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SOFT SEMI-TOPOLOGICAL POLYGROUPS

Rasoul MOUSAREZAEI and Bijan DAVVAZ

Department of Mathematical Sciences, Yazd University, Yazd, IRAN

Abstract. By removing the condition that the inverse function is continuous in soft topological polygroups, we will have less constraint to obtain the results. We offer different definitions for soft topological polygroups and eliminate the inverse function continuity condition to have more freedom of action.

1. Introduction

To answer the types of uncertainties that abound in various sciences, we insert soft sets into mathematical structures. Specifically, we equip topological polygroups with soft sets. This is a process that began in 1934 by Marty [16] with the introduction of hypergroups and continued with the introduction of soft sets by Molodtsov in 1999 [17]. Since then, many efforts have been made to deepen the discussion, some of which we can mention below.

A good description of the Groupoides, demi-hypergroupes et hypergroupes is given by M. Koskas in [14], also useful information about the Soft subsets and soft product operations is provided by F. Feng, Y.M. Li in [8]. There is a beautiful writing about the topological spaces from the S. Nazmul, SK. Samanta under the name Neighbourhood properties of soft topological spaces in [20], also about Soft set theory by P. K. Maji, R. Biswas and A. R. Roy in [15], Soft topological groups and rings by T. Shah and S. Shaheen in [27], On soft topological hypergroups by G. Oguz in [24], On soft topological spaces by M. Shabir and M. Naz in [26]. Only a genius like T. Hida can write such a beautiful story about the Soft topological group in [11], also G. Oguz with article Soft topological hyperstructure in [25] and M. Shabir, M. Naz With their own handwriting about the On soft topological spaces in [26]. If you want to read interesting articles about the topological polygroups,

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rasoul.mousarezaei@stu.yazd.ac.ir; 0000-0002-6163-4037
davvaz@yazd.ac.ir-Corresponding author; 0000-0003-1941-5372.

you can read Heidari's article about the Topological polygroups in [9], also about the Idealistic soft topological hyperrings by G. Oguz in [23] and A new view on topological polygroups by G. Oguz in [22], Soft sets and soft groups by H. Aktas and N. Cagman in [1], Prolegomena of Hypergroup Theory by P. Corsini in [5].

2. Preliminaries

2.1. **Soft Sets.** Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and A be a non-empty subset of E. A pair $(\mathbb{F}, A)$ is called a soft set over U, where $\mathbb{F}$ is a mapping given by $\mathbb{F} : A \to P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For $a \in A, \mathbb{F}(a)$ may be considered as the set of approximate elements of the soft set$(\mathbb{F}, A)$. Clearly a soft set is not a set. For two soft sets $(\mathbb{F}, A)$ and $(\mathbb{G}, B)$ over a common universe U, we say that $(\mathbb{F}, A)$ is a soft subset of $(\mathbb{G}, B)(i.e., (\mathbb{F}, A)\widehat{\subset}(\mathbb{G}, B))$ if $A \subseteq B$ and $\mathbb{F}(a) \subseteq \mathbb{G}(a)$ for all $a \in A$. $(\mathbb{F}, A)$ is said to be a soft super set of $(\mathbb{G}, B)$, if $(\mathbb{G}, B)$ is a soft subset of $(\mathbb{F}, A)$ and it is denoted by $(\mathbb{F}, A)\widehat{\supset}(\mathbb{G}, B)$. Two soft sets $(\mathbb{F}, A)$ and $(\mathbb{G}, B)$ over a common universe U are said to be soft equal if $(\mathbb{F}, A)$ is a soft subset of $(\mathbb{G}, B)$ and $(\mathbb{G}, B)$ is a soft subset of $(\mathbb{F}, A)$. A soft set $(\mathbb{F}, A)$ over U is said to be a NULL soft set, denoted by $\widehat{\varnothing}$, if $\mathbb{F}(a) = \varnothing$ (null set) for all $a \in A$. A soft set $(\mathbb{F}, A)$ over U is said to be ABSOLUTE soft set, denoted by $\widehat{A}$, if $\mathbb{F}(a) = U$ for all $a \in A$. $(\mathbb{F}, A)$ AND $(\mathbb{G}, B)$ denoted by $(\mathbb{F}, A)\widehat{\wedge}(\mathbb{G}, B)$ is defined by $(\mathbb{F}, A)\widehat{\wedge}(\mathbb{G}, B) = (\mathbb{H}, A \times B)$, where $\mathbb{H}((a, b)) = \mathbb{F}(a) \cap \mathbb{G}(b)$ for all $(a, b) \in A \times B$. $(\mathbb{F}, A)$ OR $(\mathbb{G}, B)$ denoted by $(\mathbb{F}, A)\widehat{\vee}(\mathbb{G}, B)$ is defined by $(\mathbb{F}, A)\widehat{\vee}(\mathbb{G}, B) = (O, A \times B)$ where, $O((a, b)) = \mathbb{F}(a) \cup \mathbb{G}(b)$ for all $(a, b) \in A \times B$. Union of two soft sets $(\mathbb{F}, A)$ and $(\mathbb{G}, B)$ over the common universe U denoted by $(\mathbb{F}, A)\widehat{\cup}(\mathbb{G}, B)$ is defined by $(\mathbb{H}, C)$, where $C = A \cup B$ and for all $a \in C$,

$$\mathbb{H}(a) = \begin{cases} \mathbb{F}(a) & \text{if } a \in A - B \\ \mathbb{G}(a) & \text{if } a \in B - A \\ \mathbb{F}(a) \cup \mathbb{G}(a) & \text{if } a \in A \cap B. \end{cases}$$

Bi-intersection of two soft sets $(\mathbb{F}, A)$ and $(\mathbb{G}, B)$ over the common universe U is the soft set $(\mathbb{H}, C)$ is defined by $(\mathbb{F}, A)\widehat{\cap}(\mathbb{G}, B) = (\mathbb{H}, C)$, where $C = A \cap B$ and $\mathbb{H}(a) = \mathbb{F}(a) \cap \mathbb{G}(a)$ for all $a \in C$. Extended intersection of two soft sets $(\mathbb{F}, A)$ and $(\mathbb{G}, B)$ over the common universe U denoted by $(\mathbb{F}, A) \cap_E (\mathbb{G}, B)$ and is defined by $(\mathbb{H}, C)$, where $C = A \cup B$ and for all $a \in C$,

$$\mathbb{H}(a) = \begin{cases} \mathbb{F}(a) & \text{if } a \in A - B \\ \mathbb{G}(a) & \text{if } a \in B - A \\ \mathbb{F}(a) \cap \mathbb{G}(a) & \text{if } a \in A \cap B. \end{cases}$$

Let $(\mathbb{F}, A)$ be a soft set. The set $Supp(\mathbb{F}, A) = \{a \in A : \mathbb{F}(a) \neq \varnothing\}$ is called the support of the soft set $(\mathbb{F}, A)$. A soft set is said to be non-null if its support is not equal to the empty set. If A is equal to E we write $\mathbb{F}$ instead of $(\mathbb{F}, A)$. Let $\theta : U \longmapsto U'$ be a function and $\mathbb{F}(resp.\mathbb{F}')$ be a soft set over $U(resp.U')$ with a parameter set E. Then $\theta(\mathbb{F})(resp.\theta^{-1}(\mathbb{F}'))$ is the soft set on $U'(resp.U)$ is defined by $(\theta(\mathbb{F}))(e) = \theta(\mathbb{F}(e))(resp.(\theta^{-1}(\mathbb{F}'))(e) = \theta^{-1}(\mathbb{F}'(e)))$. We will use the symbol $\mathbb{F}^{\widehat{c}}$ to denote soft complement of $\mathbb{F}$ and is defined by $\mathbb{F}^{\widehat{c}}(e) = U \setminus \mathbb{F}(e)(e \in E)$. Let $\mathbb{F}$ be a soft set over U and x be an element of U we call x is a soft element of $\mathbb{F}$, if $x \in \mathbb{F}(e)$ for all parameters $e \in E$ and denoted by $x\widehat{\in}\mathbb{F}$. We recall the above definitions from [11, 27].

2.2. **Polygroups.** Let H be a non-empty set. A mapping $\circ : H \times H \longmapsto P^*(H)$ is called a hyperoperation, where $P^*(H)$ is the family of non-empty subsets of H. The couple $(H, \circ)$ is called a hypergroupoid. In the above definition,if A and B are two non-empty subsets of H and $x \in H$, then we define:

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} a \circ b,\ x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

A hypergroupoid $(H, \circ)$ is called a semihypergroup if for every $x, y, z \in H$, we have $x \circ (y \circ z) = (x \circ y) \circ z$ and is called a quasihypergroup if for every $x \in H$, we have $x \circ H = H = H \circ x$. This condition is called the reproduction axiom. The couple $(H, \circ)$ is called a hypergroup if it is a semihypergroup and a quasihypergroup [5].

