SAÜ Fen Bilimleri Enstitüsü Dergisi
.Cilt 1.ve 2. Sayı (2000) 109-113

TIME TRADEOFF TRANSPORTATION PROBLEMS in CASE CHANGE of COSTS in CERTAIN INTERVALS

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Abstract- Objective function has constant cofficient in most of transportation problems. However in problems which comes to face in real life, costs can not be constant. Also, it is fact that, in transportation problems time is important. [2]

In this paper, we propose a solution with time-tradeoff in case change of costs in certain intervals, making transportation problems as a multiobjective transportation problem by order relations.

I. INTRODUCTION

Let S be a feasible region, A_{ij} intervals where c_{ij}'s changes on A_{ij}, t_{ij} represents transportation time from i to j. Then formulation of transportation problem can be given as follows

Objective function :

$$\text{Min } Z(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}x_{ij} \tag{1}$$

$$\text{Min } T=\{ t_{ij} : x_{ij}, \quad x_{ij} \neq 0 \} \tag{2}$$

Constraints :

$$\sum_{j=1}^{n} x_{ij} = a_i \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \tag{4}$$

$$x_{ij} \geq 0, \qquad t_{ij} \geq 0 \tag{5}$$

In addition, our model can be converted to multiobjective transportation problem by order relation. Then (1), (2) objective functions can be written

$$\text{Min } Z^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^k_{ij}x_{ij} \tag{6}$$

$$\text{Min } T_k = \{\min_p \max_{i,j} \; t_{ij} : x_{ij} \neq 0 \} \tag{7}$$

where p, is the number of alternative solution of k^{-th} solution.

Besides, It is accepted that the model is balanced model, that is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ equality holds.

As in all of multiobjective programming problems, instead of optimum solution in multiobjective transportation problems it is proposed the best feasible solution or set of solutions, for decision maker.

One of the most important factors of transportation is time. Either for in deformation of transporting goods or for responding of demands in time. Time is very important. For transportation problems, solution techniques which refletcs interaction with time of objectives, have been developed [3,6,7].

II. INTERVAL ANALYSIS

Let a_L be left-limit and a_R right-limit. Then an interval is defined by ordered pair as

$$A = [a_L , a_R] = \{ a : a_L \leq a \leq a_R , \; a \in R\} \tag{9}$$

Similarly, intervals is also denoted by its center and width as

$$A=\langle a_C , a_W\rangle=\{a: a_C-a_W \le a \le a_C+a_W,\ a\in R\} \quad (10)$$

Where, a_C is center and a_W is width. It is clear that

$$a_C = .5\,(a_R+a_L) \qquad a_W = .5\,(a_R-a_L) \quad (11)$$

The operations on intervals used in this paper may be explicitly calculated from defination (9) as

$$A+B = [a_L, a_R]+[b_L , b_R]=[a_L+ b_L, a_R+b_R] \quad (12)$$

$$A+B=\langle a_C,a_W\rangle+\langle b_C ,b_W\rangle=\langle a_C+ b_C , a_W+b_W\rangle \quad (13)$$

$$kA = k[a_L , a_R] = \begin{cases} [ka_L , ka_R] & \text{for } k \ge 0 \\ [ka_R , ka_L] & \text{for } k \langle 0 \end{cases} \quad (14)$$

$$kA = k\langle a_C , a_W\rangle = \langle ka_C , |k|\, a_W \rangle \quad (15)$$

III. ORDER RELATIONS for MINIMIZATION PROBLEM

Definition: Order relation $\le^*_{LR}$ between $A=[a_L , a_R]$ and $B = [b_L , b_R]$ is defined as

$$A\le^*_{LR}B \iff a_L\le b_L \quad \text{and} \quad a_R\le b_R \quad (16)$$

$$A<_{LR} B \iff A\le^*_{LR}B \quad \text{and} \quad A \ne B \quad (17)$$

Similarly, order relation $\le^*_{CW}$ between $A=\langle a_C, a_W\rangle$ and $B = \langle b_C , b_W \rangle$ is defined as

$$A \le^*_{CW} B \iff a_C \le b_C \quad \text{and} \quad a_W\le b_W \quad (18)$$

$$A\le^*_{CW} B \iff A \le^*_{CW} B \quad \text{and} \quad A \ne B \quad (19)$$

Here, It is notice that A is preferable to B if $A \le^*_{LR} B$ or $A \le^*_{CW} B$ and there is no pair A, B which satisfies condition $B \le^*_{CW} A$ [9].

