



# Amplitude Responses at Flexural Eigenmodes in Dynamic Acoustic Force Measurement Using Multimodal Excitation Schemes

Cagri Yilmaz<sup>1\*</sup>, Eyup Sabri Topal<sup>2</sup>

<sup>1\*</sup> Akdeniz University, Faculty of Engineering, Department of Mechanical Engineering, Antalya, Turkey, (ORCID: 0000-0002-2976-1044), [cagriyilmaz@akdeniz.edu.tr](mailto:cagriyilmaz@akdeniz.edu.tr)

<sup>2</sup> Akdeniz University, Faculty of Engineering, Department of Mechanical Engineering, Antalya, Turkey, (ORCID: 0000-0002-3974-060X), [estopal@akdeniz.edu.tr](mailto:estopal@akdeniz.edu.tr)

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## Abstract

In this paper, we present a computational investigation to study amplitude sensitivities to acoustic forces in a wide frequency range. We utilize bimodal, trimodal, and tetramodal excitation schemes for the actuation of the Atomic Force Microscopy (AFM) micro-cantilever in the presence of dynamic acoustic forces. In multimodal operations, the micro-cantilever is driven by applying external excitation forces at eigenmode angular frequencies. The Equation of Motion (EOM) is constructed in the consideration of the driven and damped harmonic oscillator as a model and solved numerically to obtain the deflections of the micro-cantilever at flexural eigenmodes. In this current work, time-domain responses of the micro-cantilever to acoustic forces are introduced for different excitation schemes so that free oscillations are compared with the oscillations of the driven micro-cantilever undergoing acoustic forces. Then, we evaluate the results of amplitude responses at the first four eigenmodes with respect to acoustic force frequencies for diverse force strengths. For our case, we obtain the amplitude response at the fourth eigenmode of around 0.303 nm for the force strength of 2900 pN at the frequency of 1740 kHz. This result proves that acoustic forces at megahertz frequencies are measured by using resonant AFM micro-cantilever under tetramodal operation. Therefore, multimodal excitation schemes can be applied to enhance the amplitude sensitivities to dynamic acoustic forces.

**Keywords:** Dynamic acoustic force measurement, AFM micro-cantilever, amplitude responses, multifrequency excitation, eigenmode frequency.

## Çok Modlu Tahrik Şemalarının Kullanıldığı Dinamik Akustik Kuvvet Ölçümünde Eğilme Özmodlarındaki Genlik Tepkileri

### Öz

Bu makalede geniş bir frekans aralığındaki akustik kuvvetlere karşı genlik duyarlılıklarını incelemek için hesaplamalı bir araştırma sunmaktayız. Dinamik akustik kuvvetlerin varlığında Atomik Kuvvet Mikroskobu (AKM) mikro konsolunun çalıştırılması için iki modlu, üç modlu ve dört modlu tahrik şemalarını kullanmaktayız. Çok modlu işlemlerde, mikro konsol, özmod açılmalık frekanslarındaki dış tahrik kuvvetleri uygulanarak sürülmektedir. Hareket Denklemi (HD), sürülen ve sönümlü harmonik osilatörün bir model olarak dikkate alınmasıyla oluşturulmuştur ve mikro konsolun eğilme özmodlarındaki sapmalarını elde etmek için sayısal olarak çözülmüştür. Bu güncel çalışmada mikro konsolun akustik kuvvetlere olan zaman alanındaki tepkileri, farklı tahrik şemaları için sunulmuştur, böylece serbest salınımlar, akustik kuvvetlere maruz kalan ve tahrik edilen mikro konsolun salınımları ile karşılaştırılmıştır. Ardından, çeşitli kuvvet büyüklükleri için akustik kuvvet frekanslarına göre ilk dört özmoddaki genlik tepkilerinin sonuçlarını değerlendirmekteyiz. Bizim durumumuz için 1740 kHz frekansındaki 2900 pN'lik kuvvet büyüklüğü için dördüncü öz modda yaklaşık 0.303 nm'lik bir genlik tepkisi elde etmekteyiz. Bu sonuç megahertz frekanslarındaki akustik kuvvetlerin, tetramodal operasyon altındaki rezonanslı AFM mikro konsol kullanılarak ölçüldüğünü kanıtlamaktadır. Sonuç olarak, dinamik akustik kuvvetlere karşı genlik duyarlılıklarını arttırmak için çok modlu tahrik şemaları uygulanabilmektedir.

