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OPTIMAL CONTROL FOR FRACTIONAL STOCHASTIC DIFFERENTIAL SYSTEM DRIVEN BY FRACTIONAL BROWNIAN MOTION WITH POISSON JUMPS

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Abstract. The purpose of this article is to study the optimal control problem for fractional stochastic differential system driven by fractional Brownian motion with Poisson jumps in Hilbert space. Initially, the sufficient conditions for existence of mild solution results are formulated and proved by virtue of fractional calculus, solution operator and stochastic analysis techniques. Further, we formulated and proved the existence results for optimal control of the proposed system with corresponding cost function by using Balder's theorem. Finally an example is provided to illustrate the main results.

1. Introduction

Fractional Calculus (FC) has been introduced since the end of the nineteenth century by famous mathematicians Riemann and Liouville, but the concept of non-integer calculus as a generalization of the traditional integer order calculus was mentioned already in 1695 by Leibnitz and L'Hospital. The subject of FC has become a rapidly growing area in the field of system physics, chemistry, biology, medicine and finance etc. On the other, fractional derivatives and integrals enable the description of the memory and hereditary properties inherent in various materials and processes. Hence, there is a growing need to find the behavior of solution of the fractional differential equations (FDEs). For more details on FDEs, the reader may refer to the monographs [3, 4, 5, 6, 2] and references therein.

The fractional Brownian motion (fBm) is usual candidate to model phenomena due to its self-similarity of increments and long-range dependence. This fBm $B^{\mathtt{H}}$ is the continuous centered Gaussian process with covariance function described by

$$R^{\mathtt{H}}(t,s) \quad = \quad \mathbf{E}\left[B^{\mathtt{H}}(t)B^{\mathtt{H}}(s)\right] = \frac{1}{2}(t^{2\mathtt{H}} + s^{2\mathtt{H}} - |t-s|^{2\mathtt{H}})$$

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The parameter $\mathtt{H}$ characterizes all the important properties of the process, when $\mathtt{H} < \frac{1}{2}$, the increments are negatively correlated and the correlation decays more slowly the quadratically; when $\mathtt{H} > \frac{1}{2}$, the increments are positively correlated and the correlation decays so slowly that they are not summable, a situation which is commonly known as the long memory property. The fBm can be expressed as Wiener integral with respect to the standard Wiener process, i.e. the integral of a deterministic kernal with respect to a standard Brownian motion, the Hermite process of order 1 is fBm and of order 2 is the Rosenblatt process. However, there exist only a few papers in this field, for more details (see [13, 14, 15, 16] and reference therein).

On the other hand, the Poisson jumps have become very popular one in recent years, the Poisson jumps are generally based on the Poisson random measure in aspects of applications in many real life phenomena such as, finance, biology and any other field of science see [7, 8, 10]. For example Poisson jump models that are very popular in financial modeling sice Merton first derived an option pricing formula based on a stock price process generated by a mixture of a Brownian motion and a Poisson process. This mixed process is also called the jump diffusion process. The requirement for a jump component in a stock price process is intuitive, and supported by the big crashes in stock markets: The Black Monday on October 17, 1987 and the recent market crashes in the financial crisis since 2008 are two prominent examples. To model jump events, we need two quantities: jump frequency and jump size. The first one specifies how many times jumps happed in a given time period, and the second one determines low large a jump is if it occurs. It is natural and necessary to include a jump term in the stochastic differential equation. Recently, Balasubramaniam et al. [1] and Muthukumar et al. [10] have studied, respectively, fractional stochastic differential equations driven by Poisson jumps and fractional stochastic differential equations with Poisson jumps. Very recently Rihan et al. [11] extended to study the existence of solutions of fractional stochastic differential equations with Hilfer fractional derivative and Poisson jumps.

An optimal control problem (OCP) describes the path of control variables concerned with minimizing the cost functional or maximizing a payoff to the corresponding system over a set of admissible control functions. Nowadays, optimal control theory has a considerable development and have fruitful applications in many fields like science and engineering (see [17, 18]). Stochastic optimal control problem (SOCP) makes to design the time path of the controlled variables which performs the desired control task with minimum cost despite the presence of noise. SOCPs and its applications have extensive attention in the literature see [24, 25, 26, 9]. The main goal of optimal control is to find, in an open-loop control, the optimal values of the control variables for the dynamic system which maximize or minimize a given performance index. If a fractional differential equation describes the performance index and system dynamics, then an optimal control problem is known as a fractional optimal control problem. Using the fractional variational principle and lagrange multiplier technique, Agrawal [21] discussed the general formulation and solution scheme for Riemann-Liouville fractional optimal control problems. It is remarkable thathe fixed point technique, which is used to establish the existence results for abstract fractional differential equations, could be extended to address the fractional optimal control problems. Recently, Aicha Harrat et al. [19] studied the optimal controls of impulsive fractional system with

Clarke subdifferential. Very recently, using the LeraySchauder fixed point theorem, Balasubramaniam et al. [1] studied the solvability and optimal controls for impulsive fractional stochastic integrodifferential equations. Tamilalagan et al. [20] investigated the solvability and optimal controls for fractional stochastic differential equations driven by Poisson jumps in Hilbert space via analytic resolvent operators and Banach contraction mapping principle.

Motivated by the aforementioned research works, in this manuscript we drive the sufficient conditions for the existence of solutions of the following class of optimal control for fractional stochastic differential system driven by fractional Brownian motion with Poisson jumps

$$\begin{aligned} {}^{C}D_t^{\alpha}x(t) &= Ax(t) + B(t)u(t) + f(t,x(t)) + \sigma(t,x(t))\frac{dw^{\mathtt{H}}(t)}{dt} \\ &+ \int_{\mathcal{U}} h(t,x(t),u)\widetilde{N}(ds,du),\ t \in]0,\tau], \\ x(0) &= x_0 \in \mathrm{X}, \end{aligned} \tag{1.1}$$

where the integral $l = [0,\tau] \times \mathrm{X} \times \mathrm{U} \to \mathbb{R} \cup \{\infty\}$ is specified latter; ${}^{C}D_t^{\alpha}$ is the Caputo fractional derivative of order $0 < \alpha < 1$, the state $x(\cdot)$ is X-valued stochastic process; Suppose $\{w^{\mathtt{H}}(t)\}_{t\geq 0}$ is a fractional Brownian motion with Hurst parameter $\mathtt{H} \in (\frac{1}{2}, 1)$ defined on $(\Omega, \Im, \{\Im_t\}_{t\geq 0}, \mathbb{P})$ with values in Hilbert space Y. The control function $u(\cdot)$ takes its values from a separable reflexive Hilbert space U; $A : \mathscr{D}(A) \subseteq \mathrm{X} \to \mathrm{X}$ is the infinitesimal generator of a resolvent $S_\alpha(t)$, $t \geq 0$ on X; $\{B(t) : t \geq 0\}$ is a family of linear operator from U to X; the functions $f : [0,\tau] \times \mathrm{X} \to \mathrm{X}$, $\sigma : [0,\tau] \times \mathrm{X} \to \mathcal{L}_2^0(\mathrm{Y},\mathrm{X})$ and $h : [0,\tau] \times \mathrm{X} \times \mathcal{U} \to \mathrm{X}$ are nonlinear, where $\mathcal{L}_2^0(\mathrm{Y},\mathrm{X})$ be the space of all Q-Hilbert-Schmidt operators from Y into X.