Let $(H, \circ)$ be a semihypergroup and A be a non-empty subset of H. We say that A is a complete part of H if for any non-zero natural number n and for all $a_1, \dots, a_n$ of H, the following implication holds:

$$A \cap \prod_{i=1}^{n} a_i \neq \varnothing \Rightarrow \prod_{i=1}^{n} a_i \subseteq A.$$

The complete parts were introduced for the first time by Koskas [14]. Let $(G, \circ)$ and $(H, *)$ be two hypergroups. A map $f : G \longmapsto H$, is called a homomorphism if for all x, y of G, we have $f(x \circ y) \subseteq f(x) * f(y)$; a good homomorphism if for all x, y of G, we have $f(x \circ y) = f(x) * f(y)$;f is an isomorphism if it is a good homomorphism ,and its inverse f^{-1} is a homomorphism, too.

Definition 1. *A special sub class of hypergroups is the class of polygroups.A polygroup is a system $P =< P, \circ, e, -1 >$,where $\circ : P \times P \longmapsto P^*(P)$, $e \in P$, -1 is a unitary operation on P and the following axioms hold for all $x, y, z \in P$:*

(1) $(x \circ y) \circ z = x \circ (y \circ z)$;
(2) $e \circ x = x \circ e = x$;
(3) $x \in y \circ z$ *implies* $y \in x \circ z^{-1}$ *and* $z \in y^{-1} \circ x$.

The following elementary facts about polygroups follow easily from the axioms: $e \in x \circ x^{-1} \cap x^{-1} \circ x, e^{-1} = e, (x^{-1})^{-1} = x$, and $(x \circ y)^{-1} = y^{-1} \circ x^{-1}$. A non-empty subset K of a polygroup P is a subpolygroup of P if and only if $a, b \in K$ implies $a \circ b \subseteq K$ and $a \in K$ implies $a^{-1} \in K$.

The subpolygroup N of P is normal in P if and only if $a^{-1} \circ N \circ a \subseteq N$ for all $a \in P$.

Theorem 1. *Let N be a normal subpolygroup of P then:*

(1) *$Na = aN$ for all $a \in P$;*
(2) *$(aN)(bN) = abN$ for all $a, b \in P$;*
(3) *$aN = bN$ for all $b \in aN$.*

EXAMPLE 1. *Let P be $\{1, 2\}$ and hyperoperation $\circledast$ be as follow:*

$\circledast$	1	2
1	1	2
2	2	$\{1, 2\}$

With the above multiplication table, P is a polygroup [7].

Let P is polygroup and $(\mathbb{F}, A)$ be a soft set on P. Then $(\mathbb{F}, A)$ is called a (normal)soft polygroup on P if $\mathbb{F}(x)$ be a (normal)subpolygroup of P for all $x \in Supp(\mathbb{F}, A)$.

EXAMPLE 2. *Let P be $\{e, a, b\}$ and multiplication table be:*

$\circ$	e	a	b
e	e	a	b
a	a	e	b
b	b	b	$\{e, a\}$

Subpolygroups of P are $\varnothing, P, \{e\}, \{e, a\}$. Let A be equal with P and define soft set $\mathbb{F}$ as follow:

$$\mathbb{F}(x) = \begin{cases} \{e\} & \text{if } x = e \\ \{e, a\} & \text{if } x = a \\ \{e, a, b\} & \text{if } x = b \end{cases}$$

Therefore $(\mathbb{F}, A)$ is a soft polygroup. We recall the above definitions and theorems from [7].

2.3. **Topological Hyperstructure.** Suppose that T is a topology on G, where G is a group, then (G, T) is called a topological group over G if φ and ${}^{-1}$ are continuous, where φ and ${}^{-1}$ are as follow:

(1) The mapping $\varphi : G \times G \longmapsto G$ is defined by $\varphi(g, h) = gh$ and $G \times G$ is endowed with the product topology.
(2) The mapping ${}^{-1} : G \longmapsto G$ is defined by ${}^{-1}(g) = g^{-1}$ [10].

If the condition (2) of previous definition is not met, then the (G, T) is called semi-topological group over G.

Let $(\mathbb{F}, A)$ be a soft set over G. Then the $(\mathbb{F}, A, T)$ is called soft topological group over G if the following conditions hold:

(1) $\mathbb{F}(a)$ be a subgroup of G for all $a \in A$.
(2) The mapping $\varphi : (x, y) \longmapsto xy$ of the topological space $\mathbb{F}(a) \times \mathbb{F}(a)$ onto $\mathbb{F}(a)$ be continuous for all $a \in A$.
(3) The mapping ${}^{-1} : \mathbb{F}(a) \longmapsto \mathbb{F}(a)$ is defined by ${}^{-1}(g) = g^{-1}$ be continuous for all $a \in A$.

If the condition (3) of previous definition is not met, then the $(\mathbb{F}, A, T)$ is called soft semi-topological group over G.

In [9] is proved that condition continuity φ is equivalent to following statement;

If $U \subseteq G$ is open, and $gh \in U$, then there exist open sets V_g and V_h with the property that $g \in V_g$,$h \in V_h$, and $V_gV_h = \{v_1v_2 | v_1 \in V_g, v_2 \in V_h\} \subseteq U$.

Also, condition continuity $^{-1}$ is equivalent to following statement; If U subset of G is open, then $U^{-1} = \{g^{-1} | g \in U\}$ be open.

Let (H, T) be a topological space. The following theorem give us a topology on $P^*(H)$ that is induced by T.

Theorem 2. *Let (H, T) be a topological space. Then the family β consisting of all sets $S_V = \{U \in P^*(H) \mid U \subseteq V\}, V \in T$ is a base for a topology on $P^*(H)$. This topology is denoted by T^* [12].*

Let (H, T) be a topological space, where $(H, \circ)$ be a hypergroup. Then the triple $(H, \circ, T)$ is called a topological hypergroup if the following functions are continuous:

(1) The mapping $\varphi : (x, y) \longmapsto x \circ y$, from $H \times H$ onto $P^*(H)$;
(2) The mapping $\psi : (x, y) \longmapsto x/y$, from $H \times H$ onto $P^*(H)$, where $x/y = \{z \in H | x \in z \circ y\}$.

If the condition (2) of previous definition is not met, then $(H, \circ, T)$ is called a semi-topological hypergroup.

Let (P, T) be a topological space, where $(P, \circ, e, ^{-1})$ be a polygroup. Then the (P, T) is called a topological polygroup (in short TP) if the following axioms hold:

(1) The mapping $\circ : P \times P \longmapsto P^*(P)$ be continuous, where $\circ(x, y) = x \circ y$;
(2) The mapping $^{-1} : P \longmapsto P$ be continuous, where $^{-1}(x) = -x$.

We can combine items (1),(2) and present the following case:

The mapping $\varphi : P \times P \longmapsto P^*(P)$ be continuous, where $\varphi(x, y) = x \circ y^{-1}$.

The following theorem help us to determine the continuity of hyperoperation. We us to use the following theorem for the continuity test.

Theorem 3. *The hyperoperation $\circ : P \times P \longmapsto P^*(P)$ is continuous, where P is a polygroup $\Longleftrightarrow \forall a, b \in P$ and $C \in T$ with the property that $a \circ b \subseteq C$ then there exist $A, B \in T$ with the property that $a \in A$ and $b \in B$ and $A \circ B \subseteq C$ [9].*

Example 3. *[18] Let P be $\{e, a, b, c\}$ and multiplication table be:*

$\circ$	e	a	b	c
e	e	a	b	c
a	a	$\{e, a\}$	c	$\{b, c\}$
b	b	c	e	a
c	c	$\{b, c\}$	a	$\{e, a\}$

Hyperoperation $\circ : P \times P \longmapsto P^*(P)$ is continuous with topologies:

T_{dis},
T_{ndis},
$T_1 = \{\varnothing, P, \{e, b\}\}$,
$T_2 = \{\varnothing, P, \{e\}, \{b\}\}$,
since $x^{-1} = x$ for all $x \in P$, inverse operation is identity and identity function is continuous with every topology, it follows that P with topologies T_1, T_2 is topological polygroup.

Hyperoperation $\circ : P \times P \longmapsto P^*(P)$ with below topologies is not continuous.

$T_3 = \{\varnothing, P, \{e\}\}$,
$T_4 = \{\varnothing, P, \{a\}\}$,
$T_5 = \{\varnothing, P, \{b\}\}$,
$T_6 = \{\varnothing, P, \{c\}\}$,
$T_7 = \{\varnothing, P, \{e, a\}\}$,
$T_8 = \{\varnothing, P, \{e, c\}\}$,
$T_9 = \{\varnothing, P, \{a, b\}\}$,
$T_{10} = \{\varnothing, P, \{a, c\}\}$,
$T_{11} = \{\varnothing, P, \{b, c\}\}$,
$T_{12} = \{\varnothing, P, \{e, a, b\}\}$,
$T_{13} = \{\varnothing, P, \{e, a, c\}\}$,
$T_{14} = \{\varnothing, P, \{e, b, c\}\}$,
$T_{15} = \{\varnothing, P, \{a, b, c\}\}$,
$T_{16} = \{\varnothing, P, \{e\}, \{a\}\}$.

