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Bulanık çok yanıtlı deneyler için bulanık pareto çözüm kümesinin bulanık ilişkiye dayalı siniflandirma yaklaşimi ile değerlendirilmesi

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ÖZET

Çok yanıtlı bir deneyin çözüm kümesi Pareto çözüm kümesi ile karakterize edilir. Bu çalışmada, çok yanıtlı deney bulanık çerçevede ele alınmıştır. Yanıtlar ve model parametreleri, veri setindeki belirsizliği tanımlayan üçgensel bulanık sayılar olarak göz önünde bulundurulmuştur. Modelleme ve optimizasyon için sırasıyla bulanık en küçük kareler yaklaşımı ve bulanık uyarlanmış BSGA-II (BBSGA-II) kullanılmıştır. Elde edilen bulanık Pareto çözüm kümesi, bulanık ilişkili sınıflandırma yaklaşımı kullanılarak gruplandırılmıştır. Böylece, daha iyi karar verebilmek için alternatif çözümlerin seçimi kolaylaştırılmıştır. Bulanık yanıt değerlerinden oluşan gerçek bir veri seti uygulama olarak kullanılmıştır.

Anahtar Kelimeler: Bulanık çok yanıtlı problem, bulanık modelleme, bulanık BSGA-II, bulanık Pareto çözüm kümesi, bulanık ilişkili sınıflandırma.

Evaluatıon of fuzzy pareto solutıon set by usıng fuzzy relatıon based clusterıng approach for fuzzy multı-response experıments

ABSTRACT

The solution set of a multi-response experiment is characterized by Pareto solution set. In this paper, the multi-response experiment is dealed in a fuzzy framework. The responses and model parameters are considered as triangular fuzzy numbers which indicate the uncertainty of the data set. Fuzzy least square approach and fuzzy modified NSGA-II (FNSGA-II) are used for modeling and optimization, respectively. The obtained fuzzy Pareto solution set is grouped by using fuzzy relational clustering approach. Therefore, it could be easier to choose the alternative solutions to make better decision. A fuzzy response valued real data set is used as an application.

Keywords: Fuzzy multi-response problem, fuzzy modeling, fuzzy NSGA-II, fuzzy Pareto solution set, fuzzy relational clustering.

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1. INTRODUCTION

The analysis of data from a multi-response experiment, in which a number of responses are measured simultaneously, requires a careful consideration because of the multiple response nature of the data. It is often hard to find an optimal setting of the input variables that provide the best compromise of the multiple responses simultaneously which is called multi-response problem. In general, a multi-response problem is evaluated in two stages which are modeling and optimization. Response surface methodology (RSM) is widely used for modeling and optimizing the multi-response problems [1-3]. The optimization stage is specifically called multi-response optimization (MRO). In order to obtain compromise solution set for multi-response problems, the decision making stage can be considered as the third stage to make better decision for the process.

The solutions of multi-response problems are mainly depend on an approximating model of the unknown responses. The approximating model is based on an observed data from the process or system collected via a multi-response experiment. Sometimes, however, it is not possible to get exact numerical data because of the complexity of the process. In fact, there are many cases where observations cannot be known or quantified exactly, and thus, it is possible to provide an approximate description of them, or interval as close them. And also, it can be difficult to find a probability distribution of the responses in some situations. In recent years, fuzzy sets, firstly introduced in [4], has been used for modeling systems which are non-linear, complex, ill-defined and not well understood due to its ease of implementation, flexibility, tolerant nature to imprecise data, and ability to model non-linear behavior of arbitrary complexity [5]. The fuzzy set theory has found many applications in multi-response problems. In the studies of [6-9], fuzzy models are generated using if-then fuzzy-rule based reasoning for tackling MRO. A fuzzy modeling approach is proposed to optimize dual response systems in [10]. In [11-12], fuzzy regression models based on replicates of responses are constituted and a fuzzy programming method is expressed to solve the problem. In [13], a fuzzy regression model based on possibility distribution is used and a fuzzy MRO procedure is proposed to search an appropriate combination of process parameter setting. In [14], second order response surface model in fuzzy setting has been derived for the modeling of the responses.

In this paper, the uncertainties of the collected data are inserted into the modeling of the unknown responses by means of fuzzy model coefficients. Firstly, fuzzy least square method with Diamond's distance metric is used to compose the fuzzy response models in which the response values and model coefficients are assumed as triangular fuzzy numbers and input variables are considered crisp. Secondly, multi-response problem with fuzzy responses is considered as a multi-objective optimization (MOO) problem with fuzzy objectives. In order to achieve the optimization, without dimensionality reduction of the objectives, a well-known multi-objective algorithm, Non-dominated Sorting Genetic Algorithm-II (NSGA-II), is used with the modification of centroid index based fuzzy ranking approach which makes easier to decide the domination status between the fuzzy solutions. As a result, fuzzy Pareto solution set is obtained by using proposed fuzzy NSGA-II (FNSGA-II) which is firstly introduced in [15-17]. Hence, a bigger objective space could be provided with the fuzzy response values which bring flexibility to the decision maker.