Let $(\Omega, \Im, \{\Im_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space equipped with a normal filtration $(\Im_t)$, $t \in [0,a]$ and Let X, Y be real separable Hilbert spaces and $\mathcal{L}(\mathrm{Y},\mathrm{X})$ denote the space of all bounded linear operator from Y into X. Let $Q \in \mathcal{L}(\mathrm{Y},\mathrm{Y})$ be an operator defined by $Qe_n - \lambda_n e_n$ with finite trace $tr(Q) = \sum_{n=1}^{\infty} \lambda_n < \infty$ where $\lambda_n \geq 0$ $(n = 1, 2, ...)$ are non-negative real numbers and $\{e_n\}$ $(n = 1, 2, ...)$ is a complete orthonormal basis in $\mathcal{Y}$.

We define the infiite dimensional fractional Brownian motion on Y with covariance Q as

$$w^{\mathtt{H}}(t) = w_Q^{\mathtt{H}}(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n} e_n \beta_n^{\mathtt{H}}(t)$$

where $\beta_n^{\mathtt{H}}$ are real, independent fractional Brownian motions.

In order to define Wiener integrals with respect to the Q-fractional Brownian motion, we introduce the space $\mathcal{L}_2^0 = \mathcal{L}_2^0(\mathrm{Y},\mathrm{X})$ of all Q-Hilbert-Schmidt operators $\psi : \mathrm{Y} \to \mathrm{X}$. We recall that $\psi \in \mathcal{L}(\mathrm{Y},\mathrm{X})$ is called a Q-Hilbert-Schmidt operator, if

$$\|\psi\|^2_{\mathcal{L}_2^0(\mathrm{Y},\mathrm{X})} = \sum_{n=1}^{\infty} \left\| \sqrt{\lambda_n} \psi e_n \right\|^2 < \infty$$

and that the space $\mathcal{L}_2^0$ equipped with the inner product $< \upsilon, \psi >_{\mathcal{L}_2^0} = \sum_{n=1}^{\infty} < \upsilon e_n, \psi e_n >$ is a separable Hilbert space. Let $\phi(s); s \in [0,a]$ be a function with values in $\mathcal{L}_2^0(\mathrm{Y},\mathrm{X})$, the Wiener integral of ϕ with respect to $w^{\mathtt{H}}$ is defined by

$$\begin{aligned}\int_0^t \phi(s)dw^{\mathtt{H}}(s) &= \sum_{n=1}^{\infty}\int_0^t \sqrt{\lambda}\phi(s)e_n d\beta_n^{\mathtt{H}} \\ &= \sum_{n=1}^{\infty}\int_0^t \sqrt{\lambda}K^*(\phi e_n)(s)d\beta_n(s) \end{aligned} \tag{1.2}$$

where β_n is the standard Brownian motion. Let $\mathscr{C}([0,\tau],\mathcal{L}_2(\Omega,\mathrm{X}))$ be the Banach space of continuous maps from $[0,\tau]$ into $\mathcal{L}_2(\Omega,\mathrm{X})$ satisfying $\sup_{0\le t\le\tau}\mathbf{E}\left\|x(t)\right\|^2 < \infty$. Let X_2 be the closed subspace of $\mathscr{C}([0,\tau],\mathcal{L}_2(\Omega,\mathrm{X}))$ consistingof measurable, $\Im_t$-adapted, X-valued processes $x \in \mathscr{C}([0,\tau],\mathcal{L}_2(\Omega,\mathrm{X}))$ equipped with the norm

$$\|x\|_{\mathrm{X}_2} = \left(\sup_{0\le t\le\tau}\mathbf{E}\left\|x(t)\right\|^2\right)^{1/2}.$$

Suppose that $\{q(t); t\in[0,\tau]\}$ is the Poisson point process, taking its value in a measurable space $(\mathcal{U},\mathcal{B}(\mathcal{U}))$ with a σ-finite intensity measure $v(du)$. The compensating martingle measure and Poisson counting measure are defined by

$$\widetilde{N}(ds,du) = N(ds,du) - v(du)ds.$$

Let us assume that the filteration $\Im_t = \sigma\left\{N((0,s],\Lambda) : s\le t, \Lambda\in\mathcal{B}(\mathcal{U})\right\}\vee N$, $t\in[0,\tau]$, produced by $q(\cdot)$ Poisson point process and is augmented, where N is the class of $\mathbb{P}$-null sets. Let $p_2([0,\tau]\times\mathcal{U};\mathrm{X})$ be the space of all predictable mappings $h:[0,\tau]\times\mathcal{U}\to\mathrm{X}$ for

$$\int_0^\tau\int_{\mathcal{U}}\mathbf{E}\left\|h(t,u)\right\|_{\mathrm{X}}^2 dtv(du) < \infty.$$

Consider the following integral cost functional

$$\jmath\{x,u\} = \mathbf{E}\left\{\int_0^\tau l(t,x^u(t),u(t))dt\right\}, \tag{1.3}$$

Define the admissible set U_{ad}, the set of all $v(\cdot):[0,\tau]\times\Omega\to\mathrm{U}$ such that v is $\Im_t$-adapted stochastic process and $\mathbf{E}\int_0^\tau\left\|v(t)\right\|^p dt < \infty$. Clearly $\mathrm{U}_{ad}\ne\emptyset$ and $\mathrm{U}_{ad}\subset\mathcal{L}^p([0,\tau];\mathrm{U})$ $(1<p<+\infty)$ is bounded, closed and convex.