If the condition (2) of previous defintion is not met, then $(P, \circ, e,^{-1}, T)$ is called a semi-topological polygroup.

3. Soft Semi-Topological Polygroups

The first definition we provide for soft semi-topological polygroups is as follows, and the examples and results that follow from this definition will be given below.

Definition 2. *Let T be a topology on a polygroup P. Let $(\mathbb{F}, A)$ be a soft set over P. Then the system $(\mathbb{F}, A, T)$ said to be soft semi-topological polygroup over P if the following axioms hold:*

(a) *$\mathbb{F}(a)$ is a subpolygroup of P for all $a \in A$.*
(b) *The mapping $(x, y) \longmapsto x \circ y$ of the topological space $\mathbb{F}(a) \times \mathbb{F}(a)$ onto $P^*(\mathbb{F}(a))$ is continuous for all $a \in A$.*

Topology T on P induces topologies on $\mathbb{F}(a)$, $\mathbb{F}(a) \times \mathbb{F}(a)$ and by Theorem 2 on $P^*(\mathbb{F}(a))$.

If A be $\{e, a_1, a_2, ...\}$, B be $\{e, b_1, b_2, ...\}$, and the table for $*$ in $A[B]$ be the following form:

	e	a_1	a_2	...	b_1	b_2	...
e	e	a_1	a_2	...	b_1	b_2	...
a_1	a_1	a_1a_1	a_1a_2	...	b_1	b_2	...
a_2	a_2	a_2a_1	a_2a_2	...	b_1	b_2	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
b_1	b_1	b_1	b_1	...	$b_1 * b_1$	$b_1 * b_2$	...
b_2	b_2	b_2	b_2	...	$b_2 * b_1$	$b_2 * b_2$	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then several special cases of the algebra $A[B]$ are useful [6, 7]. Before describing them we need to assign names to the 2-elements polygroups. Let 2 denotes the group $\mathbb{Z}_2$ and let 3 denotes the polygroup $\mathbb{S}_3//\langle(12)\rangle \cong \mathbb{Z}_3/T$, where T is the special conjugation with blocks $\{0\}, \{1, 2\}$. The multiplication table for 3 is

	0	1
0	0	2
1	1	$\{0, 1\}$

The system $3[M]$ is the result of adding a new identity to the polygroup $[M]$. The system $2[M]$ is almost as good. For example, suppose that R is the system with table

	0	1	2
0	0	1	2
1	1	$\{0, 2\}$	$\{1, 2\}$
2	2	$\{1, 2\}$	$\{0, 1\}$

Example 4. *With the above description, polygroup* $2[R]$ *will be as follows:*

$\circ$	0	a	1	2
0	0	a	1	2
a	a	0	1	2
1	1	1	$\{0, a, 2\}$	$\{1, 2\}$
2	2	2	$\{1, 2\}$	$\{0, a, 1\}$

Hyperoperation $\circ : 2[R] \times 2[R] \longmapsto P^*(2[R])$ *is not continuous with the following topologies:*

$T_1 = \{\varnothing, 2[R], \{0\}\}$,
$T_2 = \{\varnothing, 2[R], \{a\}\}$,
$T_3 = \{\varnothing, 2[R], \{1\}\}$,
$T_4 = \{\varnothing, 2[R], \{2\}\}$,
$T_5 = \{\varnothing, 2[R], \{0, 1\}\}$,
$T_6 = \{\varnothing, 2[R], \{0, 2\}\}$,

$T_7 = \{\varnothing, 2[R], \{a, 1\}\}$,
$T_8 = \{\varnothing, 2[R], \{a, 2\}\}$,
$T_9 = \{\varnothing, 2[R], \{1, 2\}\}$,
$T_{10} = \{\varnothing, 2[R], \{0, a, 1\}\}$,
$T_{11} = \{\varnothing, 2[R], \{0, a, 2\}\}$,
$T_{12} = \{\varnothing, 2[R], \{a, 1, 2\}\}$,
$T_{13} = \{\varnothing, 2[R], \{0, 1, 2\}\}$.
But $\circ : 2[R] \times 2[R] \longmapsto P^*(2[R])$ *is continuous with*

$$T_{14} = \{\varnothing, 2[R], \{0, a\}\}, T_{15} = \{\varnothing, 2[R], \{0\}, \{a\}\}$$

This means that $(2[R], T_{dis})$, $(2[R], T_{ndis})$, $(2[R], T_{14})$ *and* $(2[R], T_{15})$ *are semi-topological polygroups. Subpolygroups of* $2[R]$ *are* $\varnothing, 2[R], \{0\}, \{0, a\}$. *Let* A *be a arbitrary set and* $a_1, a_2, a_3 \in A$ *and define a soft set* $\mathbb{F}$ *by*

$$\mathbb{F}(x) = \begin{cases} \{0\} & \textit{if } x = a_1 \\ \{0, a\} & \textit{if } x = a_2 \\ 2[R] & \textit{if } x = a_3 \\ \varnothing & \textit{otherwise.} \end{cases}$$

In conclusion $(\mathbb{F}, A, T_{14})$ *and* $(\mathbb{F}, A, T_{15})$ *are soft semi-topological polygroups [18].*

EXAMPLE 5. *Polygroup* $3[R]$ *will be as follows:*

$\circ$	0	a	1	2
0	0	a	1	2
a	a	$\{0, a\}$	1	2
1	1	1	$\{0, a, 2\}$	$\{1, 2\}$
2	2	2	$\{1, 2\}$	$\{0, a, 1\}$

Hyperoperation $\circ : 3[R] \times 3[R] \longmapsto P^*(3[R])$ *is not continuous with the following topologies:*
$T_1 = \{\varnothing, 3[R], \{a\}\}$,
$T_2 = \{\varnothing, 3[R], \{1\}\}$,
$T_3 = \{\varnothing, 3[R], \{2\}\}$,
$T_4 = \{\varnothing, 3[R], \{0, 1\}\}$,
$T_5 = \{\varnothing, 3[R], \{0, 2\}\}$,
$T_6 = \{\varnothing, 3[R], \{a, 1\}\}$,
$T_7 = \{\varnothing, 3[R], \{a, 2\}\}$,
$T_8 = \{\varnothing, 3[R], \{1, 2\}\}$,
$T_9 = \{\varnothing, 3[R], \{0, a, 1\}\}$,
$T_{10} = \{\varnothing, 3[R], \{0, a, 2\}\}$,
$T_{11} = \{\varnothing, 3[R], \{a, 1, 2\}\}$.
Nevertheless hyperoperation $\circ : 3[R] \times 3[R] \longmapsto P^*(3[R])$ *is continuous with*
$T_{12} = \{\varnothing, 3[R], \{0\}\}$,
$T_{13} = \{\varnothing, 3[R], \{0, a\}\}$,
$T_{14} = \{\varnothing, 3[R], \{0\}, \{a\}\}$.

Therefore, $(3[R], (T_i)_{i=12,13,14})$ *are semi-topological polygroups. Subpolygroups of* $3[R]$ *are* $\varnothing, 3[R], \{0\}, \{0, a\}$*. Let* A *be* $3[R]$ *and define a soft set* $\mathbb{F}$ *by*

$$\mathbb{F}(x) = \begin{cases} \{0\} & \text{if } x = 0 \\ \{0, a\} & \text{if } x = a \\ 3[R] & \text{if } x = 1 \\ \varnothing & \text{if } x = 2. \end{cases}$$

Then, $(\mathbb{F}, A, (T_i)_{i=12,13,14}))$ *is a soft semi-topological polygroup. Now, let* A *be arbitrary set and* $a_1, a_2 \in A$ *and define a soft set* $\mathbb{F}$ *by*

$$\mathbb{F}(x) = \begin{cases} \varnothing & \text{if } x = a_1 \\ \{0, a\} & \text{if } x = a_2 \\ \{0\} & \text{otherwise.} \end{cases}$$

In this case $(\mathbb{F}, A, (T_i)_{i=3,4,5,8,9,10})$ *are soft semi-topological polygroups.*

Theorem 4. *[22] Let* $(\mathbb{F}, A)$ *be a soft polygroup over* P *and* (P, T) *be a semi-topological polygroup. then* $(\mathbb{F}, A, T)$ *is a soft semi-topological polygroup over* P.