The evaluation of the solutions can be considered as a final stage to make decision for the process. Since the fuzzy Pareto solution set can have extremely large number of solutions, the analysis of the fuzzy Pareto solution set can provide better decision process. This method is based on clustering methods. Recently, fuzzy clustering approach is used for the evaluation of Pareto solutions in the multi-objective framework. In [18], NSGA-II is used to obtain Pareto solution set and the solution set is clustered by using Fuzzy C-Means (FCM), a fuzzy clustering approach, on a calibration problem. Similarly in [19], NSGA-II is used for optimization and two fuzzy clustering methods, FCM and FLVQ (Fuzzy Learning Vector Quantization), are used for clustering of the Pareto solution set of an economic emission dispatch problem. In this work, since there are some difficulties to define the optimal cluster numbers, fuzzy relation based clustering approach [20] is used to cluster the fuzzy Pareto solution set. A case study in food engineering is used as an application.

This paper is organized as follows. Section 2 presents a brief description about fuzzy modeling, fuzzy MOO problem and some explanations about fuzzy set theory. Section 3 contains fuzzy clustering definitions and information about fuzzy relational clustering approach for fuzzy Pareto solution set. In Section 4, modeling, optimization and clustering approaches are applied on a fuzzy response valued data set in food engineering field and the results are given. Some conclusions are presented in Section 5.

2.MODELING AND OPTIMIZING OF A FUZZY MULTI-RESPONSE PROBLEM

Assume that the multi-response problem involves fuzzy responses $\tilde{Y}_j, (j=1,2,...,r)$ which depend on the given k input variables $(X_1, X_2, ..., X_k)$. Each fuzzy response variable is considered as a triangular fuzzy number and denoted as $\tilde{Y}_t = (Y_t - \underline{Y_t}, Y_t, Y_t + \overline{Y_t})$, $t=1,2,...,n$, in which $\underline{Y_t}$ and $\overline{Y_t}$ are left and right spreads, respectively. The input variables are considered crisp numbers and n denotes the number of experiments.

The relation between inputs and responses can be defined with a polynomial regression model which is used as an approximation function to an unknown response surface. Taking into account that the response surface problems have non-linear structure, second order polynomial models have an important place in the modeling strategy of the responses [2]. The second order predicted fuzzy response surface model can be written in the form

$$\hat{\tilde{Y}}_t = \tilde{\beta}_0^t + \sum_{i=1}^{k} \tilde{\beta}_i^t X_i + \sum_{i=1}^{k} \tilde{\beta}_{ii}^t X_i^2 + \sum_{i=1}^{k} \sum_{i<j}^{k} \tilde{\beta}_{ij}^t X_i X_j, \quad t=1,2,...,n, \; s=1,2,...,r \tag{1}$$

where $\tilde{\beta}_0, \tilde{\beta}_i, \tilde{\beta}_{ii}$, and $\tilde{\beta}_{ij}$ are triangular fuzzy numbers, denoting $\tilde{\beta}_i = (\beta_i - \underline{\beta_i}, \beta_i, \beta_i + \overline{\beta_i})$ and $\tilde{\beta}_{ij} = (\beta_{ij} - \underline{\beta_{ij}}, \beta_{ij}, \beta_{ij} + \overline{\beta_{ij}})$ where (β_i, β_{ij}) are center values with left spreads $(\underline{\beta_i}, \underline{\beta_{ij}})$ and right spreads $(\overline{\beta_i}, \overline{\beta_{ij}})$. The modeling error is incorporated into the fuzzy model coefficients, $\tilde{\boldsymbol{\beta}} = (\tilde{\beta}_0, \tilde{\beta}_i, \tilde{\beta}_{ii}, \tilde{\beta}_{ij})$ instead of random error term. The main aim of the fitting is to determine the fuzzy coefficients with the minimization of the distances, $\tilde{d}$, between the observed and predicted fuzzy response values. Hence, the minimization problem can be defined as

$$\min \varphi(\tilde{\boldsymbol{\beta}}) = \min \sum_{t=1}^{n} \tilde{d}^2 \left(\tilde{Y}_t, \hat{\tilde{Y}}_t\right). \tag{2}$$

The fuzzy least square (FLS) method with the Diamond's distance metric [21] is used and the optimization problem is transformed into

$$\begin{aligned} \min \varphi(\tilde{\boldsymbol{\beta}}) &= \min \sum_{t=1}^{n} \tilde{d}_D^2 \left(\tilde{Y}_t, \hat{\tilde{Y}}_t\right) \\ &= \min \sum_{t=1}^{n} \tilde{d}_D^2 \left(\left(Y_t, \underline{Y_t}, \overline{Y_t}\right), \left(\mathbf{X}_t^{'}\boldsymbol{\beta}, \mathbf{X}_t^{'}\underline{\boldsymbol{\beta}}, \mathbf{X}_t^{'}\overline{\boldsymbol{\beta}}\right)\right) \\ &= \min \sum_{t=1}^{n} \tilde{d}_D^2 \left(\left(\mathbf{X}_t^{'}\boldsymbol{\beta} - Y_t\right)^2\right. \end{aligned} \tag{3}$$
$$\left. + \left(\left(\mathbf{X}_t^{'}\boldsymbol{\beta} - \mathbf{X}_t^{'}\underline{\boldsymbol{\beta}}\right) - \left(Y_t - \underline{Y_t}\right)\right)^2 + \left(\left(\mathbf{X}_t^{'}\boldsymbol{\beta} - \mathbf{X}_t^{'}\overline{\boldsymbol{\beta}}\right) - \left(Y_t - \overline{Y_t}\right)\right)\right)^2$$

