatology	Fuzzy kNN	91.33±1.22	77.82±4.26	72.07±4.41	72.21±4.34	74.00±3.67	0.00582±0.00973
	FSSC	99.17±0.53	**97.42±1.70**	97.22±1.77	**97.21±1.79**	97.52±1.60	0.00186±0.00201
	FussCyier	98.66±0.68	95.94±1.94	96.42±1.92	95.90±2.11	95.97±2.05	0.00064±0.00041
	HDFSSC	98.84±0.67	96.39±2.03	96.42±2.05	96.22±2.17	96.52±2.02	0.00126±0.00225
	FPFS-EC	97.89±0.78	93.73±2.32	93.49±2.45	93.20±2.49	93.68±2.34	0.05192±0.02404
	IFPIFS-HC(5,10)	**99.20±0.56**	97.38±1.95	**97.30±1.89**	97.20±2.01	**97.59±1.69**	0.19764±0.05524
German Credit	Fuzzy kNN	61.71±2.80	52.33±3.50	52.08±3.15	51.97±3.31	61.71±2.80	0.02643±0.01081
	FSSC	63.44±3.02	62.09±2.82	**64.22±3.33**	61.20±2.99	63.44±3.02	0.00273±0.00403
	FussCyier	63.54±3.04	62.05±2.81	64.16±3.31	61.23±2.99	63.54±3.04	0.00103±0.00064
	HDFSSC	64.74±3.40	61.79±3.08	63.53±3.53	61.59±3.34	64.74±3.40	0.0025±0.00745
	FPFS-EC	69.00±2.84	63.11±3.31	62.96±3.33	62.94±3.29	69.00±2.84	0.44546±0.26546
	IFPIFS-HC(20,0.5)	**69.91±2.66**	**63.69±3.41**	62.83±3.20	**63.09±3.26**	**69.91±2.66**	1.79845±0.57353
Glass	Fuzzy kNN	89.18±1.74	56.95±9.72	55.93±8.62	67.87±7.07	63.66±5.95	0.00178±0.0015
	FSSC	79.87±1.92	48.22±7.77	49.36±7.12	48.08±6.44	39.61±5.75	0.00135±0.00091
	FussCyier	79.80±1.94	49.09±8.05	49.33±6.82	47.89±6.53	39.39±5.83	0.00047±0.00031
	HDFSSC	79.95±2.24	45.66±8.23	45.48±8.50	48.97±6.95	39.85±6.73	0.00079±0.00041
	FPFS-EC	89.33±1.99	67.50±9.26	65.56±8.76	69.56±7.73	68.00±5.97	0.02374±0.01285
	IFPIFS-HC(5,0.5)	**90.87±1.96**	**70.72±9.6**	**69.3±9.17**	**72.47±7.78**	**72.62±5.89**	0.09159±0.04064
HCV Data	Fuzzy kNN	97.12±0.59	54.67±10.84	47.26±11.14	66.49±9.29	92.81±1.48	0.00642±0.00076
	FSSC	97.18±0.87	63.90±11.02	62.87±11.36	69.93±8.95	92.94±2.17	0.00215±0.00068
	FussCyier	97.19±0.83	64.54±11.77	61.40±11.48	70.36±8.71	92.98±2.08	0.00068±0.00023
	HDFSSC	96.53±1.34	62.55±10.48	**64.21±11.34**	68.29±8.89	91.32±3.34	0.00125±0.00037
	FPFS-EC	97.03±0.54	59.19±13.22	45.85±9.26	**80.01±9.97**	92.58±1.34	0.12004±0.00989
	IFPIFS-HC(20,0.5)	**97.29±0.54**	**72.68±13.76**	49.61±11.06	78.21±9.39	**93.22±1.34**	0.49525±0.0394
