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Reevaluating Froebel Gifts: A Tool That Embodies Three Dimensions in Space to Enhance Pre-school Children's Geometry Skills

Ayşe Pınar Şen[1], Çiğdem Kotil

Brock University, Canada

ABSTRACT

Froebel Gifts are the first educational materials designed for kindergarten children in the education history. After Froebel introduced the first construction and design materials for early childhood mathematics education in 1850's, several companies such as Lego Bricks, Lincoln Logs, and K'nex were influenced by his design. Considering the influence of Froebel Gifts on mathematics and geometry education tools for young children, it is important to analyze their impact on pre-school children's geometry skills. This study investigated the effects of a geometry education program mediated with Froebel Gifts on 5-6 years-old children's geometric skill development. The participants consisted of 40 pre-school children in Istanbul, Turkey. Twenty children in the experimental group received an 8-week intervention with Froebel Gifts while the control group continued their regular program. Early Geometry Skill Test (EGST) was used before and after the intervention and the collected data were analyzed with independent sample T-test. The results indicate a significant change in favor of the experimental group in the skills of building with blocks, recognizing the three-dimensional objects, predicting the surface shape of the three-dimensional objects, and recognizing the side and corner properties of two-dimensional shapes. The teacher's instructional strategies and the physical and representational properties of Froebel Gifts are discussed and evaluated in the light of these results to provide insight about the features of an effective geometry education.

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1. Introduction

Concrete instructional materials such as Lego Bricks, Lincoln Logs, K'nex, unit blocks, tangrams, and pattern blocks (Read, 1992) are widely used in early childhood geometry education in order to facilitate mathematical thinking (Figueira-Sampaio et al., 2013). These materials introduce the basic concepts in geometry (e.g., point, line and plane, etc.) however, given that geometry is a coherent and cumulative set of concepts, the materials used should also allow the formation of geometric integrity in the child's mind (Clements et al., 2004). Therefore, introducing the basic geometry concepts is not enough for a meaningful learning in pre-school, but the relationship between those concepts must be highlighted. According to Froebel, tangible objects are fairly easier to comprehend compared to the relationships between those objects and their properties because they are very abstract. "A correct comprehension of external, material things is a preliminary to a just comprehension of intellectual relations" (Smith and Wiggin, 1895, p. 1). This means that the children could learn the abstract

[1] Corresponding author's address: Brock University, Canada
e-mail: apinarsen@hotmail.com
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geometry concepts and relations only after they understand the tangible objects with their properties such as their edges, vertices, or surface.

Researches on the subject also emphasize the importance of concretizing abstract concepts in mathematics education (Koğ and Başer, 2011; Yaman and Şahin, 2014). However, the use of concrete objects does not in itself guarantee a meaningful educational experience where children learn abstract geometry concepts. Goutard (1964) mentions that there has been a strong tendency to use concrete objects in math education which may limit children in an empirical mode of thinking. For a holistic understanding of geometry, the concrete objects must be used to symbolize the abstract relationships between concepts and objects and the instructions must lead children to think beyond the concrete or empirical level of thought. For example, Gattegno (1963) used Cuisenaire rods in mathematics curriculum to provide children with experiences with objects of different lengths to make comparisons. The activities included aligning the lengths for comparison and adding a rod of appropriate length when one rod's length is longer than the other which is associated with addition and subtraction. Davydov (1990) also focused on the comparison of measures in teaching math and geometry and approached the number as a dynamic relation, a relation between quantities (Coles, 2017). The use of concrete objects to move the children away from concrete thinking may sound ironic, however the key point is about how the concrete objects and the relationships between them are being symbolized. For example, if a geometric shape is always constructed using the rods, even though the lengths of the rods might differ, the children may remain in an empirical thinking level because the abstract relationship between the point, line and plane is not symbolized explicitly. However, lining beads to form the sides of a two-dimensional geometric shape or combining rods side by side to form a two-dimensional plane would concretize the relationship between the concepts of point, line and plane.

The existing literature emphasizes that most of the geometry materials including activity books and videos published for kindergarten children fail to introduce the geometry content adequately since they involve only typical geometric shapes such as equilateral triangle, square, circle and rectangle (Aktas-Arnas and Aslan, 2007; Dagli and Halat, 2016; Oberdorf and Taylor-Cox, 1999). If children are not introduced to atypical examples of geometric shapes such as isosceles triangle, right-angled triangle, trapezoid, rhombus, semi-circle, their shape perception would be limited to the prototypes. Another problem is that the geometry content is usually presented in isolation without establishing a contextualized understanding (Aktas-Arnas and Aslan, 2007; Walker et al., 2018). Abstracting the definitive properties may be very difficult for kindergarten children unless efficient instruction which concretizes the relationships of the geometry concepts and a variety-rich geometry content are provided (Barth, Mont, Lipton, and Spelke, 2005; Cetin and Dubinsky, 2017; Dubinsky, 2002; Mitchelmore and White, 2007). Manipulatives and instructions that do not relate and integrate with one another may cause the children to perceive geometrical concepts irrelevant to each other and thus lead to a non-permanent learning (Son, Smith, and Goldstone, 2008). Gray and Tall (2007) indicated that abstraction has significant role in learning geometry, especially in the formation of triangle and quadrilateral concepts, because children identify the shapes by observing the similarities, doing classification based on the characteristics of the objects, finding the observable properties of the concepts, and finally generating a concept of each shape by abstracting. Research indicates that the content, the instruction and the educational materials provided to children in pre-school mathematics activities must be contextualized to facilitate children's abstract thinking (Aktas-Arnas and Aslan, 2007; Sarama and Clements, 2009; Walker, 2018).

