Journal of Naval Science and Engineering
2016, Vol. 12, No.1, pp.80-89
Makine Mühendisliği/Mechanical Engineering

EFFECT OF MISSING PATH AND FORCE ON VIBRATION RESPONSE PREDICTION

Hakan UÇAR

Design Project Office, Istanbul Naval Shipyard, Pendik, Istanbul, Türkiye
hkucar@gmail.com

Date of Receive: 11.12.2015 **Date of Acceptance: 02.03.2016**

ABSTRACT

Accurate prediction of the vibration responses is an essential requirement during the design and optimization stages of mechanical structures. For this reason, a vibration response prediction (VRP) methodology, based on a set of frequency response functions (FRF) and operational responses, is established. In this methodology, defining the sources and transmission paths of the structure accurately is critical. Since it is based on the FRFs of the system, unconsidered sources and/or missing path result in inaccurate predictions. The goal of this paper is to present the effect of a missing path on the prediction results. In accordance with this purpose, a three degree-of-freedom system will be investigated analytically and VRP methodology will be applied along with the calculated receptances and displacements of the system.

EKSIK PATİKA VE KUVVETİN TİTREŞİM CEVABI ÖNGÖRÜSÜNE ETKİSİ

ÖZ

Mekanik yapılarınd dizayn ve optimizasyon aşamalarında yapı üzerindeki titreşimi doğru olarak öngörülmesi önemli bir ihityaçtır Bu nedenle, operasyonel cevap ve frekans cevap fonknsiyonuna (FCF) bağlı olan bir titreşim cevabı öngörüsü (TCÖ) metodu kullanılmaktadır. Bu metotta,titreşim kaynaklarını ve iletim patikalarını doğru olarak tanımlamak çok kiritiktir. Metot sistemin FCF'lerine bağlı olduğundan, hesaba katılmayan kaynak ve/veya eksik patikalar doğru olmayan öngörülere neden olacaktır. Bu makalenin amacı eksik patikanın öngörü son uçlarına olan etkisini sunmaktır. Bu amaç doğrultusunda üç sebrstlik dereceli bir system analitik olarak incelenecek ve TCÖ metodu hesaplanan deplasman ve FCF'ler ileuygulanacaktır.

Keywords: vibration response, matrix inversion, cross-coupling, 3 DOF systems.
Anahtar Kelimeler: Titreşim cevabı, matris tersine çevirme, bağıl etki, üç serbestlik dereceli system.

1. INTRODUCTION

Accurate prediction of the vibration responses is an essential requirement during the design and optimization stages of mechanical structures. The first step of the vibration response prediction (VRP) is to identify the exciting forces acting on the structure. The force data is usually not available as the direct measurement of these forces is impractical or almost impossible. Therefore, the identification of these forces by using the vibration data has been attracted a lot of attention. Verheij [1] presented dynamic stiffness method which seems to be the most basic way, especially for structures having elastomer components. However, accurate complex dynamic stiffness data is seldom available and even if there is, it is only valid for a given load condition. Hillary and Ewins [2] investigated the problem of sinusoidal force identification of a cantilever beam using a least-squares method. Dascotte and Desanghere [3] presented a methodology based on the Bayesian force identification procedure. Janssens et al. [4] investigated the use of an equivalent force method to determine the sound transmission in ships. Transmissibility method has been used to predict the vibration response at a point of interest without identifying the forces [5-8]. This approach is much simpler and faster compared to other methods but unconsidered potential cross-coupling between the paths lead to incorrect predictions. Matrix inversion method has been developed in the early 1980s [9, 10]. This technique uses an inverted matrix composed of the frequency response functions (FRFs) and a vector of measured vibration responses. In this method, defining the sources and paths accurately is critical. Thus, the decoupling program is usually created in order to avoid missing paths. A missing path means that a force is acting on the structure at a given point but no path input (e.g. an acceleration signal) is measured at that point [8]. Missing paths result in inaccurate prediction of the total force and hence, the vibration response. The goal of this paper is to present the effect of a missing path on the prediction results. For this reason, a three degree-of-freedom system will be investigated analytically and the matrix inversion method will be applied based on the calculated receptances and displacements of the system.