\* Corresponding Author: [cagriyilmaz@akdeniz.edu.tr](mailto:cagriyilmaz@akdeniz.edu.tr)

**Anahtar Kelimeler:** Dinamik akustik kuvvet ölçümü, AKM mikro konsol, genlik tepkileri, çok frekanslı tahrik, özmod frekansı.

## 1. Introduction

Localized stress-strain fields in materials generate transient elastic waves. Acoustic emissions rely on rapid release of strain energy derived from material deformations (Saboonchi & Ozevin, 2013). Phase transformation, dislocation in crystal lattice, and crack nucleation are some of the structural irregularities considered as source mechanisms of acoustic emissions (Feng, Tsai, & Jeng, 2012; Feng & Tsai, 2010). Moreover, the failure modes of composite materials are distinguished based on the frequencies of acoustic waves (Lee & Tsuda, 2005). To illustrate, for the carbon/epoxy material, matrix cracking generates acoustic emissions at frequencies in the range of 90-180 kHz. In addition, acoustic emissions at frequencies in the range of 240-310 kHz are released due to failure mechanisms of debonding (de Groot, Wijnen & Janssen, 1995). The studies on the characterization of acoustic emissions provide essential insights into the researches dealing with measurement sensitivities to acoustic emissions.

Acoustic transducers typically operate in a broadband frequency range from 100 kHz to 1 MHz (Guo, Song, Ghalambor & Lin, 2014). In this current work, we investigate the amplitude sensitivities to the acoustic force frequency up to 1.8 MHz using resonant AFM micro-cantilever under multimodal operations. Accordingly, acoustic force strengths can be determined by converting amplitudes of acoustic waves to sound pressure levels (Pa) which is effective on the one-side surface of the micro-cantilever (Yilmaz, Sahin & Topal, 2021). The following studies deal with the actuation of the micro-cantilever by acoustic forces and the detection of acoustic emissions. Laser-induced photo-acoustic emissions are utilized to excite two micro-cantilevers and their oscillation characteristics are explored (Demirkiran, Karakuzu, Erkol, Torun & Unlu, 2018). In another work, responses of the acoustically driven micro-cantilever are investigated for different environmental conditions (Xu & Raman, 2007). Furthermore, acoustic emissions are measured by using the micro-cantilever excited at resonance frequencies in the consideration of ambient air pressures (Takata et al., 2014).

In this study, we utilize multimodal operations such as bimodal, trimodal, and tetramodal excitations to increase amplitude sensitivities to dynamic acoustic forces. In our earlier work, amplitude sensitivity at the second eigenmode to acoustic force strength was enhanced by using the bimodal-frequency excitation scheme (Yilmaz et al., 2021). In the following works, multifrequency excitation schemes are utilized to improve the observable sensitivities in AFM operations. Amplitude and phase responses obtained utilizing the pentamodal excitation scheme are introduced for the tapping-mode AFM (Solares, An & Long, 2014). In another work, computational simulation of tetramodal and pentamodal micro-cantilever dynamics is performed to acquire oscillation observables at the first four or five flexural eigenmodes (An, Solares, Santos & Ebeling, 2014). Besides, changes in the frequency responses at higher eigenmodes are obtained in bimodal and trimodal operations (Solares & Chawla, 2010).

In this paper, we introduce the time-domain oscillations and amplitude responses to dynamic acoustic forces at the first four eigenmodes of the micro-cantilever. Bimodal, trimodal, and tetramodal operations are performed to investigate amplitude sensitivities for diverse strengths of acoustic forces in a wide frequency range. The results are evaluated in the consideration of flexural mode interactions in multifrequency excitations.