Now,define the following order relation $\le^*_{RC}$ related two above relations

$$A \le^*_{RC} B \iff a_R\le b_R \quad \text{and} \quad a_C \le b_C \quad (20)$$

on the other words

$$A \le^*_{RC} B \iff A \le^*_{LR}B \quad \text{and} \quad A\le^*_{CW}B \quad (21)$$

Definition : $x \in S$ is a solution of (1) if and only if there is no $x' \in S$ which satisfies $Z(x') \le^*_{RC}Z(x)$. The right limit $Z_R(x)$ of interval objective function $Z(x)$ in (1) may be calculated from (13) and (15) as

In the case of $x \ge 0$

$$Z_R(x) = (a_{C1}x_1+a_{C2}x_2+ \dots +a_{Cn}x_n) + (a_{W1}x_1+ a_{W2}x_2+ \dots +a_{Wn}x_n) \quad (22)$$

Where a_{Ci} is the center and a_{Wi} is the width of the cofficient A_i of $Z(x)$. At the same time, the center $Z_C(x)$ of $Z(x)$ in (1) may be calculated from as

$$Z_C(x) = a_{C1}x_1+a_{C2}x_2+ \dots +a_{Cn}x_n \quad (23)$$

The solution set of (1) can be obtained as the Pareto optimal solutions of the following problem

$$\min \{ ((Z_R(x) , Z_C(x)) : x \in S \subset R^n \} \quad (24)$$

It is should be noted that the objective functions (24) are to minimize $Z_R(x)$ and $Z_C(x)$. It seems that our model converted to multiobjective model.

IV. MULTIOBJECTIVE with TIME TRADEOFF TRANSPORTATION PROBLEM

In this section, following symbols and concepts will use:

x : decision vector
$Z^k(x)$: k-th objective function
$Z(x)$: objectives vector

Let ${}^P x_k^*$ be p-th alternative optimum solution for k-th objective of problem. Let us denote objectives taken values vector for ${}^P x_k^*$ solution with

$${}^P Z_k=[Z^1({}^P x^*), Z^2({}^P x^*), \dots , Z^l({}^P x^*)] \quad (25)$$

and corresponding transportation time with ${}^P T_k$.

Let N(k) be number of nondominated pairs for k^{-th} objective after elimination of dominated from pairs $({}^{P}Z_k, {}^{P}T_k)$. Operation is finished if there is solution set which decision maker accepts from these solutions. If, any solution is not accepted by decision maker releases the solution to us; we may construct joint objective

$$^{q}Z(x) = \sum_{k=1}^{l} w_k^q Z^k(x) \qquad (26)$$

where $w_k{}^q$ is the weight of $q(q=1, 2, \ldots, \prod_{k=1}^{l} N(k))^{-th}$ solution set of $Z^k(x)^{-th}$ objective. $(\sum_{k=1}^{l} w_k^q = 1)$

Theorem : x is to be a solution of system (3) - (4) - (5) - (6) iff it is an optimal solution of LP problem (3) - (4) - (5) - (26) .

Besides. as in Ringuest and Rinks's article[10], weights $w_k{}^q$ (k=1, 2, ... l) for every joint objective can be written;

$$w_1^q = \frac{w_{l+1}^q}{\sum_{j=1}^{l} Z^j(x_1^*)}, \quad w_2^q = \frac{w_{l+1}^q}{\sum_{j=1}^{l} Z^j(x_2^*)}$$

$$, \ldots, \; w_l^q = \frac{w_{l+1}^q}{\sum_{j=1}^{l} Z^j(x_l^*)} \qquad (27)$$

from the equality

$$\sum_{j=1}^{l} w_1^q Z^j(x_1^*) = \sum_{j=1}^{l} w_2^q Z^j(x_2^*) = \ldots = \sum_{j=1}^{l} w_l^q Z^j(x_l^*) = w_{l+1}^q$$