2. THEORY OF VIBRATION RESPONSE PREDICTION

In a dynamic system, the vibration response of a point strongly depends on not only the corresponding point force but also the other operating forces. This effect is called cross-couplings and should be considered by including all FRFs between path inputs, as in Equation (1).

$$\bar{X}_1 = F_1 H_{11} + F_2 H_{21} + \cdots + F_n H_{n1} \tag{1}$$

where n is the number of path inputs or internal forces, F is the exciting force and H is FRF between the force and the path inputs.

A square FRF matrix, $n \times n$, is created since the number of forces and responses are equal to each other. By taking the inverse of this FRF matrix and multiplying the vector of vibration responses, the exciting forces are identified, as shown in Equation (2).

$$\{F_i(\omega)\} = [H_{ij}(\omega)]^{-1}\{\bar{X}_i(\omega)\} \tag{2}$$

where $i=j$ which denotes the number of paths and forces.

Vibration response at the point of interest, k is predicted once the exciting forces acting on each path are calculated. Assuming that the system is linear and time-invariant, partial contributions of each path to the response are added up and the vibration response at the target is calculated, as seen in Equation (3).

$$\bar{X}_k(\omega) = \sum_{i=1}^{n} \bar{X}_{ki}(\omega) = \sum_{i=1}^{n} F_i(\omega) H_{ki}(\omega) \tag{3}$$

3. THREE DEGREE-OF-FREEDOM (DOF) SYSTEM

In order to illustrate the vibration response prediction based on the matrix inversion method and the transmissibility approach, consider the following 3 DOF system.

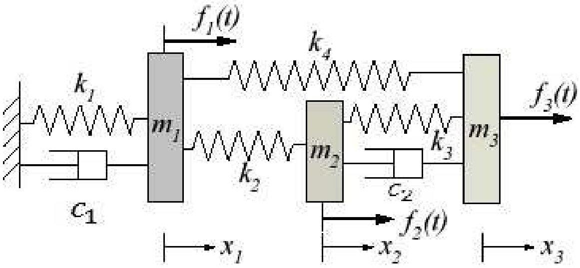


Figure 1. Three DOF system [11]

The degrees of freedom of a lumped mass system are equal to the number of vibrating mass points and the number of natural frequencies. In this system, there are three coordinates defining the movement for each moving mass; x_1, x_2, and x_3. The moving masses m_1, m_2, and m_3 are connected to each other through a series of spring and viscous dampers with the coefficients k_1, k_2, k_3 and c_1, c_2, respectively. The system is excited by the external forces f_1, f_2 and f_3.

Using the free body diagrams for each mass and analyzing the forces acting on them by applying Newton's Second Law of motion, the equation of motion for the system in matrix notation is as follows;

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & -c_2 \\ 0 & -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2+k_4 & -k_2 & -k_4 \\ -k_2 & k_2+k_3 & -k_3 \\ -k_4 & -k_3 & k_3+k_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} \quad (4)$$

As shown in Equation (4), the matrix containing the masses which is multiplied by a vector of accelerations is called mass matrix, *M*, the matrix containing the damping coefficients which is multiplied with the vector of velocity is called the structural damping matrix, *C* and similarly, the matrix containing the spring coefficients which is multiplied with the vector of displacements is called the stiffness matrix, *K*.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \tag{5}$$

Removing the damping effects and considering the free vibration, Equation (5) becomes;

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{6}$$

The solution of the undamped system can be expressed as a set of harmonic functions, which is simply amplitude multiplied by a complex exponential.