## 2. Multimodal Excitation Model

In this section, we introduce a multifrequency excitation model for the measurement of dynamic acoustic forces. In multimodal operations, the micro-cantilever is driven by external excitation forces at eigenmode angular frequencies. The micro-cantilever is regarded as a point-mass object. The EOM is constructed considering the driven and damped harmonic oscillator model and solved numerically to obtain deflections at flexural eigenmodes. The second order differential equation describing dynamics of the driven micro-cantilever undergoing acoustic force is given below.

$$m_e \ddot{z}_i(t) = -k_i z_i(t) - \frac{m_e \omega_i}{Q_i} \dot{z}_i(t) + F_{exc} + F_{Acoustic} \quad (1)$$

In Eq. (1),  $z_i$ ,  $m_e$ ,  $k_i$ ,  $\omega_i$ , and  $Q_i$  are the instantaneous deflection, the effective mass, the stiffness, the angular resonance frequency, and the quality factor of the flexural eigenmode  $i$  of the micro-cantilever respectively. The AFM micro-cantilever operates in air of which influence is represented by the quality factor (Yilmaz et al., 2021). The external excitation force  $F_{exc,i}$  is introduced as follows.

$$F_{exc} = \sum_{i=1}^m F_{exc,i} = \sum_{i=1}^m F_i \cos(\omega_i t) \quad (2)$$

In Eq. (2),  $m$  is set to 2, 3, and 4 for the bimodal, trimodal, and tetramodal excitations respectively.  $F_i$  is the acoustic force strength at the eigenmode angular frequency ( $\omega_i = 2\pi f_i$ ). For our case, responses of the resonant micro-cantilever to dynamic acoustic forces are explored for multimodal operations. Dynamic acoustic force  $F_{Acoustic}$  is formulated as cosine wave at angular frequency ( $\omega_{Ac} = 2\pi f_{Ac}$ ) with acoustic force strength  $F_{Ac}$  (Eq. (3)).

$$F_{Acoustic} = F_{Ac} \cos(\omega_{Ac} t) \quad (3)$$

Total deflection  $z_{tot}$  of the micro-cantilever under multimodal operations is approximated as in Eq. (4) (Lozano & Garcia, 2008).

$$z_{tot}(t) = \sum_{i=1}^m z_i(t) + E(\epsilon) \approx \sum_{i=1}^m A_i \cos(\omega_i t - \phi_i) \quad (4)$$

$A_i$  is the amplitude and  $\phi_i$  is the phase shift at flexural eigenmode  $i$  in Eq. (4).  $E(\epsilon)$  represents contributions of higher modes. Since the micro-cantilever is driven at resonance frequencies, the influences of the dynamic acoustic forces on periods of oscillations are ignored. Instantaneous deflection at flexural eigenmode  $i$  can be extracted from total deflection to determine amplitude and phase shift at flexural eigenmode  $i$ .

## 3. Numerical Simulation

In numerical calculations, the EOM (Eq. (1)) is solved using the Fourth order Runge-Kutta method. Amplitude responses to acoustic forces are determined for the flexural eigenmodes in bimodal, trimodal, and tetramodal operations. The AFM micro-cantilever has a rectangular cross-section with a length ( $l$ ) of 225  $\mu\text{m}$ , width ( $w$ ) of 40  $\mu\text{m}$ , and thickness ( $t$ ) of 1.8  $\mu\text{m}$  as given in (Lozano & Garcia, 2008).