At this point instructional strategies and manipulatives gain prominence in facilitating children's understanding of the relationships between shapes and their definitive properties. Teachers' structurally aligned and guided instructions are crucial for children to form relational abstraction in shape recognition as well as the supporting manipulatives (Christie and Gentner, 2010; Yuan et al., 2017). Studies indicate that children's initial tendency in focusing on the visual attributes of geometric shapes generally results in overshadowing the properties such as side length, vertices, and angle

(Kalénine et al., 2011; Son et al., 2008; Walker et al., 2018). Froebel Gifts provide a diversity of geometric shapes that children can compare in far and close similarity options. For example, the diversity of triangles has the potential to enrich the children's triangle classification and allows them to make the triangle definition more accurate while representing the determinant aspects of triangles in each example (e.g., isosceles triangle, right-angled triangle, equilateral triangle). It is hypothesized that these comparison activities in Froebel Gifts Geometry Education Program would expand children's mental shape imagery and thus result in more in-depth considerations regarding the geometric shape classifications. In a geometry education mediated with Froebel Gifts, teachers' instructions are aligned with the structural design of Froebel Gifts. The teachers gradually introduce the geometry activities and manipulatives, beginning with three dimensional shapes and gradually proceeding to two dimensional shapes and to the concept of point. It is hypothesized that the gradual introduction of geometric shapes from solids to planes, from planes to lines and from lines to points with a rich geometric shape content included in Froebel Gifts have the potential to facilitate children's shape recognition and geometry skills. The aim of this study is to evaluate the use of Froebel Gifts in promoting geometry skills of 5-6 years-old pre-school children. This assessment was conducted by comparing the score change in the pre and post geometry test scores of the experiment and control group children. The questions that lead this study are:

- Is a geometry education program mediated with Froebel Gifts effective on 5-6 years old kindergarten children's geometry skills?

- If effective, which sub-skills in particular do the program have an impact on?

2. Literature Review and Theoretical Framework

Geometrical understanding naturally occurs by interacting with objects in everyday life and thinking about the spatial relationships between the objects. Duval's (1995) cognitive model of geometrical reasoning elucidates the nature of geometrical reasoning and how it develops. In this model, he identifies four types of cognitive apprehension as the following:

1. Perceptual apprehension: This stage represents the visual perception, in other words, what is recognized as a figure at first glance. This stage can be considered as a pre-abstraction stage because the object or the figure is perceived overall without specifically focusing on the properties or the components of the figure.

2. Sequential apprehension: This stage can be described as the cognitive process used when constructing a figure or when describing its construction. In this case, the figural units are perceived as mathematical and technical constraints, rather than just a visual figure. This stage is the first stage of abstract thinking where children classify the objects based on their similarities.

3. Discursive apprehension: This stage describes that perceptual recognition of figures depends on discursive statements because mathematical aspects of a drawing cannot be determined by perceptual apprehension. Certain parts of it that cannot be represented in the drawing, or on the shown object must be explained through speech. For example, when children are able to express how they can change a shape into another shape by describing the process with appropriate vocabulary and accurate reasoning, we can say that they are at the stage of discursive apprehension (Ramatlapana and Berger, 2018). In this level of reasoning, children can suggest abstract definitions of shapes, such as "all the closed shapes that has three straight lines and three corners can be classified as triangle". At this stage, objects are operated cognitively as mental objects and appropriate vocabulary is used to represent these mental objects and their relationships.

4. Operative apprehension: This stage can be described as making meaningful changes or operations on the figure, either mentally or physically. In this stage, mental operations themselves become new objects of thought in the operation of reflective abstraction where individuals perform mental actions on mental concepts. Abstract understanding of geometric shapes and spatial relationships is essential for the mental operations to become the new object of thought in a geometry problem. This level of

reflective abstract thinking is not expected from kindergarten children who are typically classifying geometric shapes based on visual perception or on their definitive properties (Duval, 1995, as cited in Jones, 1998).

Duval also proposed three cognitive processes in geometrical reasoning which are: (i) visualization process which refers to a basic visual representation of a complex geometric shape or situation; (ii) construction process which refers to using tools to express geometric understanding, and (iii) reasoning processes which refers to discursive attempts to extend the current knowledge or proof (Halat and Dağli, 2016). In this study, Duval's cognitive processes were taken into account while designing the geometry activities. Each activity involved a free play period which allowed children to engage with the visual attributes of the objects, a block building, shape composing, and decomposing activity to facilitate the construction process, and finally discussion of the activity to foster children reasoning about the geometry concepts and operations.

2.1. Development of Geometry Skills in Children

Children understand geometry as they observe, analyze and build knowledge of shapes and their spatial relations to other shapes and objects. Children conceptualize geometric shapes by analyzing characteristics and properties of two- and three-dimensional geometric shapes and considering relationships of their definitive properties gradually as the former learnings establish a base for more complex and abstract concepts (Clements, 1999; Clements, 2003; Clements et al., 2003; Sarama and Clements, 2009; Willingham, 2009). Young children gain informal geometry knowledge through their everyday experiences; however, carefully aligned instructions and instructional materials are fundamentally important to establish an intellectual connection between children's informal and formal knowledge (Baroody, 2017). Children visualize the mental images of geometric shapes through their spatial memory and it is crucial to enrich their spatial memory (NCTM, 2000). Symbolizing the definitive properties of these shapes by using rods to represent lines, beads to represent vertices and tablets to represent the planes can lead them to recognize and represent shapes from different perspectives, and to recognize geometric shapes and their structures in different environments.