Theorem 5. *[22] Let* $(\mathbb{F}, A, T)$ *and* $(\mathbb{G}, B, T)$ *be soft semi-topological polygroups over* P*. Then* $(\mathbb{F}, A, T)\widehat{\cap}(\mathbb{G}, B, T)$ *and* $(\mathbb{F}, A, T)\cap_E(\mathbb{G}, B, T)$ *are soft semi-topological polygroup over* P.

Theorem 6. *[22] If* $(\mathbb{F}_i, A_i, T)$ *be a nonempty family of soft semi-topological polygroups, then* $\widehat{\cap}_{i\in I}(\mathbb{F}_i, A_i, T)$ *is a soft semi-topological polygroup over* P.

Theorem 7. *[22] Let* $(\mathbb{F}, A, T)$ *and* $(\mathbb{G}, B, T)$ *be soft semi-topological polygroups over* P*. Then* $(\mathbb{F}, A, T)\widehat{\wedge}(\mathbb{G}, B, T)$ *and* $(\mathbb{F}, A, T)\widehat{\cup}(\mathbb{G}, B, T)$ *are soft semi-topological polygroup.*

Theorem 8. *[22] Let* $(\mathbb{F}_i, A_i, T)$ *be a nonempty family of soft semi-topological polygroups over* P*. Then* $\widehat{\wedge}_{i\in I}(\mathbb{F}_i, A_i, T)$ *and* $\widehat{\cup}_{i\in I}(\mathbb{F}_i, A_i, T)$ *are soft semi-topological polygroup.*

Definition 3. *Let* $(\mathbb{F}, A, T)$ *be a soft semi-topological polygroup over* P *. Then* $(\mathbb{G}, B, T)$ *is called a soft semi-topological subpolygroup (resp. normal subpolygroup) of* $(\mathbb{F}, A, T)$ *if the following items hold:*

(a) B *subset of* A *and* $\mathbb{G}(b)$ *is a subpolygroup (resp. normal subpolygroup) of* $\mathbb{F}(b)$ *for every* $b \in supp(\mathbb{G}, B)$.

(b) *the mapping* $(x, y) \longmapsto x \circ y$ *of the topological space* $\mathbb{G}(b) \times \mathbb{G}(b)$ *onto* $P^*(\mathbb{G}(b))$ *is continuous for every* $b \in supp(\mathbb{G}, B)$.

Theorem 9. *Let* $(\mathbb{F}, A, T)$ *be a soft semi-topological polygroup over P, and* $(\mathbb{G}_i, B_i, T)_{i\in I}$ *be a non-empty family of (normal) soft semi-topological subpolygroups of* $(\mathbb{F}, A, T)$*. Then*

(1) *If* $\cap_{i\in I}B_i \neq \varnothing$*, then* $\widehat{\cap}_{i\in I}(\mathbb{G}_i, B_i, T)$ *is a (normal) soft subpolygroup of* $(\mathbb{F}, A, T)$.

(2) *If* $B_i \cap B_j = \varnothing$ *for all* $i, j \in I$ *and* $i \neq j$, *then* $(\cap_E)_{i\in I}(\mathbb{G}_i, B_i, T)$ *is a (normal) soft subpolygroup of* $(\mathbb{F}, A, T)$.

(3) *If* $B_i \cap B_j = \varnothing$ *for all* $i, j \in I$ *and* $i \neq j$, *then* $\widehat{\cup}_{i\in I}(\mathbb{G}_i, B_i, T)$ *is a (normal) soft subpolygroup of* $(\mathbb{F}, A, T)$.

(4) *The* $\widehat{\wedge}_{i\in I}(\mathbb{G}_i, B_i, T)$ *is a (normal) soft subpolygroup of the soft polygroup* $\widehat{\wedge}_{i\in I}(\mathbb{F}, A, T)$.

Proof.
(1) Suppose that $C = \cap_{i\in I}(B_i)$ and $\mathbb{H}(c) = \cap_{i\in I}(\mathbb{G}_i(c))$ Furthermore $C \subseteq A$ and $\mathbb{H}(c)$ is a (normal) soft subpolygroup of A and the mapping in Definition 3 (b) is continuous on $\mathbb{H}(c)$.
(2) Give $C = \cup_{i\in I}(B_i), \mathbb{H}(c) = \mathbb{G}_i(c)$ where $c \in B_i$ and $\mathbb{H}(c)$ is a (normal) soft subpolygroup of $F(c)$ and the mapping in Definition 3 (b) is continuous on $\mathbb{H}(c)$.
(3) Take $C = \cup_{i\in I} B_i, \mathbb{H}(c) = \mathbb{G}_i(c)$, where $c \in B_i$ thus $B_i \subseteq A$ notably $\cup_{i\in I}(B_i) \subseteq A$ in conclusion $\mathbb{H}(c) = \mathbb{G}_i(c)$ is a (normal) soft subpolygroup of $\mathbb{F}(c)$ and the mapping in Definition 3 (b) is continuous on $\mathbb{H}(c)$.
(4) Select $C = \times_{i\in I}(B_i), \mathbb{H}((c_i)_{i\in I}) = \cap_{i\in I}\mathbb{G}_i((c_i)_{i\in I})$ and $\mathbb{G}_i(c_i)$ is a (normal) soft subpolygroup of $\times_{i\in I}\mathbb{F}(c_i)$ in conclusion the mapping in Definition 3 (b) is continuous on $\mathbb{H}((c_i)_{i\in I})$. □

Definition 4. *Let* $(\mathbb{F}, A, T)$ *and* $(\mathbb{G}, B, \xi)$ *be the soft semi-topological polygroups over* P_1 *and* P_2, *where* T *and* ξ *are topologies are defined over* P_1 *and* P_2 *respectively. Let* $f : P_1 \longmapsto P_2$ *and* $g : A \longmapsto B$ *be two mappings. Then the pair* (f, g) *is called a soft semi-topological polygroup homomorphism if the following condition true:*

(a) f *be strong epimorphism and* g *be surjection.*
(b) $f(\mathbb{F}(a)) = \mathbb{G}(g(a))$.
(c) $f_a : (\mathbb{F}(a), T_{\mathbb{F}(a)}) \longmapsto (\mathbb{G}(g(a)), \xi_{\mathbb{G}(g(a))})$ *is continuous.*

Then $(\mathbb{F}, A, T)$ is said to be soft semi-topologically homomorphic to $(\mathbb{G}, B, \xi)$ and denoted by$(\mathbb{F}, A, T) \sim (\mathbb{G}, B, \xi)$. If f is a polygroup isomorphism, g is bijective and f_a is continuous as well as open, then the pair (f, g) is called a soft semi-topological polygroup isomorphism. In this case $(\mathbb{F}, A, T)$ is soft topologically isomorphic to $(\mathbb{G}, B, \xi)$, which is denoted by $(\mathbb{F}, A, T) \simeq (\mathbb{G}, B, \xi)$.

Theorem 10. *If* $(\mathbb{F}, A, T) \sim (\mathbb{G}, B, \xi)$ *and* $(\mathbb{F}, A, T)$ *is a normal soft polygroup over* P, *then* $(\mathbb{G}, B, \xi)$ *is a normal soft polygroup over* Q, *where* $(\mathbb{F}, A, T)$ *and* $(\mathbb{G}, B, \xi)$ *be soft semi-topological polygroups over* P *and* Q.

Proof. Let (f, g) be a soft semi-topological homomorphism from $(\mathbb{F}, A)$ to $(\mathbb{G}, B)$. For all $x \in supp(\mathbb{F}, A)$, $\mathbb{F}(x)$ is a normal subpolygroup of P; then $f(\mathbb{F}(x))$ is a normal subpolygroup of Q. For all $y \in supp(\mathbb{G}, B)$, there exists $x \in supp(\mathbb{F}, A)$ with the property that $g(x) = y$. In conclusion $\mathbb{G}(y) = \mathbb{G}(g(x)) = f(\mathbb{F}(x))$ is a normal subpolygroup of Q. Thus $(\mathbb{G}, B)$ is a normal soft polygroup on Q.