Denoted by the set of all admissible state-control pairs (x,u) by $\mathcal{A}_{ad}$, where x is the mild solution of the system (1.1) corresponding to the control $u\in\mathrm{U}_{ad}$. The main objective of this paper is to find a pair $(x^0,u^0)\in\mathcal{A}_{ad}$ such that

$$\jmath(x^0,u^0) := \inf\left\{\jmath(x,u) : (x,u)\in\mathcal{A}_{ad}\right\} = \epsilon.$$

To the best of authors knowledge, up to now, no work has been reported to derive the optimal control for fractional stochastic differential system driven by fractional Brownian motion with Poisson jumps. The main contributions are summarized as follows:

(1) Fractional stochastic differential system driven by fractional Brownian motion with Poisson jumps is formulated.
(2) Fractional calculus theory is effectively used to derive the existence and uniqueness of mild solution, a set of sufficient conditions is constructed by using fixed point theorem.
(3) The existence of fractional optimal control for stochastic system is also discussed.

(4) An example is provided to illustrate the obtained theoretical results.

2. Preliminaries

In this section, we collect basic concepts, definitions and Lemmas which will be used in the sequel to obtain the main results.

Definition 2.1. *The Riemann-Liouville fractional integral operator of order $\alpha > 0$ of a function $f : [0, \infty) \to \mathbb{R}$ with the lower limit 0 is defined as*

$$I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s) ds,$$

where Γ is the Euler gamma function.

Definition 2.2. *The Caputo fractional derivative of order $\alpha > 0$ for the function $f \in \mathscr{C}^m([0,\tau], \mathbb{R})$ is defined by*

$$^{\mathcal{C}}D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-s)^{m-\infty-1} f^{(m)}(s) ds, \ m-1 < \infty < m \in \mathbb{N}.$$

If f is an abstract function with values in X, then the integrals appearing in Definition 2.1 and Definition 2.2 are taken in the Bochner sense. Moreover, the Caputo derivative of a constant is always zero.

The two-parameter function of the Mittag-Leffler type is defined by the series expension

$$\mathrm{E}_{\alpha,\beta}(z) = \sum_{j=1}^{\infty} \frac{z^j}{\Gamma(\alpha j + \beta)} = \frac{1}{2\pi i} \int_{\mathcal{C}} e^{\lambda} \frac{\lambda^{\alpha-\beta}}{\lambda^{\alpha} - z} d\lambda; \ \alpha, \beta > 0, \ z \in \mathbb{C},$$

where $\mathcal{C}$ is a contour that start and end at $-\infty$ and encircles the disc $\|\lambda\| \leq |z|^{1/2}$ counterclockwise.

Definition 2.3. [22] *A closed and linear operator A is said to be sectorial type μ if there exist $\pi/2 \leq \theta \leq \pi$, $\widetilde{M} > 0$ and $\mu \in \mathbb{R}$ such that the following conditions are satisfied: $\rho(A) \subset \sum_{(\theta,\mu)} = \{\lambda \neq \mu, |\arg(\lambda - \mu)| < \theta\}$, and $\|R(\lambda, A)\| = \left\|(\lambda - A)^{-1}\right\| \leq \frac{\widetilde{M}}{|\lambda-\mu|}$, $\lambda \in \sum_{(\theta,\mu)}$.*

Lemma 2.1. [22] *For $0 < \alpha < 2$, a linear closed densely defined operator A belongs to $A^{\alpha}(\theta_0, \mu_0)$ iff $\lambda^{\alpha} \in \rho(A)$ for each $\lambda \in \sum_{(\theta_0+\pi/2),\mu}$ and for any $\mu > \mu_0$, $\theta < \theta_0$ there is a constant $\mathcal{C}_0 = \mathcal{C}_0(\theta, \mu)$ such that*

$$\left\|\lambda^{\alpha+1} R(\lambda^{\alpha}, A)\right\| \leq \frac{\mathcal{C}_0}{|\lambda - \mu|}, \ \textit{for } \lambda \in \sum_{(\theta_0+\pi/2),\mu}.$$

Lemma 2.2. [22] *If f satisfies the uniform Holder condition with the exponent $0 < \gamma \leq 1$ and A is a sectorial operator, then the unique solution of the Cauchy problem*

$$\begin{aligned} ^{\mathcal{C}}D_t^{\alpha} x(t) &= Ax(t) + f(t), \ 0 < \infty < 1, \ t \in (0, \tau], \\ x(0) &= x_0, \end{aligned} \tag{2.1}$$

is given by

$$x(t) = S_{\alpha}(t) x_0 + \int_0^t T_{\alpha}(t-s) F(s) ds,$$

where

$$S_\alpha(t) = \mathrm{E}_{\alpha,1}(At^\alpha) = \frac{1}{2\pi i}\int_{\hat{\mathcal{B}_\rho}} e^{\lambda t}\frac{\lambda^{\alpha-1}}{\lambda^\alpha - A}d\lambda,$$
$$T_\alpha(t) = t^{\alpha-1}\mathrm{E}_{\alpha,1}(At^\alpha) = \frac{1}{2\pi i}\int_{\hat{\mathcal{B}_\rho}} e^{\lambda t}\frac{1}{\lambda^\alpha - A}d\lambda,$$

$\hat{\mathcal{B}_\rho}$ is the Bromwich path, $T_\alpha(t)$ is called the α-resolvent family, and $S_\alpha(t)$ is the solution operator generated by A.

An operator A is said to belong to $\mathcal{D}^\alpha(\widetilde{M},\mu)$ if problem (4) with $f = 0$ has a solution operator $S_\alpha(t)$ satisfying $\|S_\alpha(t)\| \leq \widetilde{M}e^{\mu t}$. Denote $\mathcal{D}^\alpha(\mu) = \cup\{\mathcal{D}^\alpha(\widetilde{M},\mu) : \widetilde{M} \geq 1\}$, $\mathcal{D}^\alpha = \{\mathcal{D}^\alpha(\mu : \mu \geq 0)\}$, and $\mathcal{A}^\alpha(\theta_0,\mu_0) = \{A \in \mathcal{D}^\alpha : A$ generates analytic solution operators $S_\alpha(t)$ of type $(\theta_0,\mu_0)\}$.