Studies emphasize that guided alignment in instructions is an important factor to facilitate pre-school children's ability to form hypotheses about the relationships of the shapes (Yuan, Uttal and Gentner, 2017). These instructions should begin with introducing the children with a class of shapes, say cubes, and later on proceeding to rectangular prisms to help them form one or more possible hypotheses from earlier examples with cube that can be tested against later examples with rectangular prisms (Christie and Gentner, 2010). In this way, the definitive properties of cube can be formed when the first exemplars are introduced and are subsequently refined when the next exemplar appears. The process of comparison has the potential of highlighting common structures between two shapes which would then be more apparent and available to apply it on new categories of shape (Christie and Gentner, 2010).

The most typical indication of lack of geometric knowledge is that children have limited imagery of different geometric shapes and their variances (e.g., isosceles triangles, right-angled triangle, obtuse triangles, and right-angled isosceles triangles). This is because the children's shape schemes adhere to a prototype when the shapes are introduced in isolation without concretizing the relationships between the shapes. For example, 5-year-olds categorized some of the different triangles shown to them as half-triangles and combined some of them to resemble the isosceles triangle because they fit the triangle prototype in their visual memory. The most common triangle that they recognized was the equilateral triangle (Charlesworth, 2005). Five- and six-years old children generally identify shapes by pairing them with a visual prototype (Aktas-Arnas and Aslan, 2005).

Research (Clements and Sarama, 2000) indicates that a significant majority (96%) of kindergarten children, recognize and correctly classify circles, even though they occasionally add the ellipse in circle category. Eighty-seven percent of children recognize and classify squares correctly, but a small proportion of them add/include non-square rhombuses into the square category. Even though 54% of the children recognized the rectangles and categorized correctly, they usually consider all parallelogram shapes with two parallel edges and square-like shapes as rectangles (Clements and Sarama, 2000). Children's overextension and under inclusion behaviours in shape categorization depends on their knowledge about the relationship between the properties of the shapes. Research (Boswell and Green, 1982) shows that children are more successful in categorizing the shapes as strangers and familiars generally, when they are shown a prototype. However, they have more difficulty in selecting the exact prototype. Children's difficulty in considering the definitive properties of shapes might be due to the lack of comparison activities in which children can focus on the quantitative and qualitative differences between very similar and/or different shapes, while manipulating these properties of the objects.

Aktas-Arnas and Aslan (2010) examined the criteria that 3-6-year-old kindergarten children take into consideration in recognizing and differentiating geometric shapes. The results indicate that young children are more familiar with typical geometric shapes, but they were less successful in recognizing atypical geometric shapes of different sizes, skewness or kurtosis, and often categorize by considering the visual characteristics of the shapes. The results of a more recent study which examined 5-6 years old preschool children's recognition of geometric shapes show that children had difficulty in differentiating the triangle, square, rectangle, and circle when there is a distractor. Researchers recommend increasing the variety of shapes in educational content as a possible solution to this problem (Kesicioglu et al.,, 2011). Introducing children to a variety of geometric shapes gradually and explaining the relationship between the shape's properties and the shape's name is crucially important in kindergarten geometry education. Researchers indicate that very few of the elementary school students know both the names and properties of geometric objects correctly and the students generally have misconceptions about the prism and cubes, squares and rectangles (Incikabi and Kilic, 2013).

2.2. Froebel Gifts

Influenced by the educational approach of Jean-Jaques Rousseau (Rousseau, 1762) and Pestallozzi (1894), Friedrich Froebel developed a series of educational materials (the gifts), activities (the occupations) in 1850's to encourage children's autonomous learning. In Froebel's approach, teachers encourage children to imitate shapes or mechanisms in real life with the pieces in the gifts. With regards to his design principles, he designed objects that are developmentally appropriate for kindergarten children and can be introduced in a gradual way, in a modular fashion (Bruce, 2012; Bulow, 2007; Froebel, 2009; Hewitt, 2001; Zuckerman, 2006).

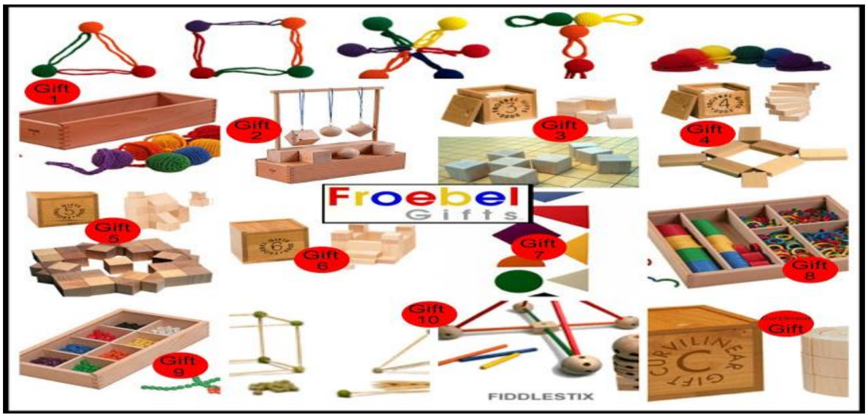

Image 1. Froebel Gifts (Retrieved on 04.21.2021 from http://www.epic-childhood.com/2016/01/froebel-gifts-original-spielgabe-or.html)