Ionosphere	Fuzzy kNN	84.88±3.22	88.99±2.92	79.48±4.35	81.58±4.41	84.88±3.22	0.00629±0.0025
	FSSC	64.10±0.36	64.10±0.36	50.00±0.00	78.13±0.27	64.10±0.36	0.00125±0.00152
	FussCyier	64.10±0.36	64.10±0.36	50.00±0.00	78.13±0.27	64.10±0.36	0.00063±0.00033
	HDFSSC	64.10±0.36	64.10±0.36	50.00±0.00	78.13±0.27	64.10±0.36	0.00088±0.0007
	FPFS-EC	89.46±3.17	91.81±2.76	85.88±4.24	87.70±3.97	89.46±3.17	0.07383±0.03865
	IFPIFS-HC(0.5,5)	**91.9±2.97**	**92.29±3.07**	**90.15±3.72**	**90.95±3.41**	**91.90±2.97**	0.29221±0.12971
Leaf	Fuzzy kNN	96.18±0.24	32.05±5.36	31.89±4.11	61.48±4.21	32.43±4.09	0.00273±0.00061
	FSSC	97.42±0.36	66.55±5.98	61.85±5.46	70.69±4.18	61.35±5.37	0.00637±0.00117
	FussCyier	97.44±0.35	66.82±5.81	62.12±5.33	70.83±3.96	61.63±5.24	0.00122±0.00119
	HDFSSC	97.53±0.36	68.16±6.21	63.60±5.64	71.83±3.79	63.00±5.47	0.00313±0.00052
	FPFS-EC	97.82±0.30	71.41±5.14	67.32±4.74	74.38±3.37	67.23±4.57	0.04165±0.00366
	IFPIFS-HC(5,0.5)	**98.1±0.32**	**76.11±4.94**	**71.08±4.86**	**76.89±3.9**	**71.46±4.85**	0.16729±0.01129
Mice Protein	Fuzzy kNN	99.90±0.1	99.62±0.39	99.60±0.42	99.60±0.42	99.60±0.42	0.05815±0.06803
	FSSC	98.66±0.41	94.95±1.56	94.85±1.59	94.78±1.61	94.64±1.65	0.01023±0.01476
	FussCyier	98.73±0.41	95.24±1.51	95.14±1.56	95.07±1.58	94.90±1.63	0.00284±0.00274
	HDFSSC	**100±0.00**	**100±0.00**	**100±0.00**	**100±0.00**	**100±0.00**	0.00717±0.01646
	FPFS-EC	**100±0.00**	**100±0.00**	**100±0.00**	**100±0.00**	**100±0.00**	0.67986±0.58764
	IFPIFS-HC(20,20)	**100±0.00**	**100±0.00**	**100±0.00**	**100±0.00**	**100±0.00**	2.5347±1.9008
Parkinson's Disease	Fuzzy kNN	70.52±3.12	60.54±4.02	59.84±3.66	60.02±3.76	70.52±3.12	0.4102±0.05782
	FSSC	37.87±6.48	46.85±4.35	47.03±4.43	37.25±6.11	37.87±6.48	0.01531±0.00212
	FussCyier	61.37±16.56	46.17±6.02	48.70±2.18	46.36±15.32	61.37±16.56	0.01376±0.0018
	HDFSSC	61.81±16.49	46.77±6.89	48.86±2.37	47.13±15.65	61.81±16.49	0.01465±0.00248
	FPFS-EC	93.76±2.07	91.94±2.83	91.70±3.00	91.75±2.75	93.76±2.07	0.86699±0.14823
	IFPIFS-HC(20,5)	**95.09±1.94**	**93.99±2.70**	**93.06±2.89**	**93.45±2.62**	**95.09±1.94**	3.60079±0.47408