in which fuzzy parameters are defined in a vector form, $\tilde{\boldsymbol{\beta}} = (\boldsymbol{\beta} - \underline{\boldsymbol{\beta}}, \boldsymbol{\beta}, \boldsymbol{\beta} + \overline{\boldsymbol{\beta}})$. Assuming $\sum_{t=1}^{n} \mathbf{X}_t \mathbf{X}_t^{'}$ is non-singular, the parameter values are given by

$$\begin{aligned} \boldsymbol{\beta} &= \left(\sum_{t=1}^{n} \mathbf{X}_t \mathbf{X}_t^{'}\right)^{-1} \sum_{t=1}^{n} \mathbf{X}_t Y_t \\ \underline{\boldsymbol{\beta}} &= \left(\sum_{t=1}^{n} \mathbf{X}_t \mathbf{X}_t^{'}\right)^{-1} \sum_{t=1}^{n} \mathbf{X}_t \underline{Y_t} \\ \overline{\boldsymbol{\beta}} &= \left(\sum_{t=1}^{n} \mathbf{X}_t \mathbf{X}_t^{'}\right)^{-1} \sum_{t=1}^{n} \mathbf{X}_t \overline{Y_t}. \end{aligned} \tag{4}$$

The parameter vector, $\tilde{\boldsymbol{\beta}} = (\tilde{\beta}_0, \tilde{\beta}_i, \tilde{\beta}_{ii}, \tilde{\beta}_{ij})$, can be easily obtained by using (4). Hence, a multi-response problem is converted to a fuzzy multi-response problem with fuzzy response functions.

Since the multi-response problems often involve incommensurate and conflicting responses, it is necessary to take into account all responses simultaneously to obtain satisfactory compromise solution set. Therefore, a fuzzy MRO problem can be formulated as following

$$\begin{aligned} &\textit{optimize} \; \left\{\hat{\tilde{Y}}_1(\mathbf{X}), \hat{\tilde{Y}}_2(\mathbf{X}), ..., \hat{\tilde{Y}}_r(\mathbf{X})\right\} \\ &\quad s.t. \qquad \mathbf{X} \in S \end{aligned} \tag{5}$$

where $\hat{\tilde{Y}}_j(\mathbf{X})$ denotes the jth fuzzy response, $j=1,2,...,r$, $\mathbf{X}$ is an input vector, and S is an experimental region. By using (5), a fuzzy MOO problem can be formulated as

$$\begin{aligned} &\textit{optimize} \; \left\{\tilde{f}_1(\mathbf{X}), \tilde{f}_2(\mathbf{X}), ..., \tilde{f}_r(\mathbf{X})\right\} \\ &\quad s.t. \qquad \mathbf{X} \in S. \end{aligned} \tag{6}$$

where each $\tilde{f}_j$, $j=1,2,...,r$ represents fuzzy response function as a fuzzy objective function. In order to optimize the fuzzy MOO given in (6), fuzzy modified NSGA-II (FNSGA-II) is used. The FNSGA-II is created by the modification of NSGA-II with centroid index based fuzzy ranking approach. The detailed explanation and algorithmic steps of the FNSGA-II are given in [15]. The solution set of the (6) is composed of fuzzy non-dominated solutions which are represented by triangular fuzzy numbers denoted as $\tilde{f} = \left(f^l, f^c, f^u \right)$ shown in Figure 1.

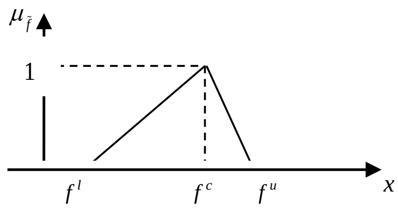

Figure 1. A triangular fuzzy number $\tilde{f}$

The fuzzy number is a subset of the real line R with the membership function $\mu_{\tilde{f}}$ is defined as

$$\mu_{\tilde{f}}(x) = \begin{cases} \mu_{\tilde{f}}^{L}(x) & , \quad f_t^l \le x < f_t^c \\ 1 & , \quad x = f_t^c \\ \mu_{\tilde{f}}^{R}(x) & , \quad f_t^c < x \le f_t^u \\ 0 & , \quad otherwise \end{cases} \tag{7}$$

where $\mu_{\tilde{f}}^{l}(x) = \dfrac{x - f^l}{f^c - f^l}$ and $\mu_{\tilde{f}}^{u}(x) = \dfrac{f^u - x}{f^u - f^c}$.