Gift 1 includes 8 wool balls and highlights the similarity and dimensions of similarity in shape and color (See image 1). Gift 2 highlights that the cube, cylinder and sphere have similar properties; however, cylinder and sphere have more similar properties than cube. Gift 3 emphasizes the multiplication and fraction through eight identical cubes that form a bigger cube (Eugene and Provenzo, 2009). Gift 4 emphasizes that rectangular parallelepiped and square prisms are the equivalent in terms of the number of edges, vertices and faces; however, they are different in terms of edge length. The comparison opportunities that Gifts 5-6 underscore that cube, oblong prism, rectangular parallelepiped and square prism are more similar in terms of the number of edges, vertices and faces than triangular prism (Reinhold et al., 2017; Stiny, 1980). Gift 7 includes square, rectangle, rhomboid, parallelogram, trapezoid, and quadrilateral, equilateral triangle, isosceles triangle, right-angled isosceles triangle, right-angled triangle, obtuse-angled isosceles triangle, circle and semicircle tablets (Correia and Fisher, 2014). With the activities in Gift 7, children can compare the properties of different shapes in general to categorize them. Then they can compare fine details of similar shapes to find differences between them and re-organize them in between each other. The figures in the 7th gift are obtained from the surfaces of the 3D objects in the first six gifts. The flat tablets emphasize the concept of face or plane; thus, this gift is important for the transition from solids to flat surfaces (planes). The first 6 gifts enable the child to form miniature forms of real three-dimensional objects. The 7th gift allows the child to comprehend the two-dimensional form of these 3D objects and create a mental image for these abstract concepts (Hill, 1908; Palmer, 1912).

Gift 8 symbolizes the transition from the surface to the line concept, embodying the boundaries, edges and lines of surfaces and three-dimensional objects. In this gift, the concept of length is particularly emphasized, and the most characteristic feature of the gift is that it deals with the concept of straight or curved lines (Correia and Fisher, 2014). The alignment of concepts in Froebel Gifts proceeds to the concept of point with Gift 9. This gift introduces the point on its own as the most basic building block of previous gifts. In fact, although there is no length, depth, or width of the point, it is transformed into a tangible object for the child to understand it in a concrete way (Wilson, 1967). Gift 9 provides an opportunity to the child to comprehend the concept of straight line and that a line is the shortest distance between two points and changes direction at each point of a curved line. All planar or curvilinear geometric shapes or all angles can be represented with dots (Woodard, 1979). The concepts that the gifts symbolize are gradual, from the relation of solids with concrete concepts, to the relations of spatial patterns with abstract concepts.

With the last gift, Gift 10, abstract concepts such as shape, form, area, line and point can take a solid and compound form. For this experience, the use of all previous gifts must be internalized (Correia and Fisher, 2014). Gift 10 includes all the components of the previous gifts such as plane, line and point (Friedman, 2018). It reifies the similarities between geometric shapes' components such as sides, planes, vertices and highlights the property similarity rather than an overall visual similarity. In this way, children can disassemble the perceived three-dimensional objects into its components such as surfaces, edges, and points physically with the gifts and create mental concepts for these operations during these activities.

3. Methods

In this study, Froebel Gifts are used to foster pre-school children's geometry skills by embodying the abstract concepts of three dimensions in geometry. To evaluate the effects of a geometry education program mediated with Froebel Gifts on preschool children's geometry skills, a quasi-experimental study was designed to control the possible independent variables on pre-school children's geometry skills. Studies suggest that parent's educational level, income status, socio-economical status have strong impacts on children's mathematics achievement (Braham and Libertus, 2017; Valero et al., 2015). Therefore, in determining the study group, a general information tool was used to analyze the socio-economic and educational level of parents. In addition to the general information tool, a consent form was sent to the parents of 60-72 months old children in the school. After collecting and analyzing the data on the general information form and consent form, 2 pre-school classes were chosen as participants of this study. Experimental group received Froebel Gifts Geometry Education Program (FGGEP) while control group continue their geometry activities based on the preschool education program of the Ministry of National Education. Experimental group's teachers received the FGGEP two weeks prior to the pre-test and adapted themselves and the classroom to the program. Twenty sets of Froebel Gifts were brought to the school and stored on the attic for teachers to bring them for children's use respectively according to the FGGEP. Both the experimental and control groups did geometry activities 4 days a week, 1 hour a day. The control group did geometry activities following the National Ministry of Education's kindergarten curriculum. The question that leads this analysis is: Is there a significant difference between the group's geometry score changes between pre-test and post- test?

3.1. Frobel Gifts Geometry Education Program

According to the Froebel Approach, the gifts must be introduced to children in a specific order from gift 1 to gift 10 respectively. In this process, the focus is on introducing children to a variety of geometric shapes and leading them to understand the fundamental geometry concepts (e.g., point, line, area, vertices, edges, etc.) and mathematical procedures (e.g. adding, subtracting, counting. etc.) while playing with gifts actively (Manning, 2005; Park and Vakalo, 2000). The experiment group teachers were trained about the Froebel Gifts Geometry Education Program (FGGEP) and the Froebel Gifts by the researcher for five days prior to the intervention. The activity book and the materials were given to the teachers 2 weeks prior to the intervention to provide enough time for teachers' engagement in the activities and the materials. The teachers guided the geometry activities and provided the gifts to the children according to the activity book. The most important role of teachers in the experiment group was to ask the right questions to lead children to think targeted geometric concepts (e. g., Why do we call it triangle? Is square a quadrilateral? Does the word imply any numbers? or Do cube, cylinder and sphere roll on the floor?) (Ransbury, 1982). The aim was to reinforce children's geometry preconceptions and facilitate their transition to more complex information processing (Tovey, 2016). The concepts and manipulatives in FGGEP gradually proceed to abstract notions with a specific alignment moving from: three dimensional to two dimensional, from surface to line, and from line to point (Eugene and Provenzo, 2009) while the manipulatives act as an example.

