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# AN INVESTIGATION OF THE ENERGY LEVELS AND MULTIPOLE MIXING RATIO OF ELECTROMAGNETIC TRANSITIONS IN THE EVEN-EVEN HAFNIUM ISOTOPES

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**Summary-** In this work some of the electromagnetic interactions of even-even Hafnium isotopes in the $150 \leq A \leq 190$ deformation region were studied in a detailed manner.

In this region, using the experimental δ(E2/M1) multipole mixing ratios the deformation parameters $\beta_0$ and the quadrupole moments $q_0$ and $q'_2$ were calculated. The obtained results are in a good agreement with the general systematic of the deformation region under consideration.

## 1-INTRODUCTION

The Theoretical and experimental investigations of the rare earth elements in the $150 \leq A \leq 190$ deformation region provide the valuable informations in explanation of the characteristic properties of the elements toward the end of the defo mation region[2,3]. Since the Hafnium isotopes (Z=72) are the important members of this deformation region they are under the intensive investigations[1-12].

## 2-THEORETICAL BACKGROUND

As it is well known, the pairing plus the quadrupole forces have an important role among the nuclear forces[1,2]. The quadrupole force produces the nuclear deformation while the pairing force tries to retain the spherical symmetry. The effect of the pairing forces decreases as the added number of valance particles increases.

In this case the forces, causing to the rotational spectra, appear a dominant effect and nucleus becomes into deformed state[3]. Dudex suggested the relation of

$$G = \frac{G_0}{A} + \frac{G_1(N - Z)}{A} \tag{1}$$

for calculation of the pairing forces. Using the values of the $G_0$ and $G_1$ for the protons and neutrons in the Eqn.(1) we obtain the folloving equations;

$$G_p = |17.90 + 0.176(N-Z)| / A$$
$$G_n = |18.95 + 0.078(N-Z)| / A \tag{2}$$

Where $G_p$ and $G_n$ represent the pairing forces for the protons and neutrons respectively. As the Eqn.(2) show, $G_p > G_n$ so that we can write $\beta_0(n) > \beta_0(p)$. It must be noted that $\beta_0(p)$ and $\beta_0(n)$ are the deformation parameters for the protons and neutrons respectively. We have calculated the deformation parameters $\beta_0$ from the second reference as a function of the f and δ(E2/M1) as given in the Eqn.(3);

$$\beta_0 = (\delta / E_\gamma)_{2^+_\gamma \to 2^+_g} \frac{10^3(1-2f)f}{(3.7185)^{1/2} A^{5/3}} \tag{3}$$

Where f is given as follows

$$f = \frac{\beta_0 - \beta_0(p)}{\beta_0} \approx \frac{N}{A}\left(\frac{\beta_0(n)}{\beta_0(p)} - 1\right) \tag{4}$$

and it can be put in a useful form as in the following Equation from ref. [14].

$$\frac{\beta_0(p)}{\beta_0} = \sqrt{\frac{G_n}{G_p}} \tag{5}$$

Using the Eqn.(2) values of the $G_p$ and $G_n$ and the Eqn.(5), we reach to the Eqn.(6) for calculation of the parameter f

$$f = \frac{N}{A}\left(\sqrt{\frac{G_p}{G_n}} - 1\right) \tag{6}$$

It should be emphasized that the Eqn(3) can be used for the calculation of $\beta_0$ the parameters of the isotopes toward the end section of the region of the $150 \leq A \leq 190$. The electric quadrupole moments are the significant factors for the nuclear deformations. They can be determined in terms of the spectroscopic studies. We have to remember that they can be used as a measure of the shape of the defined charge ditributions and the dimensional antisymmetry of the rotational nuclei. Therefore the q, electric quadrupole, quantities have a great significance in investigation of the deformed nuclei. The spectroscopic quadrupole moment of nucleus is given by;

$$q(I) = \frac{3K^2 - I(I+1)}{(I+1)(2I+1)} q_0 \qquad (7)$$

formula[6]. Here $q_0$ is the intrinsic qudrupole moment and it is defined as[5].

$$q_0 = \frac{3}{\sqrt{5\pi}} ZR^2 \beta_0 (1 + 0.36) \qquad (8)$$

On the other hand, as the Table 1. shows, the deformation has an important effect on the excitation energies of the Hf isotopes. Some of the transition energies[7] of these isotopes and their ratios were given in the Table 1. to clarify this effect. And also the variation of the excitation energies as a function of the number of neutrons was shown in the Figure 1.