Segment	Fuzzy kNN	98.30±0.28	94.31±0.95	94.06±1.00	94.06±1.00	94.06±1.00	0.09524±0.03241
	FSSC	93.78±0.43	78.90±1.69	78.22±1.49	76.80±1.65	78.22±1.49	0.01163±0.00534
	FussCyier	94.60±0.69	81.26±2.53	81.11±2.42	80.99±2.48	81.11±2.42	0.00289±0.00124
	HDFSSC	92.48±0.53	73.60±1.87	73.67±1.86	73.12±1.88	73.67±1.86	0.00673±0.00318
	FPFS-EC	**99.34±0.20**	**97.73±0.70**	**97.70±0.71**	**97.70±0.71**	**97.70±0.71**	1.91221±0.36154
	IFPIFS-HC(5,0.5)	99.28±0.18	97.53±0.63	97.50±0.64	97.50±0.64	97.5±0.64	7.78665±0.93521
Semeion	Fuzzy kNN	97.20±0.71	**97.41±1.53**	86.73±3.40	91.13±2.51	97.20±0.71	0.70981±0.0554
	FSSC	43.96±2.41	57.50±0.34	68.82±1.39	40.47±1.84	43.96±2.41	0.01385±0.00439
	FussCyier	76.21±2.37	64.03±1.21	84.33±2.06	64.52±2.18	76.21±2.37	0.01133±0.00253
	HDFSSC	89.52±1.62	73.59±2.38	88.25±2.86	78.09±2.56	89.52±1.62	0.0127±0.0028
	FPFS-EC	96.58±0.98	92.07±3.18	88.29±3.58	89.97±2.97	96.58±0.98	1.93266±0.16265
	IFPIFS-HC(20,20)	**98.06±0.62**	97.23±1.74	**91.77±2.88**	**94.22±1.95**	**98.06±0.62**	8.55762±0.5436
Sonar	Fuzzy kNN	82.57±5.02	83.38±5.11	82.14±5.14	82.23±5.21	82.57±5.02	0.00322±0.00112
	FSSC	75.00±6.65	75.81±7.08	74.55±6.66	74.49±6.78	75.00±6.65	0.00082±0.00068
	FussCyier	72.12±6.95	73.81±6.83	72.78±6.81	71.87±7.10	72.12±6.95	0.00048±0.00029
	HDFSSC	70.14±7.66	70.60±7.78	70.19±7.69	69.90±7.74	70.14±7.66	0.00059±0.00043
	FPFS-EC	**86.77±4.78**	**87.61±4.66**	**86.41±4.92**	**86.53±4.92**	**86.77±4.78**	0.0323±0.01112
	IFPIFS-HC(5,0.5)	85.27±4.85	85.87±4.86	85.01±4.97	85.07±4.97	85.27±4.85	0.1067±0.03353
Teaching Assistant Evaluation	Fuzzy kNN	72.48±5.76	60.87±9.31	58.75±8.64	58.15±9.06	58.71±8.64	0.00071±0.00019
	FSSC	63.21±4.98	48.97±11.74	45.45±7.55	43.13±7.63	44.81±7.47	0.00048±0.00015
	FussCyier	63.13±5.09	48.12±11.5	45.31±7.72	42.93±7.50	44.70±7.64	0.00024±8e-05
	HDFSSC	69.63±5.18	56.02±8.46	54.71±7.81	53.83±7.81	54.45±7.78	0.00032±0.00012
	FPFS-EC	76.58±5.07	66.06±7.88	64.80±7.59	64.32±7.71	64.87±7.60	0.00883±0.00179
	IFPIFS-HC(5,0.5)	**76.95±5.56**	**66.62±8.74**	**65.36±8.30**	**64.91±8.57**	**65.43±8.34**	0.03474±0.00639
Vehicle	Fuzzy kNN	75.65±1.46	54.12±3.98	51.58±2.89	48.69±3.05	51.31±2.92	0.01841±0.01016
	FSSC	69.67±1.75	39.68±4.79	40.06±3.49	36.58±4.10	39.33±3.49	0.00284±0.00102
	FussCyier	69.79±1.74	40.05±4.96	40.37±3.48	36.63±4.15	39.59±3.48	0.001±0.00044
	HDFSSC	70.54±1.64	42.32±4.12	41.73±3.28	39.52±3.56	41.08±3.28	0.00192±0.00126
	FPFS-EC	83.95±1.34	**67.54±2.83**	68.22±2.67	67.71±2.71	67.89±2.68	0.27599±0.11182
	IFPIFS-HC(10,5)	**83.97±1.23**	67.53±2.66	**68.28±2.45**	**67.76±2.52**	**67.94±2.47**	1.12365±0.25498
Vertebral Column 3C	Fuzzy kNN	80.28±2.99	**75.50±7.96**	60.92±5.47	62.95±6.03	70.42±4.48	0.00191±0.00033
	FSSC	81.15±3.76	68.73±6.21	69.23±6.16	68.36±6.20	71.72±5.65	0.00078±0.00016
	FussCyier	79.91±4.08	67.51±6.26	68.76±6.45	67.27±6.51	69.87±6.12	0.00032±8e-05