FGGEP gradually introduces the geometry concepts and ends with the 10th gift called scaffolding, integrating the previously learned concepts. The alignment of objects and activities in FGGEP emphasizes the fundamental abstract properties such as vertices, angles, sides, surface by introducing a rich variety of geometric shapes and their components. In this way, children can ignore the surface features such as instant visual shapes and focus on the underlying, deep structures such as the number of vertices or sides. Particularly, the richness of the geometric shape contained in gift 7 can be an effective solution to the lack of atypical shapes in educational contents which cause the children to refer only to typical prototypes in recognizing a specific class of shape emphasized by research (Aslan and Arnas, 2007; Clements and Sarama, 2011; Posnansky and Neumann, 1976).

The teachers in the experiment group introduced the gifts from 1 to 10 respectively and facilitated activities with each gift in four phases in accordance with Duval's cognitive apprehension model to help students understand geometry concepts. In first phase, students engaged with the gift in free play setting. In phase 2, teachers asked students to identify and define each shape that they encountered during their free play. In phase 3, teachers asked the students to form as many examples as possible of each type of shape (e.g., triangle, quadrilateral, etc.) that they had defined during phase 2, using the pieces of the gifts. In phase 4, teachers asked the students why they formed the shapes the way they did and invited the students evaluate each other's shapes by expressing their own reasoning.

3.2. Geometry Activities in Pre-school Program of Ministry of Education

The geometry activities that are prepared according to the Pre-school Program of Ministry of Education (PPME) are designed in line with the acquisitions (learning outcomes) and indicators in the field of cognitive development (Ministry of Education, 2013). The teachers in the control group continued to their daily activities including geometry activities 4 days a week. Teachers provided wooden blocks which were all cube shape in different sizes, and colorful cardboards of certain geometric shapes such as equilateral triangle, rectangle, square and circle in their activities. They asked children to observe the given object(s) and tell the name, color, shape, size, length, texture, sound, quantity and purpose of use of the object(s). Some other activities involved matching the objects according to their properties; matching objects with their shadows or images; distinguishing and comparing the properties of objects and grouping objects according to their properties.

The teachers often asked children to say the name of the displayed geometric figure to see if the children recognize geometric shapes and name them correctly. Usually, the activities followed by the teacher asking children to explain the properties of given geometric shapes; to show similar objects; to create patterns with objects by looking at the model; to indicate the rule in the pattern consisting of at most three elements; to identify and complete the missing item in a pattern; to create a unique pattern with objects and to display the part-whole relationship by showing a whole and its' parts. The activities were guided by the teacher and the children did not have any free play opportunity with the geometry materials such as cardboards and blocks.

3.3. Participants

The population of the study consisted of 5-6 years old pre-school children in all kindergartens affiliated to the Ministry of National Education within the provincial borders of Istanbul in 2015-2016 academic year. In determining the sample classes of the study group, the similarity of socioeconomic levels of children and similarities of class characteristics were taken into consideration. Existing natural classes were selected without disturbing the structure of the classes because all the 5–6-year-old classrooms in the school were designed almost identically including the posters and materials. The only difference between the classrooms were the number of windows, color of tables and chairs and the classrooms' locations in the building. In each classroom, there were one teacher and one assistant teacher. The study group consisted of 40 children in total, 20 children (9 female, 11 male) in the experimental group and 20 children (8 female, 12 male) in the control group attending a private kindergarten in Pendik district of Istanbul. It was decided by random assignment which class would be experimental or control group.

3.4. Data collection tools

In this study, general information form and Early Geometry Skill Test (EGST) (2016) were used as data collection tools to determine the study groups' demographics and the geometry skills of children. The general information form collects information about children's gender, date of birth, mathematics or geometry education in addition to education at school, parent's educational status, income status and number of people living in household.

EGST (Sezer and Guven, 2016) tests the children between the ages of 5-7. The scale is administered to children individually by the researcher. As a result of the analyzes, Scope Validity Index of the EGST was found to be .65, total reliability coefficient Cronbach's Alpha value was .855 and KR20 coefficient was .853. The test items are homogeneous, interrelated, and the test items are addable in terms of scores. In-group Correlation Coefficient criterion was .124, Pearson Correlation Coefficient was .697 between the two halves of the test, Spearman-Brown Coefficient was .821 and Guttman Split-Half Coefficient was .767. According to Guttman Lambda (Li) method, reliability coefficients vary between .760 and .883 values. Finally, the test-retest results of the test were .898 for the Pearson Correlation coefficient, .738 for Kendall's tau_b coefficient, and .885 for Spearman's rho coefficient. As a result of these findings, a valid and reliable test consisting of 42 items was obtained. There are fifteen sub-skills in the content of the test which are; recognition of typical examples of two-dimensional shapes, two-dimensional shape selection, two-dimensional shape drawing, recognition of the side-corner features of two-dimensional shapes, shape rotation, forming a mental shape, shape separation, shape forming with the material, forming a shape with rods, shape-ground relationship, recognition of three-dimensional shapes, pattern, perspective taking, building with blocks and surface estimation of three-dimensional shapes (Sezer, 2016).

3.5. Data Collection and Procedure

Prior to collecting the data, official permissions were obtained from the Ethics Committee of University with a report summarizing the content of the study. General Information Form and Early Geometry Skill Test were applied to experimental and control groups as pre-test prior to the intervention. Children's names were coded on the EGST sheets and recorded the same way in SPSS. The FGGEP started to be implemented in April 2016 for 8 weeks. Each child in experiment group received one set of Froebel Gifts in each activity. The post-test measurements were performed immediately after the end of the program. During the interviews, children were not given access to Froebel Gifts, however they were given a pencil and rods to demonstrate their answers to some specific questions in the test.