A fuzzy number can be represented as a family of sets called α-level set. Let $\tilde{f}_\alpha$ denotes the α-level set defined as

$$\tilde{f}_\alpha = \left\{ x \in R : \mu_{\tilde{f}}(x) \ge \alpha \right\} \tag{8}$$

where $\mu_{\tilde{f}}(x)$ is given in (7). Every α-cut of a fuzzy number is a closed interval, defined as $\tilde{f}_\alpha = \left[\tilde{f}_\alpha^L, \tilde{f}_\alpha^U \right]$ where $\tilde{f}_\alpha^L = \inf\left\{ x \in R : \mu_{\tilde{f}}(x) \ge \alpha \right\}$ and $\tilde{f}_\alpha^U = \sup\left\{ x \in R : \mu_{\tilde{f}}(x) \ge \alpha \right\}$ [22]. Assume that the fuzzy MOO problem involves two fuzzy objective functions. The α-cut representation of a fuzzy non-dominated solution for this problem will be a rectangular as in Figure 2. The Figure 2 shows a triangular fuzzy solution, obtained for several α-cut values, e.g. $\alpha = 0; 0.80; 1$. It is clear from the Figure 2 that if the α-cut values gets closer to 1, the fuzzy non-dominated solution becomes crisp valued.

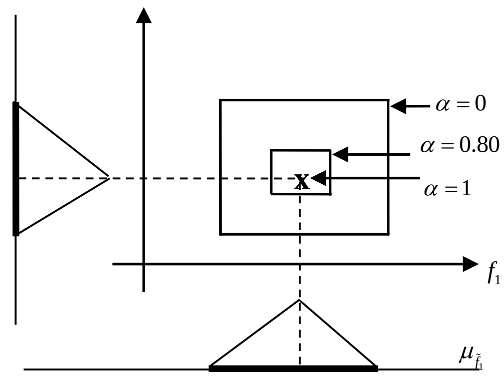

Figure 2. Representation of a triangular fuzzy solution on an objective space with the α-cut values

3. EVALUATING OF THE FUZZY PARETO SOLUTION SET WITH A FUZZY CLUSTERING APPROACH

The fuzzy Pareto set consists of many fuzzy non-dominated solutions. Each solution can be considered an important element of fuzzy set since it gives a different solution options for the decision making analysis. It is most common that the fuzzy Pareto set can be composed of similar valued fuzzy solutions. A decision maker can need to determine the reasonable Pareto solution groups which have the similar fuzzy values. Hence, a grouped set will have some alternative solutions and the dimension of solution set will be reduced which helps for decision making. It can be easily said that the grouping of fuzzy Pareto solutions can be derived as a fuzzy clustering problem.

3.1 Fuzzy Clustering

In a clustering problem, the main goal is determining the optimal number of clusters. The fuzzy clusterings can be roughly divided into two categories. One category, based on objective functions, is an effective and studied popularly in the literature. Fuzzy C-Means (FCM) algorithm is one of the most popular objective function based fuzzy clustering algorithm which is first developed by [20] and improved by [23]. The FCM algorithm has many assumptions for the data set. It can perform well only when the clusters in the data have approximately the same size and shape. It has difficulty in discovering small clusters. Many algorithms have been developed to solve these problems which are Gustafson-Kessel (GK) algorithm [24], adaptive fuzzy c-varieties (AFC) algorithm [25], and fuzzy clustering with volume prototypes algorithm [26] which extends

FCM and GK algorithm with volume prototypes [27]. Most of the proposed algorithms use cluster validity measure such as the compactness of the clusters [28]. Since in different data set clusters can be of different shapes, sizes and densities, it is difficult to devise a unique measure that is suitable for all different cases. Moreover, these procedures are computationally expensive because they require solving the optimization problems repeatedly for different values of the number of clusters over a pre-specified range.

Another category is based on a relation matrix such as correlation coefficient, equivalence relation, similarity relation, and fuzzy relation, etc. [29]. In this type of fuzzy clustering, the clustering starts with a large number of clusters and the compatible clusters are iteratively merged until the correct number of clusters are determined [26]. Clusters can be merged in several ways.

One of the merging method is based on fuzzy relational clustering which is used in this study. In fuzzy relational clustering method, a matrix of similarity measures is converged to a solution by employing the well-known max-min composition several times. A threshold value is used for merging the clusters. After cluster merging one has obtained the optimal number of clusters [20, 30].

3.2 Fuzzy Relational Clustering for Fuzzy Pareto Solution Set

Assume that the fuzzy Pareto solution set contains n number of fuzzy non-dominated solutions. A similarity relation between the fuzzy solutions is obtained in two steps: First, a $n \times n$ fuzzy compatibility relation matrix S is calculated. The elements S_{ij} of S are given by

$$S_{ij} = \begin{cases} \dfrac{s\left(A_i \cap B_j\right)}{s\left(A_i \cup B_j\right)} & , \quad i \neq j \\ 0 & , \quad i = j \end{cases} \tag{9}$$

where $i, j = 1,2,...,n$; A_i and B_j are the fuzzy solutions in the fuzzy Pareto set defined by α -cut values shown with rectangular in Figure 3. The $\cap$ and $\cup$ operators, given in Equation (9), are the intersection and the union, respectively.