	HDFSSC	80.57±3.69	69.47±5.78	69.86±5.92	69.01±5.86	70.85±5.54	0.00052±0.00015
	FPFS-EC	81.89±3.44	69.16±6.33	68.11±6.01	67.94±6.07	72.83±5.15	0.03267±0.00312
	IFPIFS-HC(10,5)	**84.59±3.10**	72.27±6.11	**71.26±5.77**	**71.25±5.91**	**76.88±4.65**	0.13379±0.00909
Wholesale Customers	Fuzzy kNN	64.81±2.81	33.01±6.99	32.08±5.42	36.07±6.41	47.21±4.21	0.0035±0.00059
	FSSC	55.40±4.27	33.80±4.05	33.46±6.27	27.98±4.86	33.10±6.41	0.00108±0.00028
	FussCyier	52.06±4.19	34.32±4.35	33.01±5.61	25.12±4.84	28.09±6.28	0.00044±0.00013
	HDFSSC	54.02±3.82	34.52±3.88	33.22±5.48	27.09±4.54	31.02±5.72	0.00073±0.00025
	FPFS-EC	70.45±2.87	32.84±4.82	33.05±4.42	38.18±8.67	55.67±4.30	0.06443±0.00536
	IFPIFS-HC(10,20)	**71.56±2.79**	**36.73±5.69**	**36.83±5.50**	**39.17±6.08**	**57.34±4.18**	0.26388±0.01403
Wine	Fuzzy kNN	82.34±4.17	74.09±7.18	72.13±6.26	72.39±6.27	73.50±6.26	0.00108±0.00035
	FSSC	96.38±2.43	94.98±3.15	95.47±3.02	94.78±3.52	94.57±3.64	0.00058±0.0002
	FussCyier	96.53±2.10	95.07±2.95	95.54±2.69	95.00±3.06	94.79±3.15	0.00027±8e-05
	HDFSSC	95.46±2.53	93.64±3.58	93.94±3.37	93.46±3.67	93.18±3.80	0.00041±0.00022
	FPFS-EC	97.51±1.96	96.43±2.72	96.89±2.46	96.41±2.90	96.27±2.94	0.01451±0.00288
	IFPIFS-HC(10,5)	**98.50±1.71**	**97.97±2.21**	**98.13±2.12**	**97.92±2.38**	**97.75±2.57**	0.05463±0.0095
Wireless Indoor Localization	Fuzzy kNN	**99.12±2.37**	**98.26±2.53**	**98.23±2.74**	**98.24±2.59**	**98.23±2.37**	0.05973±0.00203
	FSSC	97.49±2.05	95.41±2.25	94.97±2.29	94.97±2.20	94.97±2.05	0.00562±0.00191
	FussCyier	97.61±2.04	95.62±1.93	95.22±2.55	95.22±2.29	95.22±2.04	0.00155±0.00088
	HDFSSC	96.73±2.09	93.90±2.20	93.46±2.49	93.46±2.30	93.46±2.09	0.00326±0.00166
	FPFS-EC	94.76±1.66	89.63±1.77	89.53±1.93	89.53±1.80	89.53±1.66	1.37093±0.02538
	IFPIFS-HC(0.5,0.5)	99.05±1.55	98.12±1.67	98.10±1.77	98.10±1.67	98.10±1.55	5.82577±0.08281
Zoo	Fuzzy kNN	97.73±1.42	91.52±6.35	84.46±10.03	92.97±5.65	92.23±4.90	0.00131±0.00744
	FSSC	98.04±1.31	91.30±6.71	86.84±8.42	93.09±5.14	93.33±4.39	0.00095±0.00095
	FussCyier	97.78±1.37	90.66±6.74	86.52±8.39	92.51±5.28	92.43±4.63	0.00036±0.00028
	HDFSSC	98.38±1.19	92.96±5.96	88.20±8.96	93.89±4.85	94.52±3.96	0.00055±0.00051
	FPFS-EC	98.69±1.26	93.99±7.49	89.03±10.4	96.01±4.94	95.54±4.30	0.00818±0.00377
	IFPIFS-HC(0.5,0.5)	**99.25±1.04**	**97.03±4.71**	**94.74±7.39**	**97.51±3.77**	**97.49±3.44**	0.02383±0.01068
Mean Results	Fuzzy kNN	84.90±2.51	71.96±5.19	67.95±5.13	71.91±4.85	75.28±3.74	0.07005±0.01275
	FSSC	78.12±2.64	68.01±4.59	68.05±4.57	66.53±4.26	67.68±3.95	0.00411±0.00215
	FussCyier	80.76±3.16	68.63±4.81	69.07±4.56	68.36±4.82	70.65±4.51	0.00203±0.00072
	HDFSSC	81.93±3.17	69.43±4.65	69.95±4.68	70.25±4.78	72.36±4.47	0.00303±0.00208
	FPFS-EC	89.59±2.23	80.27±4.57	78.40±4.54	81.20±4.36	83.60±3.31	0.43193±0.09199
	IFPIFS-HC	**90.59±2.12**	**82.88±4.39**	**80.75±4.43**	**82.89±4.02**	**85.48±3.15**	1.79414±0.26701