$$\{x\} = \{\emptyset\}\, e^{j\omega_n t} \tag{7}$$

The term $e^{j\omega_n t}$ is a mathematical expression to solve differential equations by using the phasor form for the solution. After substituting the solution to the matrix;

$$([K] - \omega_n^2 [M])\{\emptyset\}e^{j\omega_n t} = \{0\} \tag{8}$$

In Equation (8), the exponential part, $e^{j\omega_n t}$ cannot be zero for finite *t*. Thus, the equation is reduced to the problem often encountered in mathematics which is called the eigenvalue problem, as in Equation (9)

$$([K] - \omega_n^2 [M])\{\emptyset\} = \{0\} \tag{9}$$

In this eigenvalue problem, ω_n^2 contains the eigenvalues of the system, namely natural frequencies and $\{\emptyset\}$ is the eigenvector representing the mode shapes. After solving the eigenvalue problem, the modal matrix is constructed with the eigenvectors and denoted by Φ. Modal transformation is carried out by defining the relationship between the physical coordinate, *x* and modal coordinate, *q*.

$$\{x\} = [\Phi]\{q\} \tag{10}$$

The modal matrix is used to transform into modal coordinates and uncouple the equations of motion. Using the orthogonality, diagonal modal mass, damping and stiffness matrices are created as;

$$= [\Phi]^T[M][\Phi] \quad [C_q] = [\Phi]^T[C][\Phi] \quad [K_q] = [\Phi]^T[K][\Phi] \tag{11}$$

Force vector is also transformed into modal coordinates;

$$\{f_q\} = [\Phi]^T\{f\} \tag{12}$$

Substituting Equation (10) and (11) into Equation (5), the modal equation of motion for the damped system is determined as follows;

$$[M_q]\{\ddot{q}\} + [C_q]\{\dot{q}\} + [K_q]\{q\} = \{f_q\} \tag{13}$$

After solving Equation (13), the results are back-transformed to physical coordinates and a solution exists of the form $\{x\}$ depending on $\{f\}$. The relationship between $\{x\}$ and $\{f\}$ can be expressed as;

$$\{x\} = [\alpha]\{f\} \tag{14}$$

where $[\alpha]$ is the receptance FRF matrix and defines the frequency response of the model. The individual component of the receptance matrix, $\alpha_{jk}(\omega)$ is defined as follows;

$$\alpha_{jk}(\omega) = \frac{x_j(\omega)}{f_k(\omega)} \tag{15}$$

where j represents the number of DOF and k denotes the number of forces. Individual FRF parameter, $\alpha_{jk}(\omega)$ can be calculated by using the following formula based on the modal matrix elements.

$$\alpha_{jk}(\omega) = \sum_{r=1}^{N} \frac{(\phi_{jr})\,(\phi_{kr})}{(k_r - \omega^2 m_r) + i\,(\omega\, c_r)} \tag{16}$$

where k_r, m_r and c_r are the indices of the modal mass, damping, and stiffness matrices, respectively.

3.1. Vibration Response Prediction Study

Turning now to the VRP study based on the 3-DOF system in Figure 1, the study is composed of three parts. In the first part, x_3 is aimed to be predicted when there are just two forces applied on m_1 and m_2. In this case, there is a *(2x2)* FRF receptance matrix and a *(2x1)* vector of vibration responses as displacement vector. x_3 is aimed to be predicted by applying matrix direct inversion method along with Equation (2) and (3). The following parameters are used in this section.

Table 1 Three DOF system properties

m_1	1 kg	c_1	1 Ns/m	k_1	1.4×10^5 N/m	f_1	20 N
m_2	0.5 kg	c_2	1 Ns/m	k_2	2.8×10^5 N/m	f_2	10 N
m_3	0.2 kg			k_3	1×10^3 N/m	f_3	0 N
				k_4	7×10^4 N/m		

According to the comparison between calculated and predicted response, x_3 shown in Figure 2, it can be stated that VRP based on the matrix inversion method poses accurate prediction if all sources and paths are considered.