3.6. Data Analysis

The data obtained from the study were analyzed using SPSS (Windows 22.0). Number, percentage, mean and standard deviation were used as descriptive statistical methods in the evaluation of the data. The t-test was used to compare the quantitative continuous data between two independent groups (control-experiment) and the independent t-test was used to determine the difference between the two dependent groups (pretest-posttest). The relationship between the group variables was tested by chi-square analysis. The findings were evaluated in 95% confidence interval and 5% significance level.

4. Results

Table 1 demonstrates the pre-test and post-test geometry skill scores of the experimental group children. The results indicate that there was a significant difference between the children's pre-test and post-test scores in the sub-skills of two-dimensional shape selection, recognition of the side-corner features of two-dimensional shapes, shape separation, recognition of three-dimensional shapes, surface estimation of three-dimensional shapes and building with blocks.

Table 1: The geometry skill score changes between the experimental group's pre-test and post-test scores

Sub-skills	Pre-test				Post-test				N	t	p
	Mean	Sd	Min	Max	Mean	Sd	Min	Max			
Recognition of typical examples of two-dimensional shapes	3,000	0,000	3	3	3,000	0,000	3	3	20	-	1,000
Two-dimensional shape selection	15,500	3,035	7	21	23,200	3,750	14	30	20	-11,182	**0,000**
Two-dimensional shape drawing	1,500	0,946	1	3	1,800	0,894	0	3	20	-0,972	0,343
Recognition of the side-corner features of two-dimensional shapes	0,950	0,686	0	2	1,600	0,503	1	2	20	-4,333	**0,000**
Shape rotation	2,100	0,641	1	3	2,250	0,851	1	3	20	-1,000	0,330
Forming a mental shape	1,650	0,587	1	2	1,750	0,444	1	2	20	-0,809	0,428
Shape separation	1,700	0,571	2	2	2,000	0,000	2	2	20	-2,349	**0,030**
Shape forming with the material	1,100	0,308	0	2	1,000	0,000	1	1	20	1,453	0,163
Shape forming with rods	1,450	0,887	0	2	1,800	0,834	0	3	20	-1,505	0,149
Shape-ground relationship	2,800	0,523	2	3	2,850	0,366	2	3	20	-0,438	0,666
Recognition of three-dimensional shapes	2,850	1,725	2	6	4,750	0,910	2	6	20	-5,146	**0,000**
Recognition of patterns	1,850	0,875	0	4	1,950	0,686	1	3	20	-0,623	0,541
Perspective taking	0,400	0,503	0	1	0,450	0,510	0	1	20	-0,567	0,577
Building with blocks	1,850	1,137	2	4	2,700	0,865	2	4	20	-5,667	**0,000**
Surface estimation of three-dimensional shapes	1,600	0,681	2	3	2,100	0,447	1	3	20	-4,359	**0,000**
Total score	40,250	6,592	33	50	53,400	7,044	37	66	20	-11,669	**0,000**

Table 2 presents the pre-test and post-test geometry skill scores of the control group children. The table indicates that there was a significant change in control group children's scores of shape rotation, forming a mental shape, building with blocks and surface estimation of three-dimensional shapes.

Table 2: The geometry skill score changes between the control group's pre-test and post-test scores

Sub-skills	Pre-test				Post-test				N	t	p
	Mean	Sd	Min	Max	Mean	Sd	Min	Max			
Recognition of typical examples of two-dimensional shapes	3,000	0,000	3	3	3,000	0,000	3	3	20	-	1,000
Two-dimensional shape selection	13,900	3,386	7	21	14,050	3,546	9	21	20	-0,411	0,685
Two-dimensional shape drawing	1,400	0,940	0	3	1,800	0,768	0	3	20	-1,902	0,072
Recognition of the side-corner features of two-dimensional shapes	1,050	0,510	0	2	1,100	0,447	0	2	20	-0,370	0,716
Shape rotation	2,250	0,639	1	3	2,050	0,605	1	3	20	2,179	**0,042**
Forming a mental shape	1,150	0,745	0	2	1,600	0,503	0	2	20	-3,327	**0,004**
Shape separation	1,900	0,308	1	2	2,000	0,000	0	2	20	-1,453	0,163
Shape forming with the material	1,000	0,649	0	2	1,050	0,510	1	2	20	-0,567	0,577
Shape forming with rods	1,150	0,745	0	2	1,200	0,768	0	3	20	-0,295	0,772
Shape-ground relationship	2,750	0,444	2	3	2,650	0,489	1	3	20	1,000	0,330
Recognition of three-dimensional shapes	3,200	1,240	1	5	3,450	1,099	0	6	20	-0,839	0,412
Recognition of patterns	2,200	1,196	0	4	2,450	1,099	0	3	20	-1,751	0,096
Perspective taking	0,100	0,000	0	1	0,100	0,000	0	1	20	-	1,000
Building with blocks	2,550	0,945	1	4	2,950	0,686	0	4	20	-3,559	**0,002**
Surface estimation of three-dimensional shapes	1,900	0,553	1	3	2,100	0,308	1	3	20	-2,179	**0,042**
Total score	39,550	5,186	29	48	41,750	4,711	30	51	20	-4,881	**0,000**

Table 3 shows that, FGGEP has a positive and greater impact on children's geometry skills in total compared to the PPME in which the control group was trained. In particular, FGGEP was effective only on certain geometry skills such as two-dimensional shape selection, recognition of the side-corner features of two-dimensional shapes, recognition of three-dimensional shapes, surface estimation of three-dimensional shapes, and building with blocks. However, there was no significant difference between the score changes of the experiment and control group in the skills of recognition of typical examples of two-dimensional shapes, two-dimensional shape drawing, shape rotation, forming a mental shape, shape separation, shape forming with the material, shape forming with rods, shape-ground relationship, recognition of patterns, and perspective taking. The strengths of the program were revealed by discussing the increased scores and the reasons affecting the total score in the discussion section.