Acc, Pre, Rec, MacF, MicF, and their standard deviations (SD) are presented in percentage. Running time and its SD are presented in seconds. The best results are shown in bold. In addition, IFPIFS-HC(λ_1, λ_2) denotes that IFPIFS-HC utilises the intuitionistic fuzzification values λ_1 and λ_2.

4.4. Statistical Evaluation

In this subsection, we conduct the Friedman test [35] and Nemenyi post-hoc test [36] in a procedure suggested by Demšar [41] to analyse overall performance results obtained in view of Acc, Pre, Rec, MacF, MicF, and running time. The Friedman test produces a performance-based ranking of the classifiers for each data set. Thereby, the rank of 1 refers to the best performing classifier, the rank of 2 to the second best, etc. If the performances of the classifiers are equal, then it assigns the average of their possible ranks to their ranks. Next, the Friedman test first compares the average ranks of the classifiers and secondly calculates the Friedman statistic χ_F^2, distributed according to the χ_F^2 distribution with $k-1$ degree of freedom where k is the number of classifiers. If a statistically significant difference is detected in the performance, a post-hoc test should be used to detect which difference belong to which classifier. The Nemenyi test is one of the post-hoc tests commonly used to compare all the classifiers with each other. In this test, if the distance of the average ranks of the two classifiers occurs more than the critical distance, then the test shows that their performance is considerably different.

In this subsection, firstly, since the number of classifiers compared is 6 and the number of datasets is 21, each classifier average ranking is calculated for $k = 6$ and $N = 21$ using the Friedman test. Friedman test statistics of Acc, Pre, Rec, MacF, MicF, and running time values, $\chi_2^F = 57.63$, $\chi_2^F = 45.74$, $\chi_2^F = 39.38$, $\chi_2^F = 56.39$, $\chi_2^F = 57.63$, and $\chi_2^F = 103.37$, respectively. For $k = 6$ and $N = 21$, the Friedman test critical value is 11.07 at the $\alpha = 0.05$ significance level (for more details, see [42]). Since the Friedman test statistics of Acc (57.63), Pre (45.74), Rec (39.38), MacF (56.39), MicF (57.63), and running time (103.37) are greater than the critical value 11.07, there is a significant difference between the performances of the compared classifiers. Therefore, the null hypothesis "There are no performance differences between the classifiers" is rejected, and thus the Nemenyi post-hoc test can be applied. For $k = 6$, $N = 21$, and $\alpha = 0.05$, since the value for the infinite degrees of freedom in the table Studentized Range q is 4.030, the critical distance is $\frac{4.030}{\sqrt{2}} \times \sqrt{\frac{6\times7}{6\times21}} \approx 1.645$ according to the Nemenyi post-hoc test. The critical diagrams produced by the Nemenyi post-hoc test for the five performance metrics and running time are presented in Figure 2.