by using $w_1^q + w_2^q + \ldots + w_l^q = 1$. We obtain

$$W_{l+1}^q = [\frac{1}{\sum_{j=1}^{l} Z^j(x_1^*)} + \frac{1}{\sum_{j=1}^{l} Z^j(x_2^*)} + \ldots + \frac{1}{\sum_{j=1}^{l} Z^j(x_l^*)}]^{-1} \qquad (28)$$

where $x_k{}^*$ represents only one alternative optimum solution of objective $Z^k(x)$. Then all dominated solution set $({}^{q}Z, {}^{q}T)$ submitted to decision maker by interacting objective

$$^{q}Z(x) = \sum_{k=1}^{l} w_k^q Z^k(x) = \sum_{k=1}^{l} w_k^q \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} = \ldots = \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}^q x_{ij} \qquad (29)$$

with transportation time $^{q}T = \max_{i,j} \{ t_{ij} : x_{ij} \neq 0\}$

under the constrains (2)-(3)-(4). Clearly, $f_{ij}^q = \sum_{k=1}^{l} w_k^q c_{ij}^k$.

If a solution set which decision maker admits then operation is finished. Otherwise, If decision maker doesn't choose any solution set or leave us decision, solution set corresponding to;

$$\min_q R_q = \min_q \left\{ \min_r \frac{{}^{q}Z_{r+1} - {}^{q}Z_r}{{}^{q}T_r - {}^{q}T_{r+1}} \right\} \qquad (30)$$

is proposed to decision maker since that solution set makes the least cost-time slope. Here r represents time interaction step with objective q. Thus, solution set $({}^{s}Z, {}^{s}T)$ corresponding to $\min_q R_q = R_s$ is our last proposed solution to decision maker.

V. COROLLARY

This study is supported with a computer program. It is observed that there is no difference between solutions of computer and manuel. The program includes a main menu procedures and subreports. Here, functions and procedures makes following solutions.

* Least Cost Method
* Modi Test
* Finding Alternative Solutions
* Time Tradeoff with Alternative Solutions
* Computing Weights at Compromise Objective function.
* Time Tradeoff with Compromise Solutions
* Determining of optimum solution