Table 1. Frequency, stiffness and quality factor at flexural eigenmode  $i$  of the micro-cantilever.  $K_i$  are the real roots of a characteristic equation of the micro-cantilever ( $1 + \cos(K_i) \cosh(K_i) = 0$ )

Flexural eigenmode - $i$	$K_i$	Frequency - $f_i$ (kHz) $f_i = f_1 \left(\frac{K_i}{K_1}\right)^2$	Stiffness - $k_i$ (N/m) $k_i = k_1 \left(\frac{f_i}{f_1}\right)^2$	Quality factor - $Q_i = Q_1 \frac{f_i}{f_1}$
1	1.875	48.9	0.9	255.0
2	4.694	306.7	35.4	1598.9
3	7.855	858.4	277.2	4475.3
4	10.996	1682.1	1064.7	8769.5

The mass density  $\rho$  of the micro-cantilever, made of silicon material, is equal to  $2320 \frac{kg}{m^3}$ . The effective mass  $m_e$  of the micro-cantilever is determined using the following expression as in (Skrzypacz, Nurakhmetov, & Wei, 2019).

$$m_e = \frac{33}{140} lwt\rho \quad (5)$$

Micro-cantilever properties for the first four eigenmodes are introduced in Table. 1 (Garcia & Herruzo, 2012). In the numerical simulation, the acoustic force frequencies are generated in the range of 0-1800 kHz with a step of 10 kHz. Strengths of dynamic acoustic forces are set to 60, 1400, and 2900 pN, which are in the range of sound pressure levels from 50.5 to 84.1 dB, for multimodal operations (Yilmaz et al., 2021; Svoren, Nascak, Koleda, Barcik, & Nemeč, 2021). The magnitudes of the excitation forces at the first four eigenmodes are 35.3, 66.4, 185.8, and 364.2 pN respectively. Initial conditions, i.e. initial deflections and velocities, are set to zero for solving the EOM (Eq. (1)) numerically. The total time interval of the numerical simulation is between 19.12 and 19.19 ms.

## 4. Results

Firstly, free oscillations and oscillations in the presence of dynamic acoustic forces are introduced for bimodal, trimodal, and tetramodal excitations in this section. Then, amplitude sensitivities to acoustic forces on the frequency domain are given for diverse force strengths.

The oscillations at the first four eigenmode frequencies obtained using multimodal excitations are illustrated in Fig. 1. Oscillations at eigenmode angular frequencies in the measurement of acoustic forces at diverse frequencies are obtained by solving the EOM (Eq. (1)) numerically in Matlab. Free oscillations and oscillations under dynamic acoustic forces are compared and evaluated in the Discussion section.

Time-varying values of amplitudes at flexural eigenmodes can be determined over the periods of oscillations (Lopez-Guerra, Somnath, Solares, Jesse, & Ferrini, 2019). For all calculations, we determine an amplitude per period by taking an average of first two amplitudes in the time interval between 19.14 and 19.19 ms. The amplitude responses on the domain of acoustic force frequency are depicted in in Fig. 2 and the results are assessed in the Discussion section.

## 5. Discussion

As it is illustrated in Fig. 1(a), the highest amplitude response is obtained using tetramodal frequency excitation. This result proves that amplitude sensitivity at the first eigenmode is enhanced by applying driving forces at higher eigenmode frequencies simultaneously. Moreover, slight phase shifts among multimodal excitations exist for this case. In addition, it is worth mentioning that negligible amplitude differences among free oscillations and oscillations obtained in the measurement of acoustic forces at 50 kHz are prevalent for all cases. Accordingly, the free oscillations do not respond significantly to acoustic forces with the strength of 60 pN irrespective of multimodal operation. On the other hand, when the acoustic force frequency is increased from 50 to 320 kHz, amplitude response ascents significantly considering the ratios of changes in the amplitudes to the free oscillation amplitudes (Fig. 1(b)). Herein, in the absence of acoustic force, the driven micro-cantilever oscillates in the negative range over particular periods owing to the interaction of flexural modes in multimodal excitations. More interestingly, more amplitude sensitivity is obtained using a bimodal excitation scheme rather than trimodal and tetramodal excitations. Here, we infer that excitation parameters, i.e. magnitudes of driving forces, are to be optimized to improve amplitude sensitivities to acoustic forces at particular frequencies in multimodal excitations.

As it is depicted in Fig. 1(c), significant increases in amplitude responses are evident in trimodal and tetramodal operations. We remark that notable phase shifts among these operations also exist for the measurement of acoustic force at the frequency of 890 kHz. As expectedly, the amplitude difference between the free oscillation and the oscillation in the presence of acoustic force at the frequency of 1740 kHz is clearly observed under tetramodal excitation (Fig. 1(d)). We would like to emphasize that sensitivities of oscillation observables such as amplitude and phase shift to acoustic forces become quite evident at higher frequencies.