If $0 < \alpha < 1$ and $A \in \mathcal{A}^\alpha(\theta_0,\mu_0)$, then we have $\|S_\alpha(t)\| \leq \widetilde{M}e^{\mu t}$ and $\|T_\alpha(t)\| \leq \mathcal{C}e^{\mu t}(1 + t^{\alpha-1})$, $t > 0$, $\mu > \mu_0$. If

$$M_S = \sup_{0\leq t\leq\tau}\|S_\alpha(t)\|,\ M_T = \sup_{0\leq t\leq\tau}\mathcal{C}e^{\mu t}(1 + t^{1+\alpha}),$$

then, we have

$$\|S_\alpha(t)\| \leq M_S,\ \|T_\alpha(t)\| \leq t^{\alpha-1}M_T.$$

Lemma 2.3. [13] *If* $\psi : [0,a] \to \mathcal{L}_2^0(\mathrm{Y},\mathrm{X})$ *satisfies* $\int_0^a \|\psi(s)\|^2_{\mathcal{L}_2^0} < \infty$ *then the above sum in (1.2) is well defined as* X*-valued random variable and we have*

$$\mathbf{E}\left\|\int_0^t \psi(s)dw^{\mathtt{H}}(s)\right\|^2 \leq 2\mathtt{H}t^{2\mathtt{H}-1}\int_0^t \|\psi(s)\|^2_{\mathcal{L}_2^0}\,ds.$$

By Lemma 2.2. a mild solution of the system (1.1) is defined as

Definition 2.4. *An* $\Im_t$*-adapted stochastic process* $x(t) \in \mathscr{C}([0,\tau],\mathcal{L}^2(\Omega,\Im,\mathrm{X}))$ *is called a mild solution of system (1.1) if for each* $u(\cdot) \in \mathcal{L}^p([0,\tau];\mathrm{U}), x(t)$ *is measurable and the following stochastic integral equation*

$$\begin{aligned} x(t) = S_\alpha(t)x_0 &+ \int_0^t T_\alpha(t-s)[B(s)u(s) + f(s,x(s))]ds \\ &+ \int_0^t T_\alpha(t-s)\sigma(s,x(s))dw^{\mathtt{H}}(s) \\ &+ \int_0^t\int_{\mathcal{U}} T_\alpha(t-s)h(s,x(s),u)\widetilde{N}(ds,du). \end{aligned} \tag{2.2}$$

3. Existence and Uniqueness

To prove the existence and uniqueness of mild solution of the system (1.1), we impose the following hypotheses:

(H1) The functions $f : [0,\tau] \times \mathrm{X} \to \mathrm{X}$, $\sigma : [0,\tau] \times \mathrm{X} \to \mathcal{L}_2^0(\mathrm{Y},\mathrm{X})$ and $h : [0,\tau]\times\mathrm{X}\times\mathcal{U} \to \mathrm{X}$ are continuous, and satisfying linear growth and Lipschitz conditions: there are positive constants L_f, L_σ and L_h such that

$$\|f(t,x)-f(t,y)\|^2 \leq L_f\|x-y\|^2, \ \|f(t,x)\|^2 \leq L_f(1+\|x\|^2),$$
$$\|\sigma(t,x)-\sigma(t,y)\|^2 \leq L_\sigma\|x-y\|^2, \ \|\sigma(t,x)\|^2 \leq L_\sigma(1+\|x\|^2),$$
$$\int_{\mathcal{U}}\|h(t,x,u)-h(t,y,u)\|^2 v(du) \leq L_h\|x-y\|^2,$$
$$\int_{\mathcal{U}}\|h(t,x,u)\|^2 v(du) \leq L_h(1+\|x\|^2).$$

(**H2**) The operator $B \in \mathcal{L}_\infty([0,\tau];\mathcal{L}(\mathrm{U},\mathrm{X}))$ and $\|B\|_\infty$ stand for the norm of operator B in the Banach space $\mathcal{L}_\infty([0,\tau];\mathcal{L}(\mathrm{U},\mathrm{X}))$.

(**H3**) The multi-valued map $\Xi(\cdot) : [0,\tau] \to 2^u/\{\emptyset\}$ has closed, convex and bounded values, $\Xi(\cdot)$ is graph measurable and $\Xi(\cdot) \subseteq \Phi$, where Φ is a bounded subset of U.

Theorem 3.1. *Assumptions* (**H1**) $-$ (**H3**) *the system (2.2) admits a unique mild solution on* $[0,\tau]$ *for each control function* $u(\cdot) \in \mathrm{U}_{ad}$ *and for each some* p *such that* $p\alpha > 1$.

Proof. Define an operator $\mathcal{G} : \mathrm{X}_2 \to \mathrm{X}_2$ as

$$\begin{aligned}(\mathcal{G}x)(t) &= S_\alpha(t)x_0 + \int_0^t T_\alpha(t-s)[B(s)u(s)+f(s,x(s))]ds \\ &+ \int_0^t T_\alpha(t-s)\sigma(s,x(s))dw^{\mathtt{H}}(s) \\ &+ \int_0^t\int_{\mathcal{U}} T_\alpha(t-s)h(s,x(s),u)\widetilde{N}(ds,du).\end{aligned}$$

To show that (2.2) is the mild solution of the system (1.1) on $[0,\tau]$, it is enough to prove that $\mathcal{G}$ has a fixed point in the space X_2. We first show that $\mathcal{G}(\mathrm{X}_2) \subset \mathrm{X}_2$. Let $x \in \mathrm{X}_2$, then we have

$$\mathbf{E}\|(\mathcal{G}x)(t)\|^2 \leq 5[\Gamma_1+\Gamma_2+\Gamma_3+\Gamma_4+\Gamma_5] \tag{3.1}$$

Clearly

$$\begin{aligned}\Gamma_1 &= \|S_\alpha(t)x_0\|^2 \\ &\leq M_s^2\mathbf{E}\|x_0\|^2.\end{aligned}$$

Next, using the Cauchy-Schwartz inequality, we have

$$\begin{aligned}\Gamma_2 &= \|T_\alpha(t-s)B(s)u(s)ds\|^2 \\ &\leq M_T^2\|B\|_\infty^2\left[\int_0^t (t-s)^{\alpha-1}\|u(s)\|ds\right]^2 \\ &\leq M_T^2\|B\|_\infty^2\left[\left(\int_0^t (t-s)^{\frac{p(\alpha-1)}{p-1}}ds\right)^{\frac{p-1}{p}}\left(\int_0^t \|u(s)\|_{\mathrm{U}}^p ds\right)^{\frac{1}{p}}\right]^2 \\ &\leq M_T^2\|B\|_\infty^2\|u\|^2_{\mathcal{L}([0,\tau];\mathrm{U})}\tau^{2(\frac{p\alpha-1}{p})}(\frac{p-1}{p\alpha-1})^{\frac{2(p-1)}{p}}.\end{aligned}$$