The folloving conclusion can be obtained from the close analysis of the Table.1 for the even-even Hf isotopes.

I. 89 keV $\leq E_{2_g^+} \leq$ 94 keV and 290 keV $\leq E_{4_g^+} \leq$ 298 keV relatively small energy values are measured experimentally.

II. 901 keV $\leq E_{2_\gamma^+} \leq$ 1277 keV and 1063 keV $\leq E_{4_\gamma^+} \leq$1391 keV relatively high energy values are determined experimentally for the $\gamma$ rays.

Table.1.Some of the Transition Energy Values of the Hf Isotopes

| Energies(in keV) / Isotopes | $E_{2_g^+}$ | $E_{4_g^+}$ | $E_{2_\gamma^+}$ | $E_{4_\gamma^+}$ | $E_{4_g^+} / E_{2_g^+}$ | $E_{2_\gamma^+} / E_{2_g^+}$ | $E_{4_\gamma^+} / E_{2_g^+}$ |
|---|---|---|---|---|---|---|---|
| $^{174}$Hf | 92 | 298 | 901 | 1063 | 3.24 | 9.79 | 11.55 |
| $^{176}$Hf | 89 | 290 | 1227 | 1391 | 3.26 | 13.79 | 15.63 |
| $^{178}$Hf | 94 | - | 1277 | - | - | 13.05 | - |
| $^{180}$Hf | 94 | - | 1201 | - | - | 12.78 | - |

## 3- INVESTIGATION OF THE ROTATIONAL ENERGY LEVELS OF THE Hf ISOTOPES

The deformation parameters $\beta_0$ , the pairing forces of the neutrons and protons $G_n$, $G_p$, the f parameters, the intrinsic quadrupole moments $q_0$ and the quadrupole moments $q(2^+)$ were calculated using the experimental energy values and $\delta(E2/M1)$ mixing ratios. For the calculated values see the Figures(1 trough 6).

(The variation of the energies $E_{2_\gamma^+}$ and $E_{4_\gamma^+}$ was drawn in the Figure.2 as a function of the neutron number).

III) $3.24 \leq E_{4_g^+} / E_{2_g^+} \leq 3.26$, $9.79 \leq E_{2_\gamma^+} / E_{2_g^+} \leq 13.79$ and $1.55 \leq E_{4_\gamma^+} / E_{2_g^+} \leq 15.63$ ratios can be calculated by dividing the corresponding energy values. The first ratio has a relatively low value with respect to the two last ratio values.(see the Figure.3). Since an ideal $\gamma$ instability is about the 2,5[1] and $E_{4_g^+} / E_{2_g^+}$

some what greather than 3, there is a $\gamma$ instability in this region.