Amplitude responses to acoustic forces on the frequency domain are determined for diverse strengths of acoustic forces. As it is illustrated in Fig. 2, the highest amplitude responses at the first four eigenmodes are obtained in the measurement of acoustic forces with the strength of 2900 pN at the frequency of 50, 320, 890, and 1740 kHz respectively. The oscillations in the presence of these acoustic forces are depicted in Fig. 1 for the multimodal operations. More interestingly, negative values of the amplitudes indicate that the micro-cantilever oscillates below the equilibrium point.

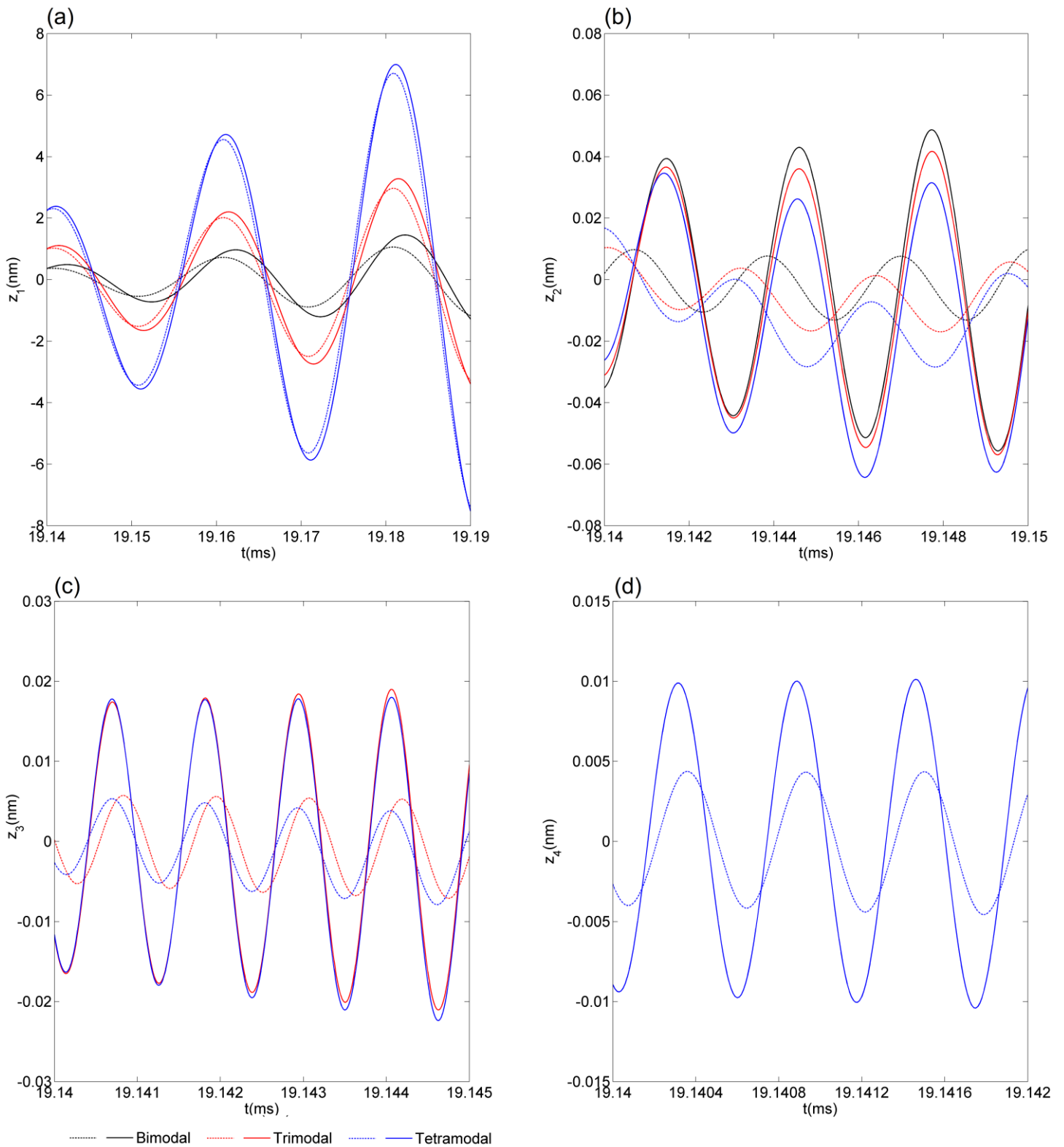


Fig. 1. Free oscillations and oscillations of the driven micro-cantilever in the presence of dynamic acoustic forces with the force strength of 60 pN for multifrequency excitations. (a) Free oscillations and oscillations in measurement of acoustic force at the frequency of 50 kHz at the first eigenmode frequency. (b) Free oscillations and oscillations in measurement of acoustic force at the frequency of 320 kHz at the second eigenmode frequency. (c) Free oscillations and oscillations in measurement of acoustic force at the frequency of 890 kHz at the third eigenmode frequency. (d) Free oscillation and oscillation in measurement of acoustic force at the frequency of 1740 kHz at the fourth eigenmode frequency. Solid lines indicate the oscillations of the micro-cantilever in the presence of acoustic forces, whereas dashed lines indicate the free oscillations, obtained in the absence of acoustic force.