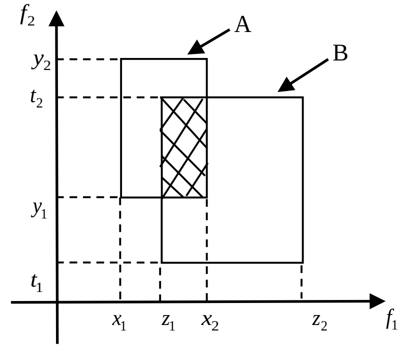

Figure 3. Representation of two fuzzy non-dominated solutions with rectangulars

The $s\left(A_i \cap B_j\right)$, $i, j = 1,2,...,n$ denotes the similarity between two fuzzy solutions by the calculation of intersection area. Hence, S_{ij} reflects the closeness amount between the two fuzzy solutions. The similarity value $S_{ij} \in [0,1]$. The compatibility matrix S is symmetric and diagonal elements of S are equal to 1 by definition. The next step is to determine which solutions one to be merged, given the similarity matrix S. This is done by using the fuzzy relational clustering method. In this method, the matrix S is converged to a solution by employing the max-min composition several times. When max-min composition and the max operator for set union are used, the resulting matrix is called transitive max-min closure. The transitive closure of a fuzzy relation S is the smallest relation that is transitive and contains S. Given a fuzzy relation S, its max-min transitive closure S_T can be calculated by using the following iterative algorithm.

1. Let $S_T^{(0)} = S$
2. Repeat for iteration number $l = 1,2,...$
   - $S_T^{(l)} = S^{(l-1)} \cup \left(S^{(l-1)} \circ S^{(l-1)}\right)$

   until $S_T^{(l)} = S_T^{(l-1)}$

where $\circ$ denotes the max-min composition of fuzzy relations [31]. The transitive matrix, S_T, indicates the groups of solutions that are similar at least to the degree denoted by the matrix elements. The matrix S_T is thresholded with pre-determined and problem dependent threshold value $\gamma\left(\gamma \in [0,1]\right)$. Hence, the groups of solutions that need to be merged are identified.

4. NUMERICAL EXAMPLE

In this section, a well-known whey protein concentrate (WPC) problem, mentioned before in the studies of [1, 3], is used to illustrate the application of the proposed fuzzy approaches. In the problem, it is aimed to define the effects of calcium chloride and cysteine on the textural and water-holding characteristics of dialyzed WPC gel systems. Table 1 shows the actual levels (in mM) and the coded levels of two input variables cysteine (X_1) and calcium chloride (X_2) used in multi response experiment to determine their effects on four properties of a food gelatin. These properties are considered as response variables which are hardness ($Y_1 - kg$), cohesiveness (Y_2), springiness ($Y_3 - mm$), and compressible water ($Y_4 - g$).

Table 1. Input variable levels and their coded values [1]

Cystein	2.6	8	21	34	39.4
Calcium Chloride	2.5	6.5	16.2	25.9	29.9
Coded level	-1.414	-1	0	1	1.414

Coding: X_1 = (Cystein-21)/13; X_2 = (Calcium Chloride-16.2)/9.7

After the checking of the correlation structure of the responses by some statistical analysis, it is decided to focus on Y_2 and Y_4 responses which are uncorrelated. Hence, the original problem is reduced to two response variables which are wanted to be maximized and minimized, respectively. Fuzzy response valued data set with a central composite design (CCD) was given in Table 2 in which the response values are fuzzified according to the confidence interval limits for each observed responses.

Table 2. Experimental coded values of input variables and observed fuzzy response variables for design points

No	X_1	X_2	$\tilde{Y}_2$			$\tilde{Y}_4$		
1	-1	-1	0.50	0.55	0.58	0.16	0.22	0.31
2	1	-1	0.46	0.52	0.54	0.58	0.67	0.74
3	-1	1	0.62	0.67	0.70	0.47	0.57	0.62
4	1	1	0.29	0.36	0.37	0.56	0.69	0.72
5	-1.414	0	0.56	0.59	0.64	0.26	0.33	0.41
6	1.414	0	0.30	0.31	0.38	0.63	0.67	0.78
7	0	-1.414	0.52	0.54	0.60	0.34	0.42	0.49
8	0	1.414	0.49	0.51	0.57	0.54	0.57	0.69
9	0	0	0.64	0.66	0.68	0.43	0.44	0.51
10	0	0	0.64	0.66	0.68	0.43	0.50	0.51
11	0	0	0.64	0.66	0.68	0.43	0.50	0.51
12	0	0	0.64	0.66	0.68	0.43	0.43	0.51
13	0	0	0.64	0.66	0.68	0.43	0.47	0.51