□

Theorem 11. *Let N be a normal subpolygroup of P, and $(\mathbb{F}, A, T)$ be a soft semi-topological polygroup over P. Then $(\mathbb{F}, A, T) \sim (\mathbb{G}, A, T)$, where $\mathbb{G}(x) = \mathbb{F}(x)/N$ for all $x \in A$, and $N \subseteq \mathbb{F}(x)$ for all $x \in supp(\mathbb{F}, A)$.*

Proof. Firstly $supp(\mathbb{G}, A) = supp(\mathbb{F}, A)$ and we know that P/N is a factor polygroup. Since for every $x \in supp(\mathbb{F}, A), \mathbb{F}(x)$ is a subpolygroup of P and $N \subseteq \mathbb{F}(x)$, it follows that $\mathbb{F}(x)/N$ is also a factor polygroup, which is a subpolygroup of P/N. Thus $(\mathbb{G}, A)$ is a soft polygroup over P/N. Therefore $f : P \longmapsto P/N, f(a) = aN$. Clearly, f is a strong epimorphism. In other words $g : A \longmapsto A, g(x) = x$. Then g is a surjective mapping. For all $x \in supp(\mathbb{F}, A), f(\mathbb{F}(x)) = \mathbb{F}(x)/N = \mathbb{G}(x) = \mathbb{G}(g(x))$. For all $x \in A - supp(\mathbb{F}, A)$, notably $f(\mathbb{F}(x)) = \varnothing = \mathbb{G}(g(x))$. Therefore, (f, g) is a soft semi-topological homomorphism, and $(\mathbb{F}, A, T) \sim (\mathbb{G}, B, \xi)$. □

Definition 5. *Closure of $(\mathbb{F}, A, T)$ denoted by $(\overline{\mathbb{F}}, A, T)$ and is defined by $\overline{\mathbb{F}}(a) = \overline{\mathbb{F}(a)}$ where $\overline{\mathbb{F}(a)}$ is the closure of $\mathbb{F}(a)$ in topology on P.*

Theorem 12. *[9] Let P be a semi-topological polygroup with the property that every open subset of P is a complete part. Then:*

(1) *If K is a subhypergroup of P, then as well as $\overline{K}$.*
(2) *If K is a subpolygroup of P, then as well as $\overline{K}$.*

Theorem 13. *Let $(\mathbb{F}, A, T)$ be a soft semi-topological polygroup over a semi-topological polygroup (P, T) and every open subset of P is a complete part Then:*

(1) *$(\overline{\mathbb{F}}, A, T)$ is also a soft semi-topological polygroup over (P, T).*
(2) *$(\mathbb{F}, A, T)\widehat{\subset}(\overline{\mathbb{F}}, A, T)$.*

Proof. (1) By Theorem 12 $\overline{\mathbb{F}(a)}$ is subpolygroup P and since (P, T) is a semi-topological polygroup, it follows that condition (b) of Definition 2 holds on $\overline{\mathbb{F}(a)}$.
(2) It is clear. □

Definition 6. *Let $(\mathbb{F}, A), (\mathbb{G}, B)$ be soft sets over polygroup $< P, e, \circ, -1 >$ define $(\mathbb{F}, A)\widehat{\circ}(\mathbb{G}, B) = (H, C)$ where $C = A \cup B$ for all $a \in C$, and*

$$H(a) = \begin{cases} \mathbb{F}(a) & \text{if } a \in A - B \\ \mathbb{G}(a) & \text{if } a \in B - A \\ \mathbb{F}(a) \circ \mathbb{G}(a) & \text{if } a \in A \cap B \end{cases}$$

Theorem 14. *[9] Let A and B be subsets of polygroup P with the property that every open subset of P is a complete part. Then:*

(1) $\overline{A} \circ \overline{B} \subseteq \overline{A \circ B}$.
(2) $(\overline{A})^{-1} = \overline{(A^{-1})}$.

Theorem 15. *[9] In every topological space (X, T) if $A, B \subseteq X$ we have:*

(1) $\overline{A} \cup \overline{B} = \overline{A \cup B}$.
(2) $\overline{A} \cap \overline{B} = \overline{A \cap B}$.

Theorem 16. *Let* $(\mathbb{F}, A, T)$, $(\mathbb{F}, B, T)$ *be soft semi-topological polygroups over a semi-topological polygroup* (P, T) *and every open subset of* P *is a complete part Then:*

(1) $(\overline{\mathbb{F}}, A, T)\widehat{\cup}(\overline{\mathbb{G}}, B, T) = \overline{(\mathbb{F}, A, T)\widehat{\cup}(\mathbb{G}, B, T)}$.
(2) $(\overline{\mathbb{F}}, A, T)\widehat{\cap}(\overline{\mathbb{G}}, B, T) = \overline{(\mathbb{F}, A, T)\widehat{\cap}(\mathbb{G}, B, T)}$.
(3) $(\overline{\mathbb{F}}, A, T)\widehat{\wedge}(\overline{\mathbb{G}}, B, T) = \overline{(\mathbb{F}, A, T)\widehat{\wedge}(\mathbb{G}, B, T)}$.
(4) $(\overline{\mathbb{F}}, A, T)\widehat{\circ}(\overline{\mathbb{G}}, B, T)\widehat{\subseteq}\overline{(\mathbb{F}, A, T)\widehat{\circ}(\mathbb{G}, B, T)}$.
(5) $(\overline{F}, A, T) \cap_E (\overline{\mathbb{G}}, B, T) = \overline{(\mathbb{F}, A, T) \cap_E (\mathbb{G}, B, T)}$.

Proof. (1) Let a be element of $A-B$. then $(\overline{\mathbb{F}}, A, T)\widehat{\cup}(\overline{\mathbb{G}}, B, T)(a) = (\overline{\mathbb{F}}, A, T)(a) = \overline{\mathbb{F}(a)}$ In conclusion, $\overline{(\mathbb{F}, A, T)\widehat{\cup}(\mathbb{G}, B, T)}(a) = \overline{\mathbb{F}}(a) = \overline{\mathbb{F}(a)}$.

Let a be element of $B-A$. Then $(\overline{\mathbb{F}}, A, T)\widehat{\cup}(\overline{\mathbb{G}}, B, T)(a) = (\overline{\mathbb{G}}, B, T)(a) = \overline{\mathbb{G}(a)}$ In conclusion, $\overline{(\mathbb{F}, A, T)\widehat{\cup}(\mathbb{G}, B, T)}(a) = \overline{G}(a) = \overline{\mathbb{G}(a)}$.

Let a be element of $A \cap B$. Then $(\overline{\mathbb{F}}, A, T)\widehat{\cup}(\overline{\mathbb{G}}, B, T)(a) = \overline{\mathbb{F}(a)} \cup \overline{\mathbb{G}(a)}$ In conclusion, $\overline{(\mathbb{F}, A, T)\widehat{\cup}(\mathbb{G}, B, T)}(a) = \overline{\mathbb{F}(a) \cup \mathbb{G}(a)}$. By Theorem 15 proof is complete.

(4) Let a be element of $A - B$. Then $(\overline{\mathbb{F}}, A, T)\widehat{\circ}(\overline{\mathbb{G}}, B, T)(a) = (\overline{\mathbb{F}}, A, T)(a) = \overline{\mathbb{F}(a)}$ In conclusion, $\overline{(\mathbb{F}, A, T)\widehat{\circ}(\mathbb{G}, B, T)}(a) = \overline{\mathbb{F}}(a) = \overline{\mathbb{F}(a)}$.

Let a be element of $B-A$. Then $(\overline{\mathbb{F}}, A, T)\widehat{\circ}(\overline{\mathbb{G}}, B, T)(a) = (\overline{\mathbb{G}}, B, T)(a) = \overline{\mathbb{G}(a)}$ In conclusion, $\overline{(\mathbb{F}, A, T)\widehat{\circ}(\mathbb{G}, B, T)}(a) = \overline{\mathbb{G}}(a) = \overline{\mathbb{G}(a)}$.

Let a be element of $A \cap B$. Then $(\overline{\mathbb{F}}, A, T)\widehat{\circ}(\overline{\mathbb{G}}, B, T)(a) = \overline{\mathbb{F}(a)} \circ \overline{\mathbb{G}(a)}$ In conclusion, $\overline{(\mathbb{F}, A, T)\widehat{\circ}(\mathbb{G}, B, T)}(a) = \overline{\mathbb{F}(a) \circ \mathbb{G}(a)}$. By Theorem 14 proof is complete.

Other items are similar (1) or (4). □

The second definition of soft semi-topological polygroups is as follows, and this definition is based on soft topologies and soft continuity. The results of this definition follow. To distinguish the latter Definition from the previous one, we use distinct symbols.

A family θ of soft sets over U is called a soft topology on U if the following axioms hold:

(1) $\widehat{\varnothing}$ and $\widehat{U}$ are in θ,
(2) θ is closed under finite soft intersection,
(3) θ is closed under (arbitrary) soft union.

We will use the symbol (U, θ, E) to denote a soft topological space and soft set $\mathbb{F}$ is called a soft close set if $\mathbb{F}^{\widehat{c}}$ is soft open set, where each member of θ said to be a soft open set [4, 26].