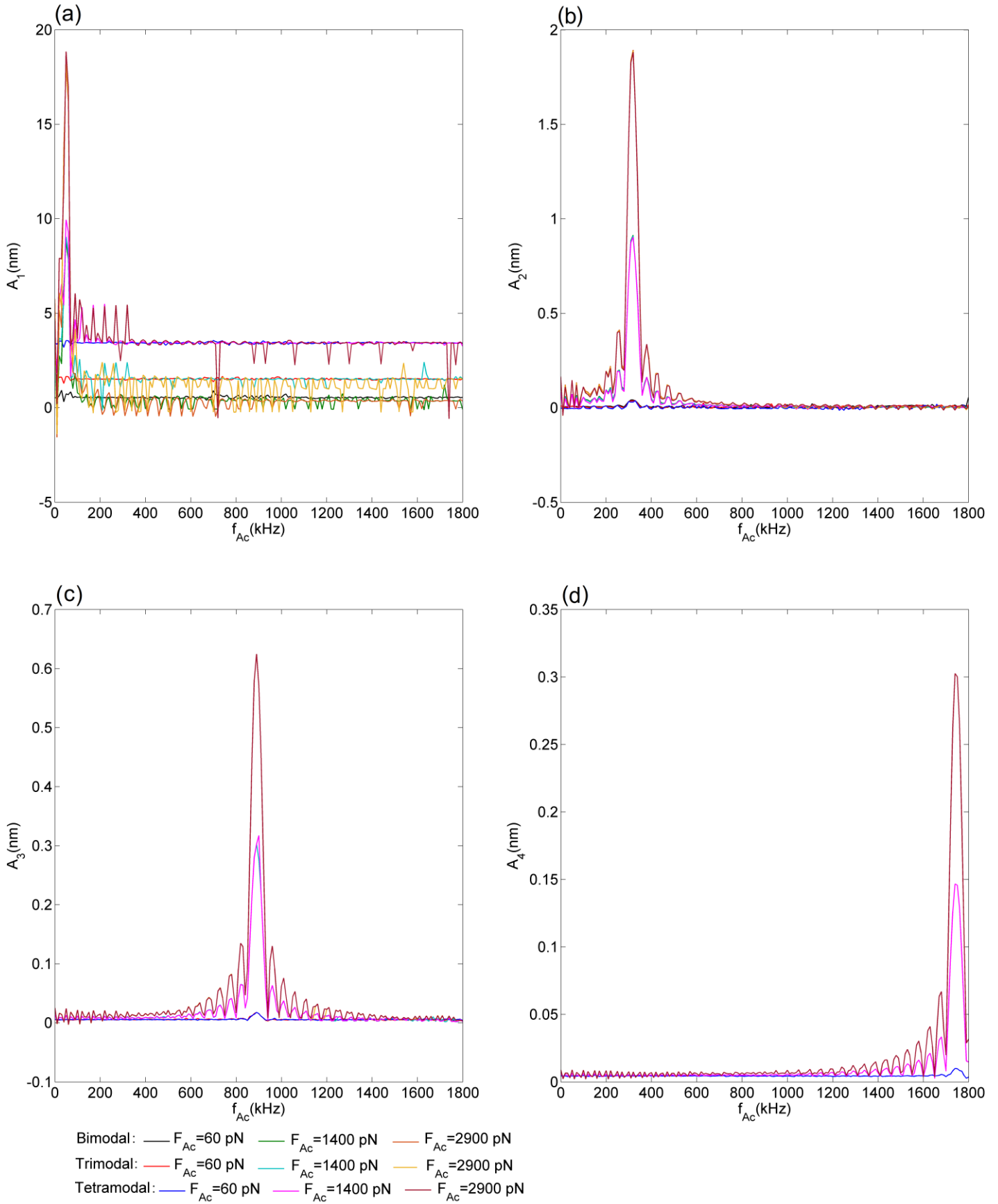


Fig. 2. Amplitude responses to acoustic forces on the frequency domain for different force strengths under multimodal operations. (a) Amplitude responses at the first eigenmode. (b) Amplitude responses at the second eigenmode. (c) Amplitude responses at the third eigenmode. (d) Amplitude responses at the fourth eigenmode.



Amplitude responses at the first eigenmode can be distinguished among different acoustic force strengths in bimodal, trimodal, and tetramodal operations. Interaction of flexural modes in each multimodal excitation has notable influences on the amplitude sensitivities at the first eigenmode on the frequency domain (Fig. 2(a)). On the other hand, differences among amplitude responses for multimodal excitations are not prevalent as it is illustrated in Fig. 2(b)(c)(d). More significantly, flexural mode interaction owing to the simultaneous application of driving forces at eigenmode frequencies does not occur at higher frequencies depending on the eigenmode frequencies and quality factors. Furthermore, an increase in force strength results in an increase in amplitude response irrespective of multimodal operation as expectedly. For our case, the amplitude response of around 0.303 nm is obtained for the acoustic force strength of 2900 pN at the frequency of 1740 kHz. This result proves that amplitude sensitivity to acoustic force at megahertz frequency is achieved by driving the AFM micro-cantilever under tetramodal excitation.