As indicated in Table 3, one of the greatest differences between the groups score change was observed in the sub-skill to select two-dimensional shapes in the favor of the experiment group. The items that measure this skill are, valid, invalid, typical and atypical geometric shapes shown to children in which they were asked to select certain geometric shape(s). It can be said that the activities of the experimental group such as the introduction of atypical shapes in the framework of the FGGEP, the discussion of the features of these shapes, the formation of typical and atypical shapes with rods and beads are effective in assisting children to understand the definitive properties of shapes.

The results demonstrate a significant change in the subskill to recognize the side-corner features of two-dimensional shapes in favor of the experimental group. Comparing the pieces of different gifts allowed the children in experiment group to reinforce the perceived similarities based on repetitive and determinant aspects of objects. The alignment of the objects and concepts in Froebel gifts represent the order of first being aware of differences between the objects, and secondly distinguishing the qualitative and quantitative properties. The gifts emphasize the qualitative differences of the objects in the earlier gifts, with a transition to quantitative aspects through the last gift. As an example of these levels, Froebel Gifts introduces concepts deductively by allowing the children to perceive the objects without being aware of the specific details such as edge, angle, vertices, face, length, or area. As they keep on focusing on the elements of the same shape class, they can further differentiate those details (Kelley, 1984). This process allows children to differentiate the side-corner features of two-dimensional shapes.

Table 3. The means of score changes between pre-test and post-test scores by groups

Sub-skills	Control (n=20)		Experiment (n=20)		T	p
	Mean	Sd	Mean	Sd		
Recognition of typical examples of two-dimensional shapes	0,000	0,000	0,000	0,000	-	-
Two-dimensional shape selection	0,150	1,631	7,700	3,080	-9,689	**0,000**
Two-dimensional shape drawing	0,400	0,940	0,300	1,380	0,268	0,790
Recognition of the side-corner features of two-dimensional shapes	0,050	0,605	0,650	0,671	-2,971	**0,005**
Shape rotation	-0,200	0,410	0,150	0,671	-1,990	0,054
Forming a mental shape	0,450	0,605	0,100	0,553	1,911	0,064
Shape separation	0,100	0,308	0,300	0,571	-1,378	0,179
Shape forming with the material	0,050	0,394	-0,100	0,308	1,342	0,188
Shape forming with rods	0,050	0,759	0,350	1,040	-1,042	0,304
Shape-ground relationship	-0,100	0,447	0,050	0,510	-0,989	0,329
Recognition of three-dimensional shapes	0,250	1,333	1,900	1,651	-3,477	**0,001**
Recognition of patterns	0,250	0,639	0,100	0,718	0,698	0,489
Perspective taking	0,000	0,000	0,050	0,394	-0,567	0,577
Building with blocks	0,400	0,503	0,850	0,671	-2,401	**0,021**
Surface estimation of three-dimensional shapes	0,200	0,410	0,500	0,513	-2,042	**0,048**
Total score change	2,200	2,016	13,150	5,040	-9,022	**0,000**

Another sub-skill that was improved by Froebel Gifts is the ability to recognize three-dimensional shapes. In the items that measure this skill, children were shown many pictures of three-dimensional shapes (e.g., cube, cylinder, rectangular prism, cone, triangular prism, and pyramid) and asked to find a certain geometric shape. The three-dimensional shape diversity of the Froebel Gifts facilitated the experimental group to learn the names, terms, properties, qualitative differences, and similarities between these shapes. I claim that when children compare certain geometric objects to find their commonalities or differences, they focus on both qualitative and quantitative properties of those objects. For instance, in FGEEP, when children were given four different angled triangles and five different sized squares, the numbers of edges and vertices would be the determinant properties of those shapes which led the children to classify them as triangles and squares. The angle and the size properties of those shapes would be ignored in this specific situation. However, in the next step, when the children are given two isosceles triangles and three equilateral triangles, the classifications were based on their different angle properties. Ignoring the quantitative differences and focusing on the qualitative and consistent similarities of the objects is the process of forming abstract concepts. This process allows them to perceive details, recognize constants and consider perspective which is supported by the post-test results.

5. Discussion

In FGGEP, children had an opportunity to develop their knowledge of geometric concepts, such as angle, length, orientation, and area. Using each gift with increasing task difficulty levels, the children realized that the exact same shape (e.g., a cube) could be composed with various compositions (e.g., 8 identical cubes or 8 identical rectangular prisms). It means that they had to examine and compare the angles, lengths, and orientations of smaller blocks in each given set-in order to try to cover the same areas or construct the same three-dimensional shape. Directive questions in the program where teachers asked children about the forms that they created when playing blocks were effective in children's building with blocks skills. Other researchers also emphasized the importance of instructions and directive questions in block play on children's mathematics skills such as numeracy, shape recognition, and mathematical language (Schmitt, et al. 2018). In another study, the fifth-grade students who were given concrete and virtual manipulative supported education, significantly succeeded in building and drawing geometric structures. (Yaman and Şahin, 2014). Considering the relevant research outcomes, I suggest that playing with concrete materials, not necessarily Froebel Gifts, has a positive effect on children's ability to build with blocks.