Next, by (**H1**) and Cauchy-Schwartz inequality, we have

$$\begin{aligned}
\Gamma_3 &= \mathbf{E}\left\|\int_0^t T_\alpha(t-s)f(s,x(s))ds\right\|^2 \\
&\leq M_T^2\left(\int_0^t (t-s)^{\alpha-1}ds\right)\left(\int_0^t (t-s)^{\alpha-1}\mathbf{E}\left\|f(s,x(s))\right\|^2 ds\right) \\
&\leq M_T^2 L_f\frac{\tau^\alpha}{\alpha}\int_0^t (t-s)^{\alpha-1}(1+\mathbf{E}\left\|x(s)\right\|^2)ds \\
&\leq M_T^2 L_f\frac{\tau^\alpha}{\alpha^2}(1+\|x\|^2_{\mathrm{X}_2}).
\end{aligned}$$

By (**H1**) and Lemma 2.3, we have

$$\begin{aligned}
\Gamma_4 &= \mathbf{E}\left\|\int_0^t T_\alpha(t-s)\sigma(s,x(s))dw^{\mathtt{H}}(s)\right\|^2 \\
&\leq M_T^2\left[\int_0^t (t-s)^{\alpha-1}2\mathtt{H}t^{2\mathtt{H}-1}\left(\mathbf{E}\left\|\sigma(s,x(s))\right\|^2_{\mathcal{L}_2^0}\right)ds\right] \\
&\leq M_T^2\left[2\mathtt{H}t^{2\mathtt{H}-1}\int_0^t (t-s)^{\alpha-1}\mathbf{E}\left\|\sigma(s,x(s))\right\|^2 ds\right] \\
&\leq M_T^2 L_\sigma 2\mathtt{H}t^{2\mathtt{H}-1}\frac{\tau^{2\alpha}}{\alpha^2}(1+\|x\|^2_{\mathrm{X}_2}).
\end{aligned}$$

and

$$\begin{aligned}
\Gamma_5 &= \mathbf{E}\left\|\int_0^t T_\alpha\int_{\mathcal{U}}(t-s)h(s,x(s),u)\widetilde{N}(ds,du)\right\|^2 \\
&\leq M_T^2\left(\int_0^t (t-s)^{\alpha-1}ds\right)\left(\int_0^t\int_{\mathcal{U}}(t-s)^{\alpha-1}\mathbf{E}\left\|h(s,x(s),u)\right\|^2 v(du)ds\right) \\
&\leq M_T^2 L_h\frac{\tau^\alpha}{\alpha^2}(1+\|x\|^2_{\mathrm{X}_2}).
\end{aligned}$$

Thus (3.1) becomes

$$\mathbf{E}\left\|(\mathcal{G}x)(t)\right\|^2 \quad\leq\quad a+b\left\|x\right\|^2_{\mathrm{X}_2},$$

where a and b are suitable positive constants. Thus $\mathcal{G}$ maps X_2 into itself.

Next, we prove that $\mathcal{G}$ is a contraction. For $x,y\in\mathrm{X}_2$, the Cauchy-Schwartz inequality, and (**H1**) yield that

$$\begin{aligned}
&\mathbf{E}\left\|(\mathcal{G}x)(t)-(\mathcal{G}y)(t)\right\|^2 \\
&\leq 3\mathbf{E}\left\|\int_0^t T_\alpha(t-s)[f(s,x(s))-f(s,y(s))]ds\right\|^2 \\
&\quad+3\mathbf{E}\left\|\int_0^t T_\alpha(t-s)[\sigma(s,x(s))-\sigma(s,y(s))]dw^{\mathtt{H}}(s)\right\|^2 \\
&\quad+3\mathbf{E}\left\|\int_0^t T_\alpha\int_{\mathcal{U}}(t-s)[h(s,x(s),u)-h(s,y(s),u)]\widetilde{N}(ds,du)\right\|^2 \\
&\leq 3M_T^2\left(L_f+L_\sigma 2\mathtt{H}t^{2\mathtt{H}-1}+L_h\right)\frac{\tau^{2\alpha}}{\alpha^2}\left\|x-y\right\|^2_{\mathrm{X}_2}.
\end{aligned}$$

Consequently if

$$3M_T^2\left[L_f + L_\sigma 2\mathtt{H}t^{2\mathtt{H}-1} + L_h\right]\frac{\tau^{2\alpha}}{\alpha^2} < 1, \tag{3.2}$$

then the operator $\mathcal{G}$ has a unique fixed point in X_2, which is a solution of the system (1.1). The extra condition on τ can be easily removed by considering the equation on intervals $[0,\tilde{\tau}]$, $[0,2\tilde{\tau}]$, ... with $\tilde{\tau}$ satisfying (3.2). □

We now obtain a priori estimate of mild solution for the system (1.1), that helps us to obtain our main results.

Lemma 3.2. *Assuming that system (2.2) is the mild solution of system (1.1) on $[0,\tau]$ corresponding to the control u. Then there exists a constant $M > 0$ such that*

$$\mathbf{E}\left\|x(t)\right\|^2 \quad \leq \quad M,\ t \in [0,\tau].$$

Proof. By (**H1**) and Holder's inequality, we obtain

$$\begin{aligned}
\mathbf{E}\left\|x(t)\right\|^2 &\leq 5\mathbf{E}\left\|S_\alpha(t)x_0\right\|^2 \\
&+ 5\mathbf{E}\left\|\int_0^t T_\alpha(t-s)B(s)u(s)ds\right\|^2 + 5\mathbf{E}\left\|\int_0^t T_\alpha(t-s)f(s,x(s))ds\right\|^2 \\
&+ 5\mathbf{E}\left\|\int_0^t T_\alpha(t-s)\sigma(s,x(s))dw^{\mathtt{H}}(s)ds\right\|^2 \\
&+ 5\mathbf{E}\left\|\int_0^t\int_{\mathcal{U}} T_\alpha(t-s)\sigma(s,x(s),u)\widetilde{N}(ds,du)\right\|^2 \\
&\leq 5M_S^2 + 5M_T^2\left\|B\right\|_\infty^2\left\|u\right\|^2_{\mathcal{L}^p([0,\tau];\mathrm{U})}\tau^{2(\frac{p\alpha-1}{p})}(\frac{p-1}{p\alpha-1})^{\frac{2(p-1)}{p}} \\
&+ 5M_T^2\left(L_f + L_\sigma 2\mathtt{H}t^{2\mathtt{H}-1} + L_h\right)\frac{\tau^{2\alpha}}{\alpha^2} \\
&+ 5M_T^2\left(L_f + L_\sigma 2\mathtt{H}t^{2\mathtt{H}-1} + L_h\right)\frac{\tau^{2\alpha}}{\alpha}\int_0^t (t-s)^{\alpha-1}\mathbf{E}\left\|x(s)\right\|^2 ds.
\end{aligned}$$