## 6. Conclusion

The simulation results indicate that amplitudes at the first four eigenmodes exhibit different responses on the domain of acoustic force frequency. Interaction of flexural modes occurs in multimodal operations in the consideration of amplitude responses at the first eigenmode. On the other hand, remarkable differences among amplitudes at higher eigenmodes are not evident for bimodal, trimodal, and tetramodal excitations. Herein, we infer that amplitude sensitivities can be improved by optimizing the excitation parameters such as magnitudes of driving forces at eigenmode frequencies. In addition, the response of the micro-cantilever to dynamic acoustic forces ascends considerably as the acoustic force strength increases irrespective of multifrequency excitation scheme. Furthermore, it is worth mentioning that we obtain high-sensitivity frequency regions near eigenmode frequencies of the micro-cantilever. Accordingly, the micro-cantilever with optimum mechanical characteristics, i.e. eigenmode frequencies, can be selected to achieve high sensitivity to acoustic force in a desired frequency band. Moreover, acoustic forces at megahertz frequency can be measured using resonant AFM micro-cantilever under tetramodal operation. For our case, we acquire significant amplitude sensitivities to acoustic forces at the frequency of 1740 kHz for the force strength in the range of 60-2900 pN. Therefore, resonant AFM micro-cantilevers driven under multifrequency excitations can be utilized to improve amplitude sensitivities to dynamic acoustic forces.

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