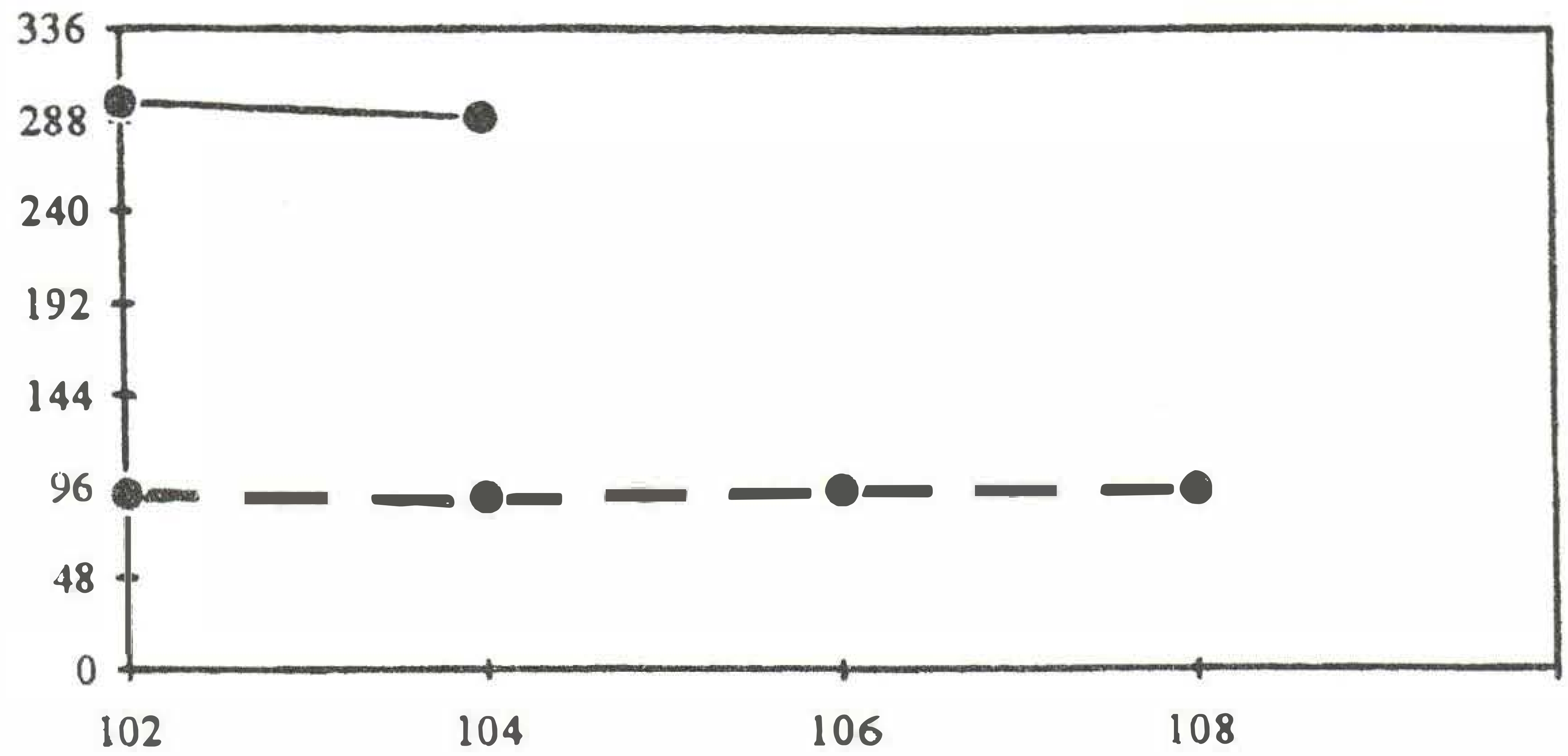


Figure. 1.The variation of the ground state $E_{2_g^+}$ and $E_{4_g^+}$ energies depending on the neutron number N for the even-even Hf Isotopes,(dashed line represents $E_{2_g^+}$ the and solid line corresponds to the $E_{4_g^+}$ )