The fuzzy response functions are obtained by applying FLS method to the experimental data set given in Table 2. The fuzzy response functions are given in (10). By considering the each predicted fuzzy response function as a fuzzy objective function, the fuzzy MOO problem can be defined as

$$\begin{array}{c} \max \tilde{f}_1(\mathbf{X}) \\ \min \tilde{f}_2(\mathbf{X}) \\ \mathbf{X} \in [-1.414, 1.414] \end{array} \quad (10)$$

where $\tilde{f}_1$ and $\tilde{f}_2$ represent the fuzzy objective function form of predicted fuzzy response functions $\hat{\tilde{Y}}_2$ and $\hat{\tilde{Y}}_4$, respectively. In order to optimize the problem given in (11), FNSGA-II is applied with the specific parameters as $n_{pop} = 50$, $n_{gen} = 50$, $\Pr_c = 0.90$, $\Pr_m = 1/\nu$, $\eta_c = 20$ where ν denotes the number of input variables The obtained fuzzy non-dominated solutions are given in Table 3. The α-cut calculations of fuzzy solutions are presented by rectangular for $\alpha = 0$ in Figure 4.

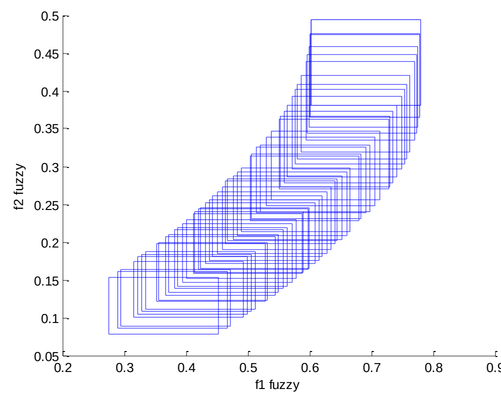

Figure 4. Fuzzy non-dominated solution set obtained by FNSGA-II ($\alpha = 0$)

The compatibility matrix S is calculated to evaluate the fuzzy Pareto solution set with the fuzzy relational clustering approach. The symmetric S matrix is given in Table 4. The iterative algorithm, given in Section 3.2, is used to obtain transitive matrix which indicates the groups of fuzzy solutions. The transitive matrix of the fuzzy Pareto solution set is given in Table 5. It can be easily seen from the Table 4-5 that both compatibility and transitive matrixes are symmetric.

$$\begin{aligned}\hat{\tilde{Y}}_2(\mathbf{X}) &= (0.6373, 0.6600, 0.6826) + (-0.092, -0.092, -0.092)X_1 + (-0.0103, -0.0103, -0.0103)X_2 \\ &+ (-0.0956, -0.0956, -0.0956)X_1^2 + (-0.0581, -0.0581, -0.0581)X_2^2 + (-0.07, -0.07, -0.07)X_1X_2 \\ \hat{\tilde{Y}}_4(\mathbf{X}) &= (0.4250, 0.4680, 0.5110) + (0.1294, 0.1314, 0.1354)X_1 + (0.0727, 0.0728, 0.0729)X_2 \\ &+ (0.0095, 0.0260, 0.0425)X_1^2 + (0.007, 0.0235, 0.04)X_2^2 + (-0.0825, -0.0825, -0.0825)X_1X_2.\end{aligned} \tag{11}$$

Table 3. The fuzzy non-dominated solution set obtained by FNSGA-II and input variable values