EXAMPLE 6. *Let* U *be* $\mathbb{Z}_2$ *and* θ *be* $\{\widehat{\varnothing}, \{e_2\} \times \mathbb{Z}_2, \widehat{\mathbb{Z}_2}\}$, *where* $E = \{e_1, e_2\}$ *and* $\{e_2\} \times \mathbb{Z}_2$ *be soft set* $\mathbb{F} : E \longmapsto P(\mathbb{Z}_2)$ *with the property that* $\mathbb{F}(e_1) = \varnothing; \mathbb{F}(e_2) = \mathbb{Z}_2$. *Then* $(\mathbb{Z}_2, \theta, E)$ *is soft topological space.*

EXAMPLE 7. *Let* P *be* $\{e, a, b, c\}$ *and hyperoparation* $\circ$ *be as follow:*

$\circ$	e	a	b	c
e	e	a	b	c
a	a	$\{e, a\}$	c	$\{b, c\}$
b	b	c	e	a
c	c	$\{b, c\}$	a	$\{e, a\}$

polygroup P *with topologies* $\theta_1 = \{\widehat{\varnothing}, \{e_1\} \times P, \widehat{P}\}$, $\theta_2 = \{\widehat{\varnothing}, \{e_2\} \times P, \widehat{P}\}$ *are soft topological spaces.*

Closure of $\mathbb{F}$ denoted by $\widehat{Cl}(\mathbb{F})$ and define soft intersection of all soft closed supersets of $\mathbb{F}$, where $\mathbb{F}$ be soft set over U.

A soft set $\mathbb{F}$ said to be a soft neighborhood of x if there exists a soft open set $\mathbb{G}$ with the property that $x \widehat{\in} \mathbb{G} \widehat{\subseteq} \mathbb{F}$, where x be an element of the universe U. The soft neighborhood system of x we will consider the collection of all soft neighborhoods of x.

Let V be a subset of the universe U. A soft set $\mathbb{F}$ said to be a soft neighborhood of V if there exists a soft open set $\mathbb{G}$ with the property that $V \widehat{\subseteq} \mathbb{G} \widehat{\subseteq} \mathbb{F}$. (i.e $\forall e \in E$: $V \subseteq \mathbb{G}(e) \subseteq \mathbb{F}(e)$).

The collection of all soft neighborhoods of V said to be the soft neighborhood system of V.

Definition 7. *Let* P_1, P_2 *be polygroups and* $(P_1, \theta_1, E), (P_2, \theta_2, E)$ *be soft topological spaces. The function* $\varphi : (P_1, \theta_1, E) \longmapsto (P_2, \theta_2, E)$ *said to be a soft continuous function if for all* $x \in P_1$ *and for all soft neighborhood* $\mathbb{F}_{\varphi(x)}$ *of* $\varphi(x)$, *there exists a soft neighborhood* $\mathbb{F}_x$ *of* x *with the property that* $\varphi(\mathbb{F}_x) \widehat{\subseteq} \mathbb{F}_{\varphi(x)}$.

Theorem 17. *The function* $\varphi : (P_1, \theta_1, E) \longmapsto (P_2, \theta_2, E)$ *is soft continuous function if and only if for every soft closed set* $\mathbb{F}'$, *the inverse image* $\varphi^{-1}(\mathbb{F}')$ *is also soft closed.*

Proof. This is easily seen to be an equivalence relation.

□

Theorem 18. *Let* $\varphi : (P_1, \theta_1, E) \longmapsto (P_2, \theta_2, E)$ *be function in this case, for every soft closed set* $\mathbb{F}'$, *the inverse image* $\varphi^{-1}(\mathbb{F}')$ *is also soft closed if and only if for all soft set* $\mathbb{F}$, *we have* $\varphi(\widehat{Cl}(\mathbb{F})) \widehat{\subseteq} \widehat{Cl}(\varphi(\mathbb{F}))$.

Proof. (i) $\Longleftarrow$ Let $\mathbb{F}'$ be soft closed set . Then we have $\varphi(\varphi^{-1}(\mathbb{F}')) \widehat{\subseteq} \mathbb{F}'$. The soft closeness of $\mathbb{F}'$, together with the assumption (for all soft set $\mathbb{F}$, we have $\varphi(\widehat{Cl}(\mathbb{F})) \widehat{\subseteq} \widehat{Cl}(\varphi(\mathbb{F}))$), proves that

$$\varphi(\widehat{Cl}(\varphi^{-1}(\mathbb{F}')))\widehat{\subseteq}\widehat{Cl}(\varphi(\varphi^{-1}(\mathbb{F}')))\widehat{\subseteq}\mathbb{F}'$$

Therefore, it holds that $\widehat{Cl}(\varphi^{-1}(\mathbb{F}'))\widehat{\subseteq}\varphi^{-1}(\mathbb{F}')\widehat{\subseteq}\widehat{Cl}(\varphi^{-1}(\mathbb{F}'))$, which shows that $\varphi^{-1}(\mathbb{F}')$ is soft closed.

(ii) $\Longrightarrow$ We have $\mathbb{F}\widehat{\subseteq}\varphi^{-1}(\widehat{Cl}(\varphi(\mathbb{F})))$ for any soft set $\mathbb{F}$. Since (for every soft closed set $\mathbb{F}'$, the inverse image $\varphi^{-1}(\mathbb{F}')$ is also soft closed), we have $\widehat{Cl}(\mathbb{F})\widehat{\subseteq}\varphi^{-1}(\widehat{Cl}(\varphi(\mathbb{F})))$. Thus, we have

$$\varphi(\widehat{Cl}(\mathbb{F}))\widehat{\subseteq}\varphi(\varphi^{-1}(\widehat{Cl}(\varphi(\mathbb{F}))))\widehat{=}\widehat{Cl}(\varphi(\mathbb{F}))$$

□

Theorem 19. *Let $\varphi : (P_1, \theta_1, E) \longmapsto (P_2, \theta_2, E)$ be a function. If for all soft open set $\mathbb{F}' \in \theta_2$, the inverse image $\varphi^{-1}(\mathbb{F}')$ is also soft open set then φ is a soft continuous function.*

Proof. For all $x \in P_1$ and a soft open neighborhood $\mathbb{F}'$ of $\varphi(x), \varphi^{-1}(\mathbb{F}')$ is a soft open set having x as a soft element. Since $\varphi(\varphi^{-1}(\mathbb{F}'))\widehat{\subseteq}\mathbb{F}'$, give $F = \varphi^{-1}(\mathbb{F}')$ in this case $\varphi(\mathbb{F})\widehat{\subseteq}\mathbb{F}'$. □

Example 8. *We prove that the opposite Theorem 19 is not true.*

Let P_1 be $< \{u\}, \theta_1, \{e_1, e_2\} >$ and P_2 be $< \{u\}, \theta_2, \{e_1, e_2\} >$, where

$$\theta_1 = \{\widehat{\varnothing}, \{(e_1, u), (e_2, u)\}\}$$
$$\theta_2 = \{\widehat{\varnothing}, \{(e_2, u)\}, \{(e_1, u), (e_2, u)\}\}$$

In soft topologies, $\{e_1, e_2\} \times \{u\}$ is the soft neighborhood of the point u. Thus $id : P_1 \longmapsto P_2$ satisfies in second part Theorem 19. However, $id^{-1}(\{(e_2, u)\})$ is not soft open in P_1, showing that the inverse images of soft open sets are, in general, not soft open. Show that, not only $id : P_1 \longmapsto P_2$ but also $id^{-1} : P_2 \longmapsto P_1$ satisfy in second part Theorem 19.

Definition 8. *A bijection $\varphi : P_1 \longmapsto P_2$ said to be a soft homeomorphism between (P_1, θ_1, E) and (P_2, θ_2, E) if φ and φ^{-1} are soft continuous.*

Theorem 20. *Let $\varphi : (P_1, \theta_1, E) \longmapsto (P_2, \theta_2, E)$ be a soft continuous function and for all soft open set $\mathbb{F}_2 \in \theta_2$, there exists a soft open set $\mathbb{F}_1 \in \theta_1$ with the property that for all $x \in P_1$; $x\widehat{\in}\mathbb{F}_1$ if and only if $x\widehat{\in}\varphi^{-1}(\mathbb{F}_2)$.*

Proof. For every $x \in P_1$ with $\varphi(x)\widehat{\in}\mathbb{F}_2$, choose a soft open $\mathbb{F}_x \in \theta_1$ with the property that $x\widehat{\in}\mathbb{F}_x$ and $\varphi(\mathbb{F}_x)\widehat{\subseteq}\mathbb{F}_2$. Then define $\mathbb{F}_1 = \widehat{\bigcup}\{\mathbb{F}_x | x \in P_1, \varphi(x)\widehat{\in}\mathbb{F}_2\}$ is the desired soft open set. □

Definition 9. *Let $(P, \circ, e, ^{-1})$ be a polygroup and θ be a soft topology on P with a parameter set E. then (P, θ, E) is a soft semi-Topological polygroup if the following item true:*

For each soft neighborhood $\mathbb{F}$ *of* $p \circ q$*, where* $(p, q) \in P \times P$ *there exist soft neighborhoods* $\mathbb{F}_p$ *and* $\mathbb{F}_q$ *of* p *and* q *with the property that* $\mathbb{F}_p \circ \mathbb{F}_q \widehat{\subseteq} \mathbb{F}$.