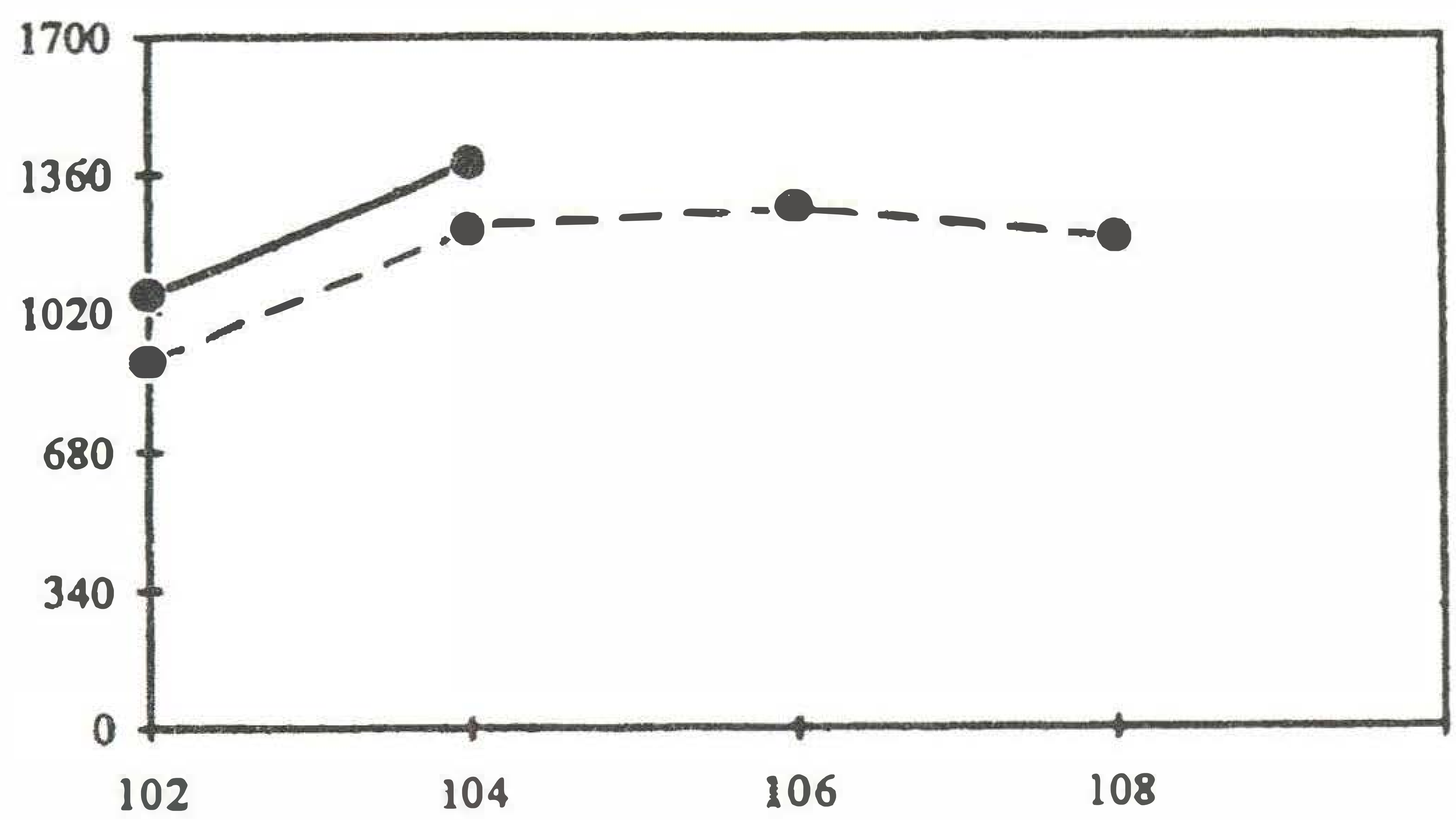


Figure. 2. Neutron number dependence of the gamma band energies $E_{2_\gamma^+}$ and $E_{4_\gamma^+}$ for the even-even Hf isotops (Dashed line corresponds to the $E_{2_\gamma^+}$ and solid line represents the $E_{4_\gamma^+}$ )