Now using the Gronwall inequality, one can easily obtain the boundedness of x in X_2. □

4. Existence of Fractional Optimal Control

In this section, we prove the existence of fractional optimal control under the hypothesis:

(**H4**) Following conditions are imposed on the integrand

$$l : [0,\tau]\times \mathrm{X}\times\mathrm{U} \to \mathbb{R}\cup\{\infty\}$$

such that
(1) The integrand $l : [0,\tau]\times\mathrm{X}\times\mathrm{U}\to\mathbb{R}\cup\{\infty\}$ is $\Im_t$-measurabl.
(2) The integrand $l(t,\cdot,\cdot)$ is sequentially lower semicontinuous on $\mathrm{X}\times\mathrm{U}$ for almost all $t\in[0,\tau]$.
(3) The integrand $l(t,x,\cdot)$ is convex on U for each $x\in\mathrm{X}$ and almost all $t\in[0,\tau]$.
(4) There exist constants $d\geq 0$, $e>0$, μ_0 is nonnegative and $\mu_0\in\mathcal{L}^1([0,\tau];\mathbb{R})$ such that

$$\mu_0(t) + d\mathbf{E}\left\|x\right\|^2 + e\mathbf{E}\left\|u\right\|_{\mathrm{U}}^p \quad\leq\quad l(t,x,u).$$

Theorem 4.1. *Suppose* (**H1**) $-$ (**H4**) *hold, then Lagrange problem (1.3) admits at least one optimal pair, that is, there exists an admissible state-control pair* $(x^0, u^0) \in \mathcal{A}_{ad}$ *such that*

$$\jmath(x^0, u^0) \quad := \quad \mathbf{E}\left\{\int_0^\tau l(t, x^0(t), u^0(t))dt\right\} \leq \jmath(x, u), \ \forall (x, u) \in \mathcal{A}_{ad}.$$

Proof. If $\inf\left\{l(x, u)|(x, u) \in \mathcal{A}_{ad}\right\} = +\infty$, then there is nothing to prove. Without any loss of generality, we may assume that $\inf\left\{l(x, u)|(x, u) \in \mathcal{A}_{ad}\right\} = +\infty$. Now assumption (**H4**) implies that $\epsilon > -\infty$. By definition of infimum, there is a minimizing sequence of feasible pairs $(x^m, u^m) \in \mathcal{A}_{ad}$, such that $l(x^m, u^m) \to \epsilon$ as $m \to +\infty$. Since $\{u^m\} \subseteq \mathrm{U}_{ad}$, $m = 1, 2, \cdots, \{u^m\}$ is a bounded subset of the separable reflexive Banach space $\mathcal{L}^p([0, \tau]; \mathrm{U})$, there exists a subsequence, relabeled as $\{u^m\}$ and $\mathcal{L}^p([0, \tau]; \mathrm{U})$ such that $u^m \overset{w}{\to} u^0$ $(u^m \to u^0)$ weakly as $m \to +\infty$ in $\mathcal{L}^p([0, \tau]; \mathrm{U})$. Since U_{ad} is closed and convex, the Mazur lemma forces us to conclude that $u^0 \in \mathrm{U}_{ad}$.

Let $\{x^m\}$ be the sequence of solution of the system (1.1) corresponding to $\{u^m\}$, that is

$$\begin{aligned} x^m(t) = S_\alpha(t)x_0 + &\int_0^t T_\alpha(t-s)[B(s)u^m(s) + f(s, x^m(s))]ds \\ &+ \int_0^t T_\alpha(t-s)\sigma(s, x^m(s))dw^{\mathtt{H}}(s) \\ &\quad + \int_0^t \int_{\mathcal{U}} T_\alpha(t-s)h(s, x^m(s), u)\widetilde{N}(ds, du). \end{aligned} \tag{4.1}$$

By Lemma 3.1, it is easy to see that there exists $\delta > 0$ such that

$$\mathbf{E}\left\|x^m\right\|^2 \quad \leq \quad \delta, \ m = 0, 1, 2, \cdots,$$

where x^0 is the mild solution of the system (1.1) corresponding to the control $u^0 \in \mathrm{U}_{ad}$ given by

$$\begin{aligned} x^0(t) = S_\alpha(t)x_0 + &\int_0^t T_\alpha(t-s)[B(s)u^0(s) + f(s, x^0(s))]ds \\ &+ \int_0^t T_\alpha(t-s)\sigma(s, x^0(s))dw^{\mathtt{H}}(s) \\ &\quad + \int_0^t \int_{\mathcal{U}} T_\alpha(t-s)h(s, x^0(s), u)\widetilde{N}(ds, du). \end{aligned}$$

For all $t \in [0, \tau]$, using (**H4**), the Cauchy-Schwartz inequality and the Holder inequality, we obtain

$$\begin{aligned} &\mathbf{E}\left\|x^m(t) - x^0(t)\right\|^2 \\ &\leq 4\mathbf{E}\left\|\int_0^t T_\alpha(t-s)[B(s)u^m(s) - B(s)u^0(s)]ds\right\|^2 \\ &\leq 4\mathbf{E}\left\|\int_0^t T_\alpha(t-s)[f(s, x^m(s)) - f(s, x^0(s))]ds\right\|^2 \end{aligned}$$

$$
\begin{aligned}
&\leq 4\mathbf{E}\left\|\int_0^t T_\alpha(t-s)[\sigma(s,x^m(s))-\sigma(s,x^0(s))]dw^{\mathtt{H}}(s)\right\|^2 \\
&\leq 4\mathbf{E}\left\|\int_0^t\int_{\mathcal{U}} T_\alpha(t-s)[h(s,x^m(s),u)-h(s,x^0(s),u)]\widetilde{N}(ds,du)\right\|^2 \\
&\leq 4M_T^2\Big(\frac{p-1}{p\alpha-1}\Big)^{\frac{2p-2}{p}}\tau^{2\alpha-\frac{2}{p}}\left(\int_0^t\left\|B(s)u^m(s)-B(s)u^0(s)\right\|^p ds\right)^{\frac{2}{p}} \\
&+ 4M_T^2\frac{\tau^\alpha}{\alpha}(L_f+L_\sigma 2\mathtt{H}t^{2\mathtt{H}-1}+L_h)\int_0^t(t-s)^{\alpha-1}\mathbf{E}\left\|x^m(s)-x^0(s)\right\|^2 ds.
\end{aligned}
$$