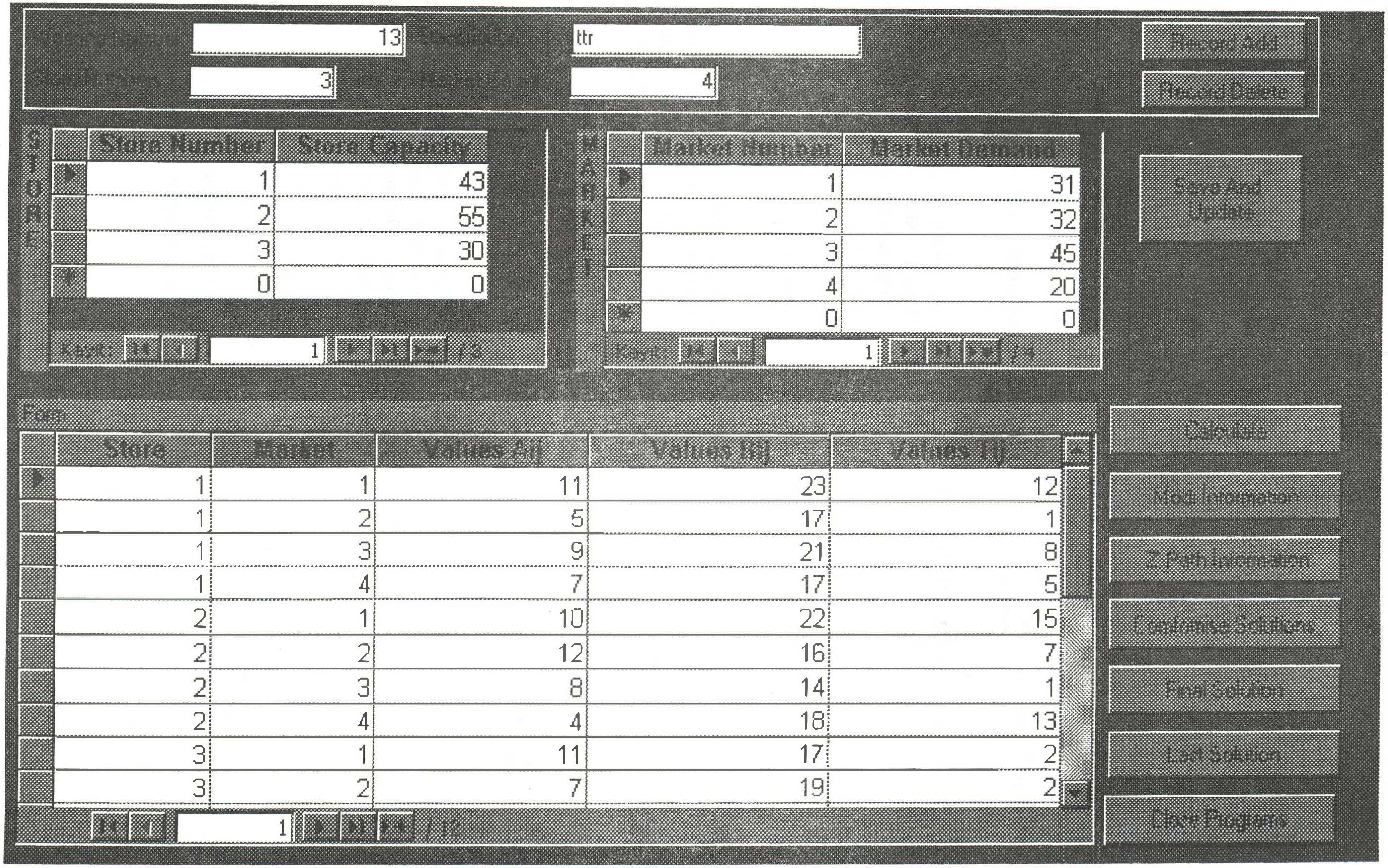

Figure 1. Main Menu

VI. A SOLUTION ALGORITHM

Step 1: Convert the single objective programming problem that changes its costs in certain intervals, to multiobjective problem by interval analysis.

Step 2: Find the all alternative optimal solutions.

Step 3: Let ${}^{P}x_k{}^{*}$ values vector of objective be

$${}^{P}Z_k = [\, Z^1 ({}^{P}x_k{}^{*}), Z^2 ({}^{P}x_k{}^{*}), \dots , Z^l ({}^{P}x_k{}^{*})]$$

and denote transportation time with ${}^{P}T_k$. As ${}^{P}T_k = \max_{i,j} t_{ij}$ for which satisfying $x_{ij} > 0$. If $({}^{P}Z_k , {}^{P}T_k)$ is a desired solution of decision maker then current solution is the best solution. STOP. Otherwise go to step 4.

Step 4: For all alternative solution of objective, If $({}^{P}Z_k , {}^{P}T_k)$ is

i) undefined, goto step3, by taking next alternative solution.
ii)defined, goto step 5.

Step 5: Eliminate undesired solution of $({}^{P}Z_k , {}^{P}T_k)$. Let N(k) be the numberof solutions which are obtained by k^{-th} objective and not eliminated.

Step 6: Considering alternative optimal solution, construct associated objective

$${}^{q}Z(k) = \sum_{k=1}^{l} w_k^q Z^k (x) \quad (q = 1, 2, \dots , \prod_{k=1}^{l} N(k))$$

By optimization of this objective, propose solution set $({}^{q}Z, {}^{q}T)$ to decision maker. If decision maker accepts this solution. STOP. Otherwise, go to step 7.

Step 7: Interact the transportation time with objective ${}^{q}Z(x)$ and determine the ratio

$$\min_q R_q = \min_q \left\{ \min_r \frac{q_{Z_{r+1}} - q_{Z_r}}{q_{T_r} - q_{T_{r+1}}} \right\}$$

Step 8: If all R_q $(q = 1, 2, \dots , \prod_{k=1}^{l} N(k))$ were constructed, go to step 9. Otherwise return step 6.

Step 9: Solution set (S_Z, S_T) corresponding to ratio $\min_q R_q = R_s$ is our final proposition to decision maker.

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