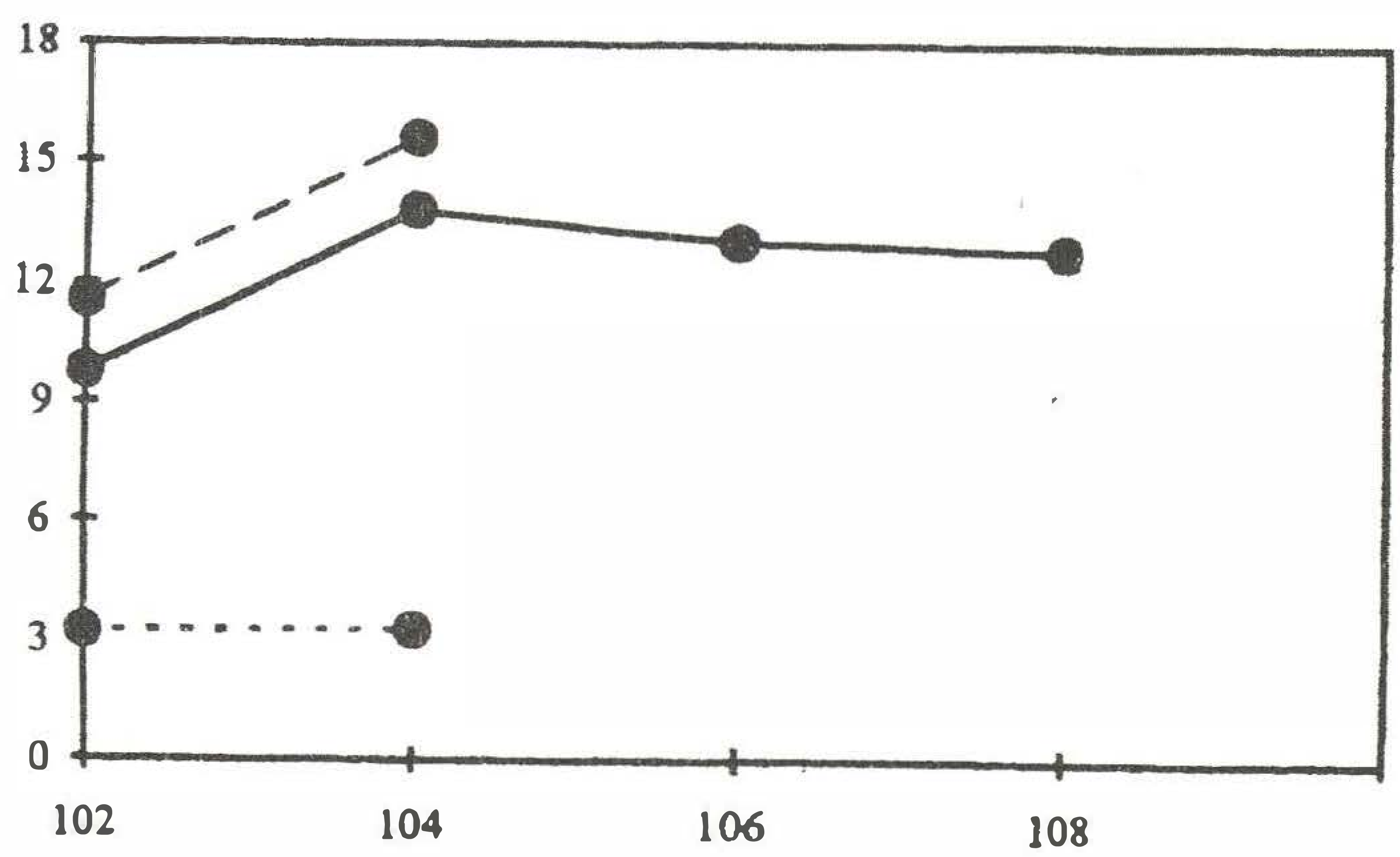


Figure 3. Behavior of the $(E_{2_\gamma^+} / E_{2_g^+})$ , $(E_{4_g^+} / E_{2_g^+})$ and $(E_{4_\gamma^+} / E_{2_g^+})$ ratios depending on the neutron number N for the even-even Hf isotops