By applying Gronwall inequality, there exists a constant $K^*(\alpha)$ independent of u, m and t such that

$$
\begin{aligned}
\mathbf{E}\left\|x^m(t)-x^0(t)\right\|^2 &\leq K^*(\alpha)\left(\int_0^\tau\left\|B(s)u^m(s)-B(s)u^0(s)\right\|^p ds\right)^{\frac{2}{p}} \\
&\leq K^*(\alpha)\left\|Bu^m-Bu^0\right\|^2_{\mathcal{L}^p([0,\tau];\mathrm{U})}. \qquad (4.2)
\end{aligned}
$$

Since B is strongly continuous, we get

$$\left\|Bu^m-Bu^0\right\|^2_{\mathcal{L}^p([0,\tau];\mathrm{U})}\xrightarrow{s} 0 \text{ as } m\to\infty. \qquad (4.3)$$

From (4.2) and (4.3), we conclude that

$$\mathbf{E}\left\|x^m(t)-x^0(t)\right\|^2\to 0 \text{ as } m\to\infty. \qquad (4.4)$$

This implies that $\mathbf{E}\left\|x^m-x^0\right\|^2\to 0$ in $\mathscr{C}([0,\tau];\mathcal{L}^2(\Omega,\mathrm{X}))$ as $m\to\infty$.

By (**H4**) implies the assumptions of Balder (see Theorem 2.1, [23]). Hence, by Balder's theorem, we get

$$(x,u)\to\mathbf{E}\int_0^\tau L(t,x(t),u(t))dt$$

is sequentially lower semicontinuous in the strong topology of $\mathcal{L}^1([0,\tau];\mathrm{X})$ and week topology of $\mathcal{L}^p([0,\tau];\mathrm{U})\subset\mathcal{L}^1([0,\tau];\mathrm{X})$. Hence, $\jmath$ is weakly lower semicontinuous on $\mathcal{L}^p([0,\tau];\mathrm{U})$, and since by (**H4**)(4), $\jmath>-\infty$, $\jmath$ attains its infimum at $u^0\in\mathrm{U}_{ad}$, that is,

$$
\begin{aligned}
\epsilon &:= \lim_{m\to\infty}\mathbf{E}\int_0^\tau l(t,x^m(t),u^m(t))dt \\
&\geq \mathbf{E}\int_0^\tau l(t,x^0(t),u^0(t))dt \\
&= \jmath(x^0,u^0)\geq\epsilon.
\end{aligned}
$$

Hence completes the proof. □

5. Application

Consider the following fractional stochastic integrodifferential system driven by Rosenblatt process with Poisson jumps

$$
\begin{aligned}
{}^{C}D_t^{\frac{2}{3}}x(t,z) &= \Delta x(t,z) + \int_0^t \tilde{B}(z,s)u(s,t)ds + \int_0^t \tilde{f}(s,z)sin(x,s)ds \\
&+ \int_0^t \frac{(x(t,z))^2}{1+(x(t,z))^2}\frac{dw^{\mathtt{H}}(t)}{dt} + \int_{\mathcal{U}}(1+e^{-t})cosy(t,x,u)\widetilde{N}(dt,du), \\
x(0,z) &= x_0(z),\ z \in \Omega_1, \\
x(t,z)|_{z\in\partial\Omega} &= 0,\ t>0, \qquad (5.1)
\end{aligned}
$$

Here Let $w^{\mathtt{H}}$ is a fractional Brownian motion with Hurst parameter $\mathtt{H} \in (\frac{1}{2},1)$. Let $\Omega_1 \subset \mathbb{R}^3$ be abounded domain and $\partial\Omega_1 \in \mathbb{C}^3$. Further let $\mathrm{X} = \mathrm{U} = \mathcal{L}^2(\Omega_1), w(t)$ is a standard cylindrical Wiener process in X defined on a stochastic space $(\Omega, \Im, \mathbb{P})$. Suppose $\mathscr{D}(A) = \mathrm{X}^2(\Omega_1)\bigcap \mathrm{X}_0^1(\Omega_1)$ and for $z \in \mathscr{D}(A)$, $Az = \left(\frac{\partial^2}{\partial z_1^2} + \frac{\partial^2}{\partial z_2^2} + \frac{\partial^2}{\partial z_3^2}\right)z$. The admissible control set $\mathrm{U}_{ad} := \left\{u \in \mathrm{U} : \|u\|_{\mathcal{L}^p([0,1];\mathrm{U})} \leq 1\right\}$. Define the fractional Brownian motion in $\mathbb{Y}$ by $w^{\mathtt{H}}(t) = \sum_{n=1}^{\infty}\sqrt{\lambda_n \beta^{\mathtt{H}}(t)}e_n$, where $\mathtt{H} \in (\frac{1}{2},1)$ and $\{\beta_n^{\mathtt{H}}\}_{n\in\mathbb{N}}$ is a sequence of one-dimensional fractional Brownian motions mutually independent.

The functions $f : [0,\tau]\times \mathrm{X} \to \mathrm{X}$, $\sigma : [0,\tau]\times\mathrm{X} \to \mathcal{L}_2^0(Q^{1/2}\mathrm{Y},\mathrm{X})$ and $h : [0,\tau]\times\mathrm{X}\times\mathcal{U} \to \mathrm{X}$ are defined by

$$
\begin{aligned}
x(t)(z) &= x(t,z),\ x(0)(z) = x(0,z) = x_0(z), \\
(Bu)(t)(z) &= \int_0^t \tilde{B}(z,s)u(s,t)ds, \\
f(t,x(t))(z) &= f(t,x(t,z)) = \int_0^t \tilde{f}(s,z)sin(x,s)ds, \\
\sigma(t,x(t))(z) &= \sigma(t,x(t,z)) = \frac{(x(t,z))^2}{1+(x(t,z))^2}, \\
\int_{\mathcal{U}} h(t,x,u)\widetilde{N}(ds,du) &= \int_{\mathcal{U}}(1+e^{-t})cosy(t,x,u)\widetilde{N}(dt,du),
\end{aligned}
$$

Thus, for $\alpha = \frac{2}{3}$ the problem (5.1) can be written as the abstract from of system (1.1) with the cost function

$$
\jmath(x,u) = \mathbf{E}\left\{\int_0^1 l(t,x(t),u(t))dt\right\},
$$

where $l(t,x(t),u(t))(z) = \int_{\Omega_1}|x(t,z)|^2dz + \int_{\Omega_1}|u(t,z)|^2dz$. It is easy to see that the assumptions $(\mathbf{H1})-(\mathbf{H4})$ are satisfied, there exists an optimal pair $(x^0,u^0) \in \mathcal{L}^0([0,1]\Omega_1\times\mathcal{L}^2[0,1]\times\Omega_1)$ such that $\jmath(x^0,u^0) \leq \jmath(x,u)$ for all $(x,u) \in \mathcal{L}^2([0,1]\times\Omega_1\times\mathcal{L}^2([0,1]\times\Omega_1)$.