According to the results, FGGEP had no significant effect on children's two-dimensional shape drawing. This result suggests that both FGGEP and MEB programs did not include adequate drawing activities which children can represent the mental image of geometric shapes in drawing. The intervention program also did not affect children's mental shape rotation skills significantly. Research suggest that mental rotation skills are closely linked to map reading, orientating, navigating and problem-solving skills and verbal and visual-spatial working memory (Bruce and Hawes, 2015). Both FGGEP and MEB programs lacked map reading and navigating activities which would enrich children's visual-spatial working memory. Mental shape rotation skill is also significantly related to forming a mental shape and shape forming with the material, since children needed to mentally rotate the two-dimensional pictures shown to them on EGST. Therefore, it is understandable that if children were having difficulty in mental rotation, they would also be having difficulty in forming a mental shape and shape forming with the material. The intervention program did not affect children's skill of shape forming with rods significantly. Even though, it seemed like an unexpected outcome as the children in the experiment group observed to be forming shapes with beads in Gift 10 and rods in Gift 8 throughout the intervention, the reason why children scored low is because the test required children to form the corners of the shape perfectly, which many children failed to do because of their undeveloped fine-motor skills. This doesn't mean that the children lacked the knowledge of corner as they scored significantly higher in recognizing the side and corner features of shapes.

The intervention program did not have a significant effect on the skill of shape separation which was measured by the test items where children were asked to mentally separate certain shapes in order to identify the requested shape. This skill is highly related to shape-ground relationship skill since it requires to identify the whole as the background and a piece as the figure. Both the control and experiment groups' score change in these two skills; shape separation and shape-ground relationship are similar. FGGEP included a variety of comparison activities using the tablet figures in Gift 7 where children had the opportunity to combine and separate these shapes and observe their own experience. Yet, the related test items showed children certain figures that are printed on a paper where children couldn't manipulate manually. The results imply that the children in both groups have difficulty in rotating, separating or organizing shapes mentally.

During the experiments, children in experimental group did activities such as calculating the surface area of geometric shapes by covering their surfaces with sticks in Gift 8 which was used a non-standard measurement unit. Through the end of the intervention, two male children stated that they can calculate the area of a triangle by dividing the number of sticks on the square into two, because two equilateral triangles constitute a square. I couldn't further analyze this unexpected development as my priority was to collect the data for my initial research questions. However, after my observations, I hypothesized that the alignment of concepts and activities in FGGEP may be promoting abstract thinking in children by chunking geometry concepts in working memory with Gift 10. Gray and Tall (2001) identified a concept called "compression" to explain the process of what Willingham (2009) called "chunking." In this process, complex and meaningless information is analyzed in terms of its essential aspects in order to re-organize the observed properties to make them available as an entity to think about (Gray and Tall, 2007). Froebel Gifts provide an opportunity for this operation of chunking, since the gifts are designed to represent the essential aspects of geometric shapes. With the 10th gift, children were able to conceive of them as whole to make them available as an entity to think about.

6. Conclusion and Suggestions

Shape recognition skills of young children develops in accordance with their abstraction processes (Son, Smith and Goldstone, 2008; Gray and Tall, 2007). Abstraction in general refers to identifying similarities in concepts, relations or operations and generate abstract ideas that highlight the deep structures rather than the surface features of the observed conditions (Willingham, 2009). Previous research indicate that children discover similarities and relations when highly similar objects are introduced in close proximity under a specific context (Kotovsky and Gentner, 1996). Therefore, the perceptual distance between the source and the target concepts must be close enough to highlight the subject abstraction. This suggests that the numerous near-comparison, far-comparison opportunities between the typical and atypical examples of geometric shapes that Froebel Gifts provide must be effective in developing children's shape abstraction. Loewenstein and Gentner (2001) state that when teaching geometry, children must initially compare very similar and highly alignable objects in order to distinguish the similarities and differences. It must slowly proceed to comparing dissimilar or less similar objects in order to help children expand the number of contexts that they consider in between far similarities (Loewenstein and Gentner, 2001). The degree of the similarity between the concepts and objects are crucial in promoting abstraction skills in shape recognition (Walker et al., 2018). As implied in this research, the most important advantage of manipulatives is to help teachers make the connection between abstract concepts and concrete experiences by using manipulatives to facilitate understanding of abstract concepts and procedural knowledge (Kamina and Iyer, 2009).

Researchers indicate that 'perceptual variability' which amounts to the use of a wide variety of materials embodying each concept is an important factor in children's abstraction processes in shape recognition. The other effective property is 'mathematical variability' which refers to the wide variation in irrelevant mathematical attributes such as size and orientation (Mitchelmore and White, 1995, p.52, Mitchelmore and White, 2000). According to this research, it is not only the width of the material's variety which promotes abstraction in children but also the contexts that the various pieces come together promote a similarity perception in between concepts. In case of Froebel Gifts, the pieces between Gift 3 and 6 are all different in terms of their unique properties such as oblong blocks, triangular prisms, rectangular prisms or cubes. However, they all come together under more general concepts such as prisms or geometric shapes. The same process can also be seen in Gift 7 where isosceles, right-angled or obtuse-angled triangles have unique properties in terms of edge length, therefore can be differentiated by assigning a concept for each of them. At the same time, they can be grouped together under the label of triangle which enables the children to abstract the fundamental properties of a triangle. These aspects of the Gifts are thought to be effective in children's shape recognition skill development; however, it is not clear if the comparison activities in general, without a certain conceptual hierarchy or the specific alignment of concepts and operations through the order of the Gifts from 1-10, have more impact on children's abstraction skill development in shape recognition. Therefore, future research can investigate the effects of conceptual alignment and comparison activities on children's abstraction processes in shape recognition.

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