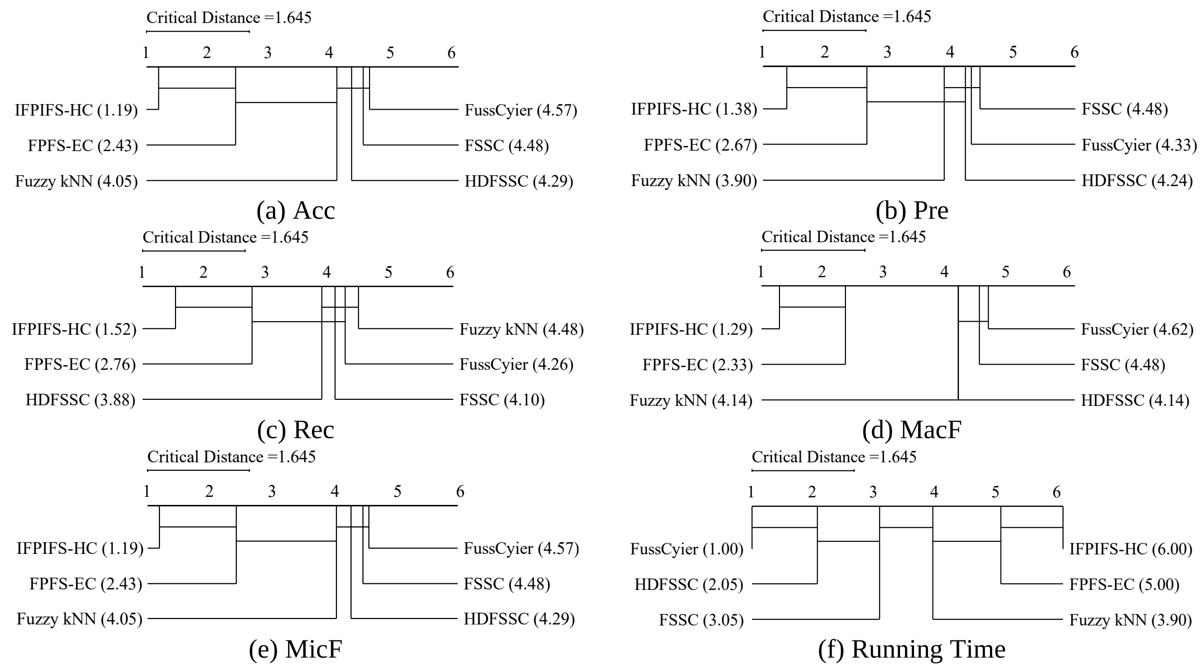

Figure 2. The critical diagrams for the five performance criteria and running time: The results from the Nemenyi post-hoc test at 0.05 significance level and average ranking from the Friedman test.

Figure 2 shows that the performance differences between the average rankings of IFPIFS-HC and the others, except for FPFS-EC, are greater than the critical distance (1.645). Moreover, Figure 2 manifests that IFPIFS-HC performs better than FPFS-EC concerning all performance metrics, even though the difference between the average rankings of IFPIFS-HC and FPFS-EC is less than the critical distance (1.645). Therefore, IFPIFS-HC outperforms the others statistically for all five performance metrics.

5. Evaluation of Computational Complexity

This section compares the classifiers' computational complexity by utilizing big O notation besides their running time results obtained in 10 runs for the 21 UCI datasets. As can be observed from Table 2, IFPIFS-HC in general seems to operate slightly slower than other classifiers. The reason is that in the pre-processing step, as with FPFS-EC, IFPIFS-HC employs all of the training samples separately, while FSSC, FussCyier, and HDFSSC employ a class-based mean of the training samples. Additionally, IFPIFS-HC's running time occurs under 1 s for 13 of the 21 datasets. Owing to its non-high running time, the proposed classifier can be used in real-time applications. From the pseudocode of IFPIFS-HC, the computational complexity is $O(mn)$ for each test sample. Here, m and n are the numbers of the training samples and attributes, respectively. The computational complexities of the compared classifiers are provided in Table 3.