Number	$\tilde{f}_1$			$\tilde{f}_2$			**x**	
1	0.2749	0.3574	0.4517	0.0788	0.1133	0.1532	2.6180	2.4842
2	0.2898	0.3723	0.4666	0.0863	0.1210	0.1610	2.7220	2.9779
3	0.2945	0.3770	0.4713	0.0892	0.1239	0.1641	2.9469	2.9770
4	0.3161	0.3986	0.4929	0.1001	0.1348	0.1750	2.6180	4.1274
5	0.3225	0.4050	0.4993	0.1045	0.1396	0.1803	3.4409	3.7074
6	0.3286	0.4111	0.5054	0.1080	0.1432	0.1839	3.5683	3.8567
7	0.3345	0.4170	0.5113	0.1117	0.1472	0.1881	3.8777	3.8558
8	0.3522	0.4347	0.5290	0.1213	0.1569	0.1981	3.9245	4.5765
9	0.3549	0.4374	0.5317	0.1225	0.1579	0.1987	3.2017	5.3059
10	0.3675	0.4500	0.5443	0.1301	0.1657	0.2068	3.4968	5.6251
11	0.3725	0.4550	0.5493	0.1341	0.1696	0.2105	2.8871	6.3904
12	0.3810	0.4635	0.5578	0.1389	0.1746	0.2159	3.3369	6.3991
13	0.3863	0.4688	0.5631	0.1416	0.1777	0.2193	4.0129	6.0674
14	0.3946	0.4771	0.5714	0.1470	0.1832	0.2249	4.0363	6.4496
15	0.4014	0.4839	0.5782	0.1518	0.1880	0.2299	3.8569	6.9433
16	0.4115	0.4940	0.5883	0.1583	0.1948	0.2370	4.1637	7.1897
17	0.4135	0.4960	0.5903	0.1598	0.1970	0.2399	5.7809	5.9442
18	0.4209	0.5034	0.5977	0.1639	0.2010	0.2438	5.2882	6.7202
19	0.4232	0.5057	0.6000	0.1655	0.2028	0.2457	5.5261	6.6436
20	0.4319	0.5144	0.6087	0.1715	0.2088	0.2517	4.9697	7.5641
21	0.4378	0.5203	0.6146	0.1756	0.2130	0.2561	5.1179	7.7600
22	0.4428	0.5253	0.6196	0.1809	0.2183	0.2613	4.4796	8.6020
23	0.4510	0.5335	0.6278	0.1846	0.2227	0.2666	6.3256	7.4875
24	0.4582	0.5407	0.6350	0.1901	0.2286	0.2730	6.8677	7.4661
25	0.4642	0.5467	0.6410	0.1985	0.2365	0.2803	4.6395	9.7320
26	0.4672	0.5497	0.6440	0.2020	0.2401	0.2840	4.5017	10.0560
27	0.4761	0.5586	0.6529	0.2033	0.2425	0.2875	7.3214	8.1665
28	0.4829	0.5654	0.6597	0.2090	0.2480	0.2929	6.3698	9.3644
29	0.4892	0.5717	0.6660	0.2141	0.2534	0.2986	6.5518	9.6234
30	0.5036	0.5861	0.6804	0.2281	0.2679	0.3137	6.3503	10.7990
31	0.5062	0.5887	0.6830	0.2312	0.2711	0.3170	6.2736	11.0648
32	0.5142	0.5967	0.6910	0.2383	0.2786	0.3250	6.5401	11.4218
33	0.5203	0.6028	0.6971	0.2398	0.2810	0.3284	8.8645	9.9328
34	0.5296	0.6121	0.7064	0.2485	0.2904	0.3384	9.3390	10.2917
35	0.5372	0.6197	0.7140	0.2560	0.2980	0.3463	8.9841	11.1890
36	0.5511	0.6336	0.7279	0.2704	0.3136	0.3631	9.9526	11.6759
37	0.5531	0.6356	0.7299	0.2727	0.3161	0.3659	10.2139	11.6759
38	0.5582	0.6407	0.7350	0.2789	0.3228	0.3733	10.8171	11.7409
39	0.5639	0.6464	0.7407	0.2858	0.3299	0.3806	10.1879	12.7924
40	0.5718	0.6543	0.7486	0.2961	0.3412	0.3928	10.8782	13.1872
41	0.5769	0.6594	0.7537	0.3036	0.3492	0.4015	11.0641	13.6916
42	0.5805	0.6630	0.7573	0.3094	0.3554	0.4082	11.1746	14.1038
43	0.5860	0.6685	0.7628	0.3189	0.3661	0.4202	12.5136	14.0272
44	0.5936	0.6761	0.7704	0.3353	0.3835	0.4389	12.1600	15.6578
45	0.5962	0.6787	0.7730	0.3436	0.3925	0.4484	11.9884	16.4270
46	0.5987	0.6812	0.7755	0.3519	0.4015	0.4584	12.3459	16.9071
47	0.6016	0.6841	0.7784	0.3653	0.4166	0.4754	13.5562	17.4096
48	0.6016	0.6841	0.7784	0.3650	0.4162	0.4749	13.5237	17.3941
49	0.6026	0.6851	0.7794	0.3809	0.4336	0.4939	13.6680	18.6328
50	0.6027	0.6852	0.7795	0.3810	0.4336	0.4939	13.5094	18.7084

Table 4. Compatibility matrix of fuzzy non-dominated solutions

Number of solutions	**1**	**2**	**3**	**4**	**5**	**6**	**...**	**48**	**49**	**50**
1	1	0.69744	0.61589	0.3751	0.31051	0.26485	...	0	0	0
2		1	0.87683	0.53029	0.44088	0.37873	...	0	0	0
3			1	0.6001	0.49892	0.42911	...	0	0	0
4				1	0.8211	0.70324	...	0	0	0
5					1	0.85238	...	0	0	0
6						1	...	0	0	0
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
48								1	0.72214	0.72067
49									1	0.99799
50										1

Table 5. Transitive matrix of fuzzy non-dominated solutions

Number of solutions	**1**	**2**	**3**	**4**	**5**	**6**	**...**	**48**	**49**	**50**
1	1	0.69744	0.69744	0.6001	0.6001	0.6001	...	0.6001	0.6001	0.6001
2		1	0.87683	0.6001	0.6001	0.6001	...	0.6001	0.6001	0.6001
3			1	0.6001	0.6001	0.6001	...	0.6001	0.6001	0.6001
4				1	0.8211	0.8211	...	0.61474	0.61474	0.61474
5					1	0.85238	...	0.61474	0.61474	0.61474
6						1	...	0.61474	0.61474	0.61474
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
48								1	0.72766	0.72766
49									1	0.99799
50										1

Table 6. Clustering results of the fuzzy non-dominated solutions for different threshold values