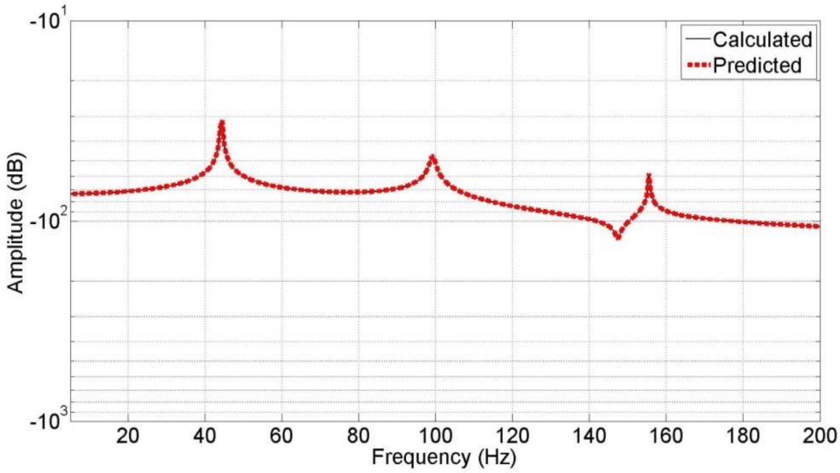


Figure 2. Vibration response, x_3

The question is now what if any path and one of the sources are not taken into account. In the second part of the study, an external force, f_3 is applied on m_3 with a magnitude of 10 N. The response, x_3 is predicted in such a case that third DOF and the external force, f_3 are not considered. Although the force also contributes to the target response, it is not taken explicitly into the calculations. In this respect, the predicted response, x_3 is shown in Figure 3.

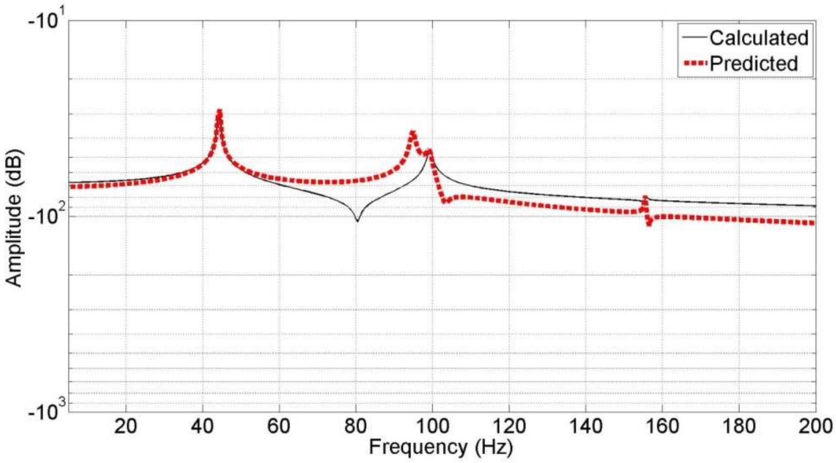
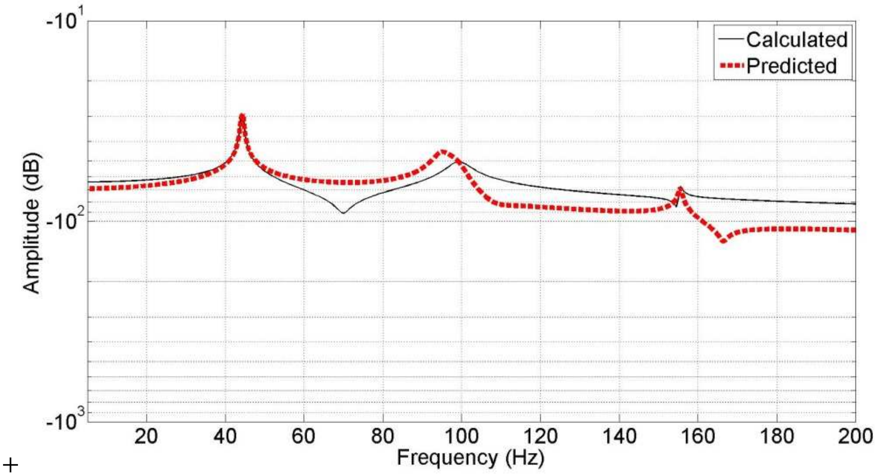