Table 3. Computational complexities of the classifiers

Classifier	Computational Complexity
Fuzzy kNN	$O(n^2 \log k)$
FSSC	$O(ml)$
FussCyier	$O(ml)$
HDFSSC	$O(ml)$
FPFS-EC	$O(mn)$
IFPIFS-HC	$O(mn)$

k is number of the nearest neighbours, m is the sample number of the training data, n is the parameter number of the training data, and l is the class number of the data.

6. Conclusion

In this study, the concept of pseudo-similarity over *ifpifs*-matrices was first defined and Hamming pseudo-similarity over *ifpifs*-matrices was suggested. Afterwards, a classifier based on the proposed Hamming pseudo-similarity of *ifpifs*-matrices (IFPIFS-HC) was proposed. Moreover, we compared IFPIFS-HC with the well-known and state-of-the-art classifiers Fuzzy kNN [30], FSSC [31], FussCyier [32], HDFSSC [33], and FPFS-EC [34] and statistically analysed the comparison results. The simulation results and statistical evaluation showed that IFPIFS-HC performed better than the others for all the performance metrics.

In recent years, various classifiers have been proposed, such as FSSC, FussCyier, and HDFSSC, which use distance and similarity measures of fuzzy soft sets [3, 5] and work by class-based averaging of training data, FPFSCC [20] using Chebyshev pseudo-similarity of *fpfs*-matrices, FPFSNHC [19] employing Hamming pseudo-similarity of *fpfs*-matrices, and FPFS-EC utilising Euclidean pseudo-similarity of *fpfs*-matrices. FPFSCC and FPFSNHC also work by averaging training data such as FSSC, FussCyier, and HDFSSC. The success of the FPFSCC and FPFSNHC has been limited, as averaging the training data leads to loss of information. To overcome this problem, utilising Euclidean pseudo-similarity of the *fpfs*-matrices and separately processing the entire training data in the training and classification phases, FPFS-EC algorithm, which outperforms Fuzzy kNN, Support Vector Machines (SVM) [43], FSSC, FussCyier, and HDFSSC for the considered data sets, was developed. These three classifiers showed that *fpfs*-matrices, which can actually consider the effect of parameters on classification, are more effective than soft sets and fuzzy sets in classification problems.

Lately, *ifpifs*-matrices [27] have been introduced to cope with a large number of data and further uncertainties than fuzzy uncertainty. This concept could model further uncertainties than *fpfs*-matrices can. The success of *fpfs*-matrices in machine learning showed that *ifpifs*-matrices, a more general concept, will also be successful in machine learning. To this end, this study focused on the application of *ifpifs*-matrices to machine learning. Besides, this study manifested that *ifpifs*-matrices could effectively model classification problems with considerable uncertainty.

This is a primary study that will lead to new research on how to construct *ifpifs*-matrices for real problems such as classification problems in machine learning. Therefore, future studies should focus on the application of *ifpifs*-matrices to real problems. Although the proposed classifier produced successful performance results, its algorithm is open to improvement. In addition, interval-valued intuitionistic fuzzy parameterized interval-valued intuitionistic fuzzy soft set [44], picture fuzzy sets [45, 46], and their hybrid versions are worth studying. Finally, the proposed classifier can be customized to produce performances close to 100%, especially for problems such as medical diagnosis.

Author Contributions

Ç. Camcı directed the project and supervised this work's findings. B. Arslan and T. Aydın devised the main conceptual ideas and developed the theoretical framework. S. Memiş and B. Arslan carried out the experiment and statistical analyses. S. Memiş wrote the manuscript with support from S. Enginoğlu, T. Aydın, and B. Arslan. S. Enginoğlu reviewed and edited the manuscript. All authors discussed the results and contributed to the final manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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