No	**0.6**	**0.70**	**0.75**	**0.80**	**0.85**	**0.90**	**1**
1	{1,2,…,50}	{1}	{1}	{1}	{1}	{1}	{1}
2		{2,3}	{2,3}	{2,3}	{2,3}	{2},{3}	{2},{3}
3		{4,5,6,7}	{4,5,6,7}	{4,5,6,7}	{4},{5,6},{7}	{4},…,{7}	{4},…,{7}
4		{8,9,…,29}	{8,9} {10,11,…,29}	{8,9}, {10,11}, {12,13}, {14,15}, {16,17,18,19}, {20,21,…,24}, {25,26,…,29}	{8,9}, {10,11}, {12,13}, {14}, {15}, {16,17}, {18,19}, {20},{21}, {22},{23},{24}, {25,26,27},{28},{29}	{8,9}, {10}, {11}, {14}, {15}, {16,17}, {18,19}, {20},{21}, {22},{23},{24}, {25},{26},{27}, {28}, {29}	{8},…,{29}
5		{30,31,…,35}	{30,31,32,33} {34,35}	{30,31},{32,33}, {34},{35}	{30,31},{32,33}, {34},{35}	{30,31},{32},{33}, {34},{35}	{30},…,{35}
6		{36,37,…,43}	{36,37,38,39} {40,41,42,43}	{36,37,38,39} {40,41,42},{43}	{36,37},{38},{39}, {40},{41},{42},{43}	{36,37},{38},{39}, {40},{41},{42},{43}	{36},…,{43}
7		{44,45,…,50}	{44,45,46} {47,48}	{44,45,46} {47,48}	{44},{45},{46}, {47,48}	{44},{45},{46}, {47,48}	{44},…,{48}
			{49,50}	{49,50}	{49,50}	{49,50}	{49},{50}

Merging of the solutions depend on the threshold values γ. In order to see the fuzzy relation clusters, by using the transitive matrix given in Table 5, different threshold values are chosen for the clustering of fuzzy non-dominated solutions such as $\gamma = 0.60; 0.70; 0.75; 0.80; 0.85; 0.90; 1$. The obtained clustering results are given in Table 6.

It can be seen from the Table 6 that if the threshold value is chosen equal to 0.70, basically the fuzzy Pareto solution set can be divided into seven clusters. The clustering results are shown in Figure 5 with different colours for $\gamma = 0.70$. Hence, it is clear to see from Figure 5 that 50 fuzzy non-dominated solutions are reduced into the solution groups which are represented with 7 clusters for $\gamma = 0.70$. If the threshold value is chosen equal to 0.80, the fourth group of the fuzzy non-dominated solutions, which is shown with the sub-figure of Figure 5, can divide different clusters, too.

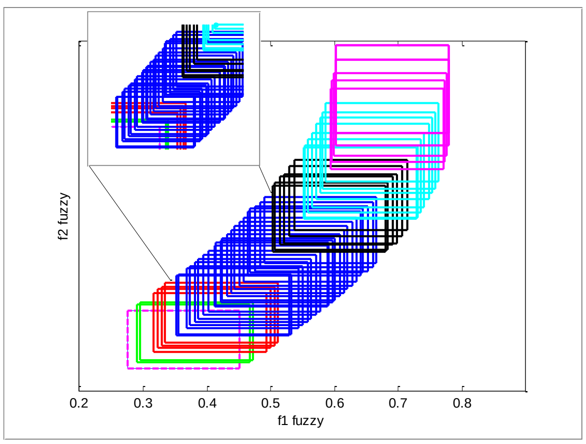

Figure 5. Clustering results of fuzzy non-dominated solutions for $\gamma = 0.70$

5. CONCLUSION AND DISCUSSION

In this study, a multi-response experiment is considered as a fuzzy response valued experiment with crisp experiment conditions. The fuzzy response values are represented as triangular fuzzy numbers. The fuzzy responses are modeled by using fuzzy least squares method and triangular fuzzy model parameters are estimated. The problem with fuzzy response functions is considered as a fuzzy MOO problem where the fuzzy objective functions are conflicting. In order to optimize the fuzzy MOO problem, FNSGA-II is used and fuzzy non-dominated solutions are obtained. Each fuzzy non-dominated solution is represented with rectangulars by using α- cut calculations where many of the fuzzy response values are covered by the rectangulars. Therefore, it could be possible to say that the fuzzy Pareto set brings flexibility for the evaluation of the fuzzy responses.

Decision making is an important step to obtain compromise solution set among many of the fuzzy alternative solutions. Clustering of the fuzzy non-dominated solutions can be considered as a final stage for the evaluation of the fuzzy multi-response problem. In this work, fuzzy non-dominated solutions are clustered by using fuzzy relational clustering approach. The clustering of the fuzzy non-dominated solutions makes easy to choose the similar fuzzy response values. Therefore, each cluster, composed with alternative solutions, can be used as a solution group of the problem. Hence, it becomes easier to decide the experimental conditions for the multi-response experiment.

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