Every soft semi-topological group is soft semi-Topological polygroup.

Example 9. *Let* E *be* $\{e_1, e_2\}$ *and* θ *be* $\{\widehat{\varnothing}, \{(e_1, \overline{1})\}, \widehat{\mathbb{Z}_2}\}$*. Conclusion* $(\mathbb{Z}_2, \theta, E)$ *is a soft semi-*T*opological polygroup.*

Example 10. *Let* P *be* $\{e, a, b, c\}$ *and hyperoparation* $\circ$ *be as follow:*

$\circ$	e	a	b	c
e	e	a	b	c
a	a	$\{e, a\}$	c	$\{b, c\}$
b	b	c	e	a
c	c	$\{b, c\}$	a	$\{e, a\}$

And E *be* $\{e_1, e_2, e_3\}$*. Then the polygroup* P *with each of the following topologies*

$$\theta_1 = \{\widehat{\varnothing}, \{e_1\} \times P, \widehat{P}\}$$
$$\theta_2 = \{\widehat{\varnothing}, \{e_2\} \times P, \widehat{P}\}$$
$$\theta_3 = \{\widehat{\varnothing}, \{e_3\} \times \{a, b\}, \widehat{P}\}$$
$$\theta_3 = \{\widehat{\varnothing}, \{e_3\} \times \{a, b\}, \{e_1\} \times \{e, b\}, \widehat{P}\}$$
$$\theta_4 = \{\widehat{\varnothing}, \{e_3\} \times \{a, b\}, \{e_1\} \times \{e, b\}, \{e_2\} \times \{e, b, c\}, \widehat{P}\}$$

is a soft semi-T*opological polygroup.*

The family of soft sets Θ, is said to be a soft indiscrete (soft discrete) topology on P if $\Theta = \{\widehat{\varnothing}, \widehat{P}\}(\Theta = SS(P))$, in this case (P, Θ) is called a soft indiscrete space (soft discrete space) over P, where $SS(P)$ is the set of all soft sets over P [26].

Example 11. *Every polygroup with soft discrete or indiscrete topology is a soft semi-*T*opological polygroup.*

If we want to merge the previous two Definitions of soft semi-topological polygroups into one Definition, it will be as follows. We will show with an example how the generalized Definition refers to the first Definition and under what conditions the second Definition.

Definition 10. *Let* (P, θ, A) *be a soft topology on* P *and* $(\mathbb{F}, E)$ *be a soft set over* P*, where* $A \neq E$ *are sets of parameters. Then* $(\mathbb{F}, \theta, A, E, \circ)$ *is called a generalized soft semi-topological polygroup over* P *if the following axioms satisfies:*

(1) $\mathbb{F}(e)$ *is a subpolygroup of* P *for all* $e \in E$.
(2) *For all* $e \in E$ *and every soft open neighborhoods* $\mathbb{F}_{p \circ q}$ *of* $p \circ q$ *subset of* $\mathbb{F}(e)$*, there exist an soft open neighborhood* $\mathbb{F}_p$ *of* p *and an soft open neighborhood* $\mathbb{F}_q$ *of* q*, such that* $\mathbb{F}_p \circ \mathbb{F}_q \widehat{\subseteq} \mathbb{F}_{p \circ q}$*, with the restricted soft topology* θ *to* $\mathbb{F}(e)$ *which is denoted by* $\theta\,|_{\mathbb{F}(e)}$.

The following example proves that the two Definitions soft semi-topological polygroup are a special case of Definition 10.

EXAMPLE 12. *Let (F,E) be $\widehat{P}$ in this case $(\mathbb{F},\theta,A,E,\circ)$ is a soft semi-Topological polygroup via Definition 9 and if A be a single member set then $(\mathbb{F},\theta,A,E,\circ)$ is a soft semi-topological polygroup via Definition 2. It should be noted that in case that set A contains a parameter, the soft topology becomes a normal topology.*

EXAMPLE 13. *Let $P=(\mathbb{Z}_4,+)$, $\theta=\{\widehat{\varnothing},\widehat{\mathbb{Z}_4},\{(a_1,\{\widehat{0},\widehat{2}\}),(a_2,\varnothing)\},\{(a_1,\{\widehat{1},\widehat{3}\}),(a_2,\mathbb{Z}_4)\}\}$, where $A=\{a_1,a_2\}$ and $E=\{e_1,e_2\}$, $(E,F)=\{(e_1,\{\widehat{0},\widehat{2}\}),(e_2,\mathbb{Z}_4)\}$. In this case we have $\theta\mid_{\mathbb{F}(e_1)}=\{\widehat{\varnothing},\{\widehat{0},\widehat{2}\},\{(a_1,\{\widehat{0},\widehat{2}\}),(a_2,\varnothing)\},\{(a_1,\varnothing),(a_2,\{\widehat{0},\widehat{2}\})\}\}$, and $\theta\mid_{\mathbb{F}(e_2)}=\theta$.*

With above condition $(\mathbb{F},\theta,A,E,+)$ is a generalized soft semi-topological polygroup over P.

Definition 11. *Let $(\mathbb{F},\theta,A,E,\circ)$ be a generalized soft semi-topological polygroup over P and $\mathbb{G}$ be a soft subset of $\mathbb{F}$. Then $(\mathbb{G},\theta,A,E,\circ)$ sub-gstp(sub-generalized soft semi-topological polygroup) of $(\mathbb{F},\theta,A,E,\circ)$ if $(\mathbb{G},\theta,A,E,\circ)$ also is a generalized soft semi-topological polygroup over P.*

EXAMPLE 14. *Let $(\mathbb{F},\theta,A,E,+)$ be in Example 13, in conclusion $(\mathbb{F},\theta,A,E,+)$ $(\mathbb{F},\theta,\{a_1\},E,+)$, $(\mathbb{Z}_4,\theta,A,E,+)$ are sub-gstp of $(\mathbb{F},\theta,A,E,+)$.*

Definition 12. *Let $(P,\circ,e_n,^{-1})$ and $(Q,\star,e'_n,^{-1})$ be polygruops if $P^*\subseteq P$, $Q\subseteq Q$ with the property that $(\widehat{P^*},\theta,A,E,\circ)$,$(\widehat{Q},\theta,A,E,\star)$ are generalized soft semi-topological polygroup over P^* and Q then $F=(f_1,f_2)$ said to be a morphism if the following conditions are true:*

(i) $f_1:(P,\theta,A)\longmapsto(Q,\theta,A)$ *is soft continuous.*
(ii) $f_2:(P,\circ)\longmapsto(Q,\star)$ *is a polygroup homomorphism.*

Theorem 21. *The image of a generalized soft semi-topological polygroup under a morphism, is also a generalized soft semi-topological polygroup.*

Proof. Let $(P,\circ,e_n,^{-1})$, $(Q,\star,e'_n,^{-1})$ be polygruops, $P^*\subseteq P$, $Q\subseteq Q$ with the property that $(\widehat{P^*},\theta,A,E,\circ)$,$(\widehat{Q},\theta,A,E,\star)$ are generalized soft semi-topological polygroup over P^* and Q and $F=(f_1,f_2)$be a morphism. since for every $e\in E$, $f_2(F(e))$ is subpolygroup of Q as f_2 is a polygroup homomorphism, it follows that $F((\widehat{P^*},\theta,A,E,\circ))$ is a generalized soft semi-topological polygroup. Furthemore the composition of two continuous functions is continuous, this proves the second and third conditions. $\square$

Definition 13. *Let $(\mathbb{F},\theta,A,E,\circ)$ be a generalized soft semi-topological polygroup over P. The $(\mathbb{F},\theta,A,E,\circ)$ is called $T_i$generalized soft semi-topological polygroup if (P,θ,A) is a soft $T_i$space.*

Theorem 22. *[11]Let $(\mathbb{F},\theta,A,E,\circ)$ be a generalized soft semi-topological polygroup over P. the following items are equivalents:*

(i) $(\mathbb{F}, \theta, A, E, \circ)$ T_0*generalized soft semi-topological polygroup.*
(ii) $(\mathbb{F}, \theta, A, E, \circ)$ T_1*generalized soft semi-topological polygroup.*
(iii) $(\mathbb{F}, \theta, A, E, \circ)$ T_2*generalized soft semi-topological polygroup.*

Let P,Q,R are polygroups and hyperoperation of polygroups is "$\circ$" and $SS(P)$ is all soft sets are defined on the set of parameters E. Note that in a polygroup, the combination of two members will be a set.