6. Conclusion

In this paper, we studied the existence of solutions and optimal control results of fractional stochastic differential system driven by fractional Brownian motion with Poisson jumps in Hilbert space. The sufficient conditions for the existence of mild solution results are formulated and proved by virtue of fractional calculus, solution operator and stochastic analysis techniques. Furthermore, the existence of optimal control of the proposed problem is presented by using Balder's theorem. The optimal control analysis for fractional stochastic differential inclusions with distributed delays, time varying delays, and impulsive effects will be our future work.

References

[1] P. Balasubramniam, P. Tamilalagan, The solvability and optimal controls for impulsive fractional stochastic integrodifferential equations via resolvent operators, Journal of Optimization Theory and Applications, 174 (2017) 139-155.

[2] G. Da prato, J. Zabczyk, Stochastic Equations in Infinite Dimensions, Cambridge University Press, Cambridge, 1992.

[3] S. Das, Functional Fractional Calculus, Springer-Verlag, Berlin, Heidelberg, 2011.

[4] A. D. Fitt, A. R. H. Goodwin, K. A. Ronaldson, W. A. Wakeham, A fractional differential equation for a MEMS viscometer used in the oil industry, Journal of Computational and Applied Mathematics, 229 (2009) 373-381.

[5] W. G. Glockle, T. F. Nonnenmacher, A fractional calculus approach of self-similar protein dynamics, Biophysical Journal, 68(1) (1995) 46-53.

[6] H. Rudolf, Applications of fractional calculus in physics, World Scientific, 2000.

[7] J. Luo, T. Taniguchi, The existence and uniqueness for non-Lipschitz stochastic neutral delay evolution equations driven by Poisson jumps, Stochastics and Dynamics, 9(1) (2009) 135-152.

[8] A. Anguraj, K. Ravikumar, Existence and stability results for impulsive stochastic functional integrodifferential equations with Poisson jumps, Journal of Applied Nonlinear Dynamics, 8(3) (2019) 407-417.

[9] K. Ramkumar, K. Ravikumar, and S. Varshini, Fractional neutral stochastic differential equations with Caputo fractional derivative: Fractional Brownian motion, Poisson jumps, and optimal control, Stochastic Analysis and Applications 39(1) (2021) 157-176.

[10] P. Muthukumar, K. Thiagu, Existence of solutions and approximate controllability of fractional nonlocal stochastic differential equations of order $1 < q \leq 2$, with infinite delay and Poisson jumps, Differential Equations and Dynamical Systems, 26(1-3) (2018) 15-36.

[11] F. A. Rihan, C. Rajivganthi, P. Muthukumar, Fractional stochastic differential equations with Hilfer fractional derivative: Poisson jumps and optimal control, Discrete Dynamics in Nature and Society, (2017) 1-11.

[12] F. Biagini, Y. Hu, B. Oksendal, T. Zhang, Stochastic calculus for fractional Brownian motion and applications, Springer Science & Business Media, 2008.

[13] C. A. Tudor, Analysis of the Rosenblatt process. ESAIM: Probability and Statistics, 12 (2018) 230-257.

[14] B. Maslowski, B. Schmalfuss, Random dynamical systems and stationary solutions of differential equations driven by the fractional Brownian motion, Stochastic analysis and applications, 22(6) (2004) 1577-1607.

[15] J. Han, L. Yan, Controllability of a stochastic functional differential equation driven by a fractional Brownian motion, Advances in Difference Equations, 2018(1) (2018) 1-18.

[16] P. Tamilalagan, P. Balasubramaniam, Approximate controllability of fractional stochastic differential equations driven by mixed fractional Brownian motion via resolvent operator, International Journal of Control, 90(8) (2017) 1713-1727.

[17] L. Urszula, H. Schattler, Antiangiogenic therapy in cancer treatment as an optimal control proble, SIAM Journal on Control and Optimization, 46(3) (2007) 1052-1079.

[18] A. Ivan, J. J. Nieto, C. J. Silva, D. F. Torres, Ebola model and optiimal control with vaccination constraints, J Ind Manag Optim., 14(2) (2018) 427-446.

[19] H. Aicha, J. J. Nieto, D. Amar, Solvability and optimal controls of impulsive Hilfer fractional delay evolution inclusions with Clarke subdifferential, Journal of Computational and Applied Mathematics, 344 (2018) 725-737.

[20] P. Tamilalagan, P. Balasubramniam, The solvability and optimal controls for fractional stochastic differential equations driven by Poisson jumps via resolvent operators, Applied mathematics and Optimization, 77(3) (2018) 443-462.

[21] O. P. Agrawal, A general formulation and solution scheme for fractional optimal control probalems, Nonlinear Dynamics, 38(1-4) (2004) 323-337.
[22] X. B. Shu, Y. Lai, Y. Chen, The existence of mild solutions for impulsive fractional partial differential equations, Nonlinear Analysis: Theory, Methods & Applications, 74(5) (2011) 2003-2011.
[23] E. J. Balder, Necessary and sufficient conditions for L1-strong-weak lower semicontinuity of integral functionals, Nonlinear Analysis 11(12) (1987) 1399-1404.
[24] C. Rajivganthi, P. Muthukumar, Almost automorphic for fractional stochastic differential equations and its optimal control, Optimal Control Appl. Methods., 37 (2016) 663-681.
[25] J. Ren, J. Wu, The optimal control problem associated with multi-valued stochastic differential equations with jumps, Nonlinear Anal., 86 (2013) 30-51.
[26] J. Shi, Optimal control for stochastic differential delay equations with Poisson jumps and applications, Random Oper. Stoch. Equ., 23 (2015) 39-52.

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