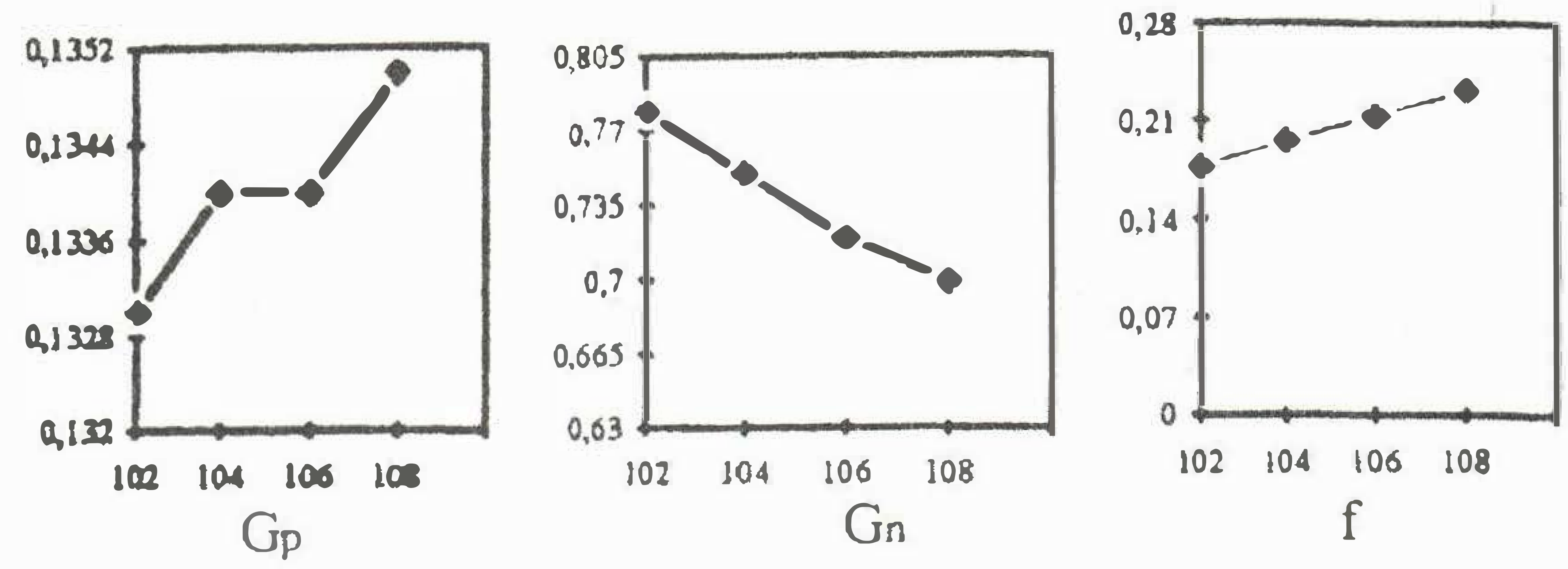


Figure 4. The variation of the parameters $G_p$ , $G_n$ and f depending on the neutron number N

δ(E2/M1) mixing ratio values of multipole mixtures for transitions from the excited higher to the lower levels in the Hf isotopes were given in the Table 2.