Figure 3. Vibration response, x_3 (without considering f_3 = 10 N)

As shown in Figure 3, unconsidered source and path result in remarkable error up to 40 dB. In the third case study, f_3 is increased up to 20 N and the response at x_3 is predicted without considering f_3, as shown in Figure 4.



\+ */+

Figure 4. Vibration response, x_3 (without considering f_3 = 20 N)

According to the analytical case studies, it is shown that the resulting error is proportional to the amplitude of the omitted force and the error increases as 2 dB when the force is doubled. Since each path is connected to each other, cross-coupling effects become more of an issue. Missing paths are one of the most important inaccuracies in the VRP study. Therefore, this issue should be given utmost importance before performing VRP study.

4. SUMMARY AND CONCLUSIONS

In this study, the effect of missing path on the prediction of vibration response was discussed with the methodology of matrix inversion. Three analytical case studies were executed. Throughout the set of these analytical case studies, it is determined that unconsidered source and missing path result in remarkable error up to 40 dB. Furthermore, the resulting error in prediction depends on the amplitude of the missing force and the overall error increases up to 2 dB when the missing force is doubled. As a consequence, cross-coupling effects become more of an issue since each path is connected to each other and missing paths are one of the most important inaccuracies in the VRP study. Therefore, this issue should be given utmost importance before performing VRP study.

5. REFERENCES

[1] Verheij, J. W., (1982), Multipath Sound Transfer from Resiliently Mounted Shipboard Machinery, TNO, Delft, Netherlands

[2] Hillary, B., (1983), Indirect Measurement Of Vibration Excitation Forces, *PhD Thesis,* Imperial College London, Dynamics Section, London.

[3] Desanghere, G., Identification *of external forces based on frequency response measurements,* Proc. 8th Int. Seminar on Modal Analysis, Leuven, Belgium, 1983.

[4] Janssens, M. H. A., Verheij, J. W., and Thompson, D. J., (1999), *The use of an equivalent forces method for the experimental quantification of structural sound transmission in ships*, Journal of Sound and Vibration, 226(2), pp. 305-328

[5] Ribeiro, A. M. R., Silva, J. M. M., and Maia, N. M. M., (2000), *On the generalisation of the transmissibility concept,* Mechanical Systems and Signal Processing, 14(1), pp. 29-35.

[6] Maia, N. M. M., Silva, J. M. M., and Ribeiro, A. M. R., (2001), *The transmissibility concept in multi-degree-of-freedom systems*, Mechanical Systems and Signal Processing, 15(1), pp. 129-137.

[7] Ribeiro, A. M. R., Fontul, M., Maia, N. M. M., and Silva, J. M. M., *Further developments on the transmissibility concept for multiple degree of freedom systems*, Proceedings of the XIth International Conference on Vibration Engineering, Timisoara, Romania, 2005.

[8] Gajdatsy, P., Janssens, K., Desmet, W., and Van der Auweraer, H., (2010), *Application of the transmissibility concept in transfer path analysis,* Mechanical Systems and Signal Processing, 24(7), pp. 1963-1976.

[9] Roesems, D., *Analysis of structure borne noise transmission paths,* Proc. 6th Int. Seminar on Modal Analysis, Leuven, Belgium , 1981.

[10] Verheij, J. W., *Experimental procedures for quantifying sound paths to the interior of road vehicles,* Proc. 2nd Int. Conference on Vehicle Comfort, pp. 483-491, Bologna, Italy, 1992.

[11] Efunda, *Multiple degree - of - freedom*, 2015.