Table 2. The Experimental δ(E2/M1) Mixing Ratios for the even-even Hf Isotopes.

| Transitions / Isotopes | $2^+_\gamma \rightarrow 2^+_g$ | $4^+_\gamma \rightarrow 4^+_g$ | $6^+_\gamma \rightarrow 6^+_g$ |
|---|---|---|---|
| $^{174}Hf$ | $-(11^{+\infty}_{-7})^{(7)}$ | $-(2.5^{+1.3}_{-0.7})^{(7)}$ | $-0.9(2)^{(7)}$ |
| $^{176}Hf$ | $\geq 4^{(7)}$ | $\geq 0.7^{(7)}$ | - |
| $^{178}Hf$ | $(0.41^{+0.09}_{-0.07})^{(8)}$ | - | - |
| $^{180}Hf$ | $0.7(2)^{(8)}$ | - | - |

The calculated deformation parameters as a function of the multipole mixing ratios, the pairing forces for the protons and neutrons, and the f parameters for the even-even Hf isotopes were presented in the Table 3.

As the Table 3. shows the pairing force of the protons increases regularly, from 0.133 to 0.135 as the neutron number increases. In a contradiction, the pairing force of neutrons regulerly decreases, from 0.078 to 0.070, as the neutron number increases. Similarly, there is a reguler increment in the f parameters of the even-even Hf isotopes. And there is a good agreement between the values calculated in this study and Kumar and Löbner values for $^{174}Hf$ isotope[3,9,13] and the obtained values for the deformation parameters $\beta_0$.

Table 3. The Computed values for the some important parameters for the even-even Hf Isotopes. *Calculated values in this work

| Isotopes | $G_p$ | $G_n$ | f | $\beta_0^*$ | $\beta_0$[ref.13] |
|---|---|---|---|---|---|
| $^{174}Hf$ | 0.133 | 0.078 | 0.179 | 0.270 | 0.250 |
| $^{176}Hf$ | 0.134 | 0.075 | 0.197 | 0.120 | 0.250 |
| $^{178}Hf$ | 0.134 | 0.072 | 0.215 | 0.020 | 0.240 |
| $^{180}Hf$ | 0.135 | 0.070 | 0.234 | 0.020 | 0.230 |

The values of the parameter $\beta_0$ are ($0.02 \leq \beta_0 \leq 0.27$) in accordance with systematic in the deformation region.

The results for the quadrupole moments $q_2^+$ and intrinsic quadrupole moments $q_0$ for the even-even Hf isotopes will be as given in the Table 4.

Table 4. calculated $q_0$ and $q_2^+$ values of the even-even Hf isotopes

| Isotopes | $^{174}Hf$ | $^{176}Hf$ | $^{178}Hf$ | $^{180}Hf$ |
|---|---|---|---|---|
| $q_0$ e.barn (this work) | 7.19 | 3.01 | 0.48 | 0.45 |
| $q_0$ e.barn [13] | 6.95 | 7.18 | 6.96 | 6.84 |
| $q_{2^+}$ e.barn | 2.05 | 0.86 | 0.14 | 0.13 |

## 4.RESULTS AND DISCUSSIONS

In the present work, variations of the energy values of the excited states, multipole mixing ratios δ(E2/M1) , the pairing forces of protons $G_p$ ,the pairing force of neutrons $G_n$, the f parameters , the deformation parameters $\beta_0$ and the quadrupole moments have been subjected in all details for the even-even Hf isotopes found in the end section of the $150 \leq A \leq 190$ deformed region. It has been observed that the pairing force of the protons $G_p$ increases as the neutron number N increases while the pairing force $G_n$ of the neutrons decreases. The parameter f has the similar character with the $G_p$ . The deformation parameter $\beta_0$ has a second order dependence on the negative of the parameter f and it decreases according to the Eq.(3). The details of the behaviours of the parameters , mentioned above.can be seen from the Table 3. The values of the parameter $\beta_0$ for $^{174}$Hf isotope is in a good agreement with the calculations given by Kumar and Löbner[3,9,13]. The Kumar's values are $-0.5 \leq \beta_0 \leq 0.5$ for the deformation region of $150 \leq A \leq 190$.

As the Table 4. shows the quadrupole moments of the Hf isotopes are good indicator of the deformation amounts of the nuclei and they rapidly decrease as the neutron numbers increase. These results are also in a good agreement with the Kumar's work[9].

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