Definition 14. *[13] Consider $\mathbb{F}_A \in SS(P)$, $\mathbb{G}_B \in SS(Q)$ and $\psi : P \longmapsto Q$, $\varphi : A \longmapsto B$ be two mappings. The (φ, ψ) is a soft mapping from $\mathbb{F}_A$ to $\mathbb{G}_B$ denoted by $(\varphi, \psi) : \mathbb{F}_A \longmapsto \mathbb{G}_B$ if and only if*

$$\psi(\mathbb{F}_A(a)) = \mathbb{G}_B(\varphi(a)), \forall a \in A.$$

We consider that all soft sets are defined on the set of parameters E and all soft mappings are defined with respect to the identity on E. Note that if $(id_E, f) : \mathbb{F} \longmapsto \mathbb{G}$ is a soft mapping we write f instead of (id_E, f).

Definition 15. *The cartesian product of $\mathbb{F}_A$ and $\mathbb{G}_B$ is shown with soft set $(\mathbb{F}_A \widehat{\times} \mathbb{G}_B) \in SS(P \times Q)$, such that $(\mathbb{F}_A \widehat{\times} \mathbb{G}_B)(a, b) = \mathbb{F}_A(a) \times \mathbb{G}_B(b), \forall (a, b) \in A \times B$, where $\mathbb{F}_A \in SS(P)$ and $\mathbb{G}_B \in SS(Q)$ [3].*

Throughout this section, we will deal with soft topological spaces defined over a soft set $\mathbb{F} \in SS(P)$. Thus, we will recall the following Definition for soft topology [26].

Definition 16. *Consider $\mathbb{F} \in SS(P)$ and Θ be a family of soft subsets of $\mathbb{F}$ and*
(i) $\widehat{\varnothing}, \mathbb{F} \in \Theta$*;*
(ii) Θ *is closed under finite intersection;*
(iii) Θ *is closed under arbitrary union.*

We say that Θ is a soft topology on $\mathbb{F}$ and $(\mathbb{F}, \Theta)$ is called the soft topological space (in short STS) and $V \in SS(P)$ is called a soft open set if $V \in \Theta$ [4].

Example 15. *Assume that $E = \mathbb{R}^+$ (the set of all positive real numbers), where $\mathbb{R}$ be the set of all real numbers. Let $\varepsilon \in E$ and $\mathbb{F}_\varepsilon \in SS(\mathbb{R})$ such that $\mathbb{F}_\varepsilon(e) = (e-\varepsilon, e+\varepsilon)$, for all $e \in E$. Consider $\Theta = \{\mathbb{F}_\varepsilon \mid \varepsilon \in E\}$. Then $(\mathbb{R}, \Theta)$ is a soft semi-topological space [2].*

Definition 17. *Assume that (P, Θ) and (Q, Λ) are soft topological spaces and f be mapping $f : P \longmapsto Q$ then*
(1) *If f satisfies in the condition $\mathbb{F} \in \Theta \Longrightarrow f(\mathbb{F}) \in \Lambda$, then f is said to be soft open;*
(2) *f is said to be soft continuous, if and only if for any $x \in P$ and any soft open neighborhoods $\mathbb{F}_{f(x)}$ of $f(x)$, there exist an soft open neighborhood $\mathbb{F}_x$ of x such that $f(x) \widehat{\in} f(\mathbb{F}_x) \widehat{\subseteq} \mathbb{F}_{f(x)}$;*

(3) *If f is bijective and f , f^{-1} are soft continuous, then f is said to be soft homeomorphism;*

(4) *Assume that $\mathbb{F} \in SS(P)$ and $\mathbb{G} \in SS(Q)$, then the mapping $f : \mathbb{F} \longmapsto \mathbb{G}$ is said to be soft continuous, if and only if for any $x \widehat{\in} \mathbb{F}$ and any soft open neighborhoods $\mathbb{F}_{f(x)}$ of $f(x)$, there exist an soft open neighborhood $\mathbb{F}_x$ of x such that$f(x) \widehat{\in} f(\mathbb{F}_x) \widehat{\subseteq} \mathbb{F}_{f(x)}$ [11].*

In the above Definition, $f(x)$ may be a set. In particular, when f is hyperoperation of polygroup.

Definition 18. *Assume that (P, Θ) and (Q, Λ) be soft topological spaces. We can make soft product topological space $(P \times Q, \Theta \widehat{\times} \Lambda)$, where the collection of all unions of soft sets in $\{\mathbb{F} \widehat{\times} \mathbb{G} \mid \mathbb{F} \in \Theta, \mathbb{G} \in \Lambda\}$ is a soft topology on $P \times Q$ and it is said to be soft product topology on $P \times Q$ and denoted by $(\Theta \widehat{\times} \Lambda)$ [19].*

Theorem 23. *Assume that (P, Θ) and (Q, Λ) is soft topological spaces. Then $proj_p : (P \times Q, \Theta \widehat{\times} \Lambda) \longmapsto (P, \Theta)$ and $proj_q : (P \times Q, \Theta \widehat{\times} \Lambda) \longmapsto (Q, \Lambda)$ are soft continuous and soft open too the smallest soft topology on $P \times Q$ for which $proj_p, proj_q$ be soft continuous is $\Theta \widehat{\times} \Lambda$ [19].*

Theorem 24. *The mapping $f : (R, \phi) \longmapsto (P \times Q, \Theta \widehat{\times} \Lambda)$ is soft continuous, if and only if the mappings $(proj_q \circ f)$ and $(proj_p \circ f)$ are soft continuous, where $(P, \Theta), (Q, \Lambda)$ and (R, ϕ) are soft topological spaces [19].*

Theorem 25. *Assume that $f : P \longmapsto Q$ and $g : Q \longmapsto R$ be soft continuous. Then the mapping $g \circ f$ is soft continuous, where (P, Θ) , (Q, Λ) and (R, ϕ) be soft topological spaces [19].*

Definition 19. *The set β is a base for a soft topological space (P, Θ) if we can make every soft open set in Θ as a union of elements of β [26].*

Definition 20. *Suppose that Q is subset of P and (P, Θ) is a soft topological space. Then the set $\Theta_{\widehat{Q}} = \{\widehat{Q} \widehat{\cap} \mathbb{F} \mid \mathbb{F} \in \Theta\}$ is said to be the soft relative topology on Q, and $(Q, \Theta_{\widehat{Q}})$ is a soft subspace of (P, Θ) [26].*

Theorem 26. *Assume that (P, Θ) is a soft topological space and $\mathbb{F} \in SS(P)$. Then the collection $\Theta_{\mathbb{F}} = \{\mathbb{F} \widehat{\cap} \mathbb{G} \mid \mathbb{G} \in \Theta\}$ is a soft topology over $\mathbb{F}$.*

Proof. The first, Θ is closed under the finite intersection and arbitrary union for all soft sets over P that is indeed $\Theta_{\mathbb{F}}$ is closed under the finite intersection and arbitrary union since the elements of $\Theta_{\mathbb{F}}$ are soft sets over P.

The second, since $\Theta_{\mathbb{F}} = \{\mathbb{F} \widehat{\cap} \mathbb{G} \mid \mathbb{G} \in \Theta\}$ and $\mathbb{F} \widehat{\cap} \mathbb{G} \widehat{\subseteq} \mathbb{F}$, it follows that element soft $\Theta_{\mathbb{F}}$ are soft subsets of $\mathbb{F}$. Moreover, since (P, Θ) be a soft topological space over P, then $\widehat{P}, \widehat{\varnothing} \in \Theta$. So, $\mathbb{F} = \mathbb{F} \widehat{\cap} \widehat{P} \in \Theta_{\mathbb{F}}$ and $\widehat{\varnothing} = \mathbb{F} \widehat{\cap} \widehat{\varnothing} \in \Theta_{\mathbb{F}}$. □

$(\mathbb{F}, \Theta_{\mathbb{F}})$ is referred to as a soft subspace of (P, Θ), where $\Theta_{\mathbb{F}}$ is said to be the soft relative topology on $\mathbb{F}$.

Theorem 27. *The union of two STS is not necessary a STS. However, the intersection of two STS is a STS [21].*

Definition 21. *Assume that Θ is a soft topology on P and $\mathbb{F} \in SS(P)$ is a soft polygroup, then the soft topological space $(\mathbb{F}, \Theta)$ is said to be soft semi-topological soft polygroup over P (in short SSTSP) if the soft mappings $f : (a, b) \longmapsto a \circ b$ from $(\mathbb{F}\widehat{\times}\mathbb{F}, \Theta\widehat{\times}\Theta)$ to $(\mathbb{F}, \Theta_{\mathbb{F}})$ is soft continuous.*

Definition 22. *The sum of $\mathbb{F}$ and $\mathbb{G}$ is the soft set $\mathbb{F}\widehat{\circ}\mathbb{G} \in SS(P)$, such that $(\mathbb{F}\widehat{\circ}\mathbb{G})(e) = \mathbb{F}(e) \circ \mathbb{G}(e)$, for all $e \in E$, where that $\mathbb{F}, \mathbb{G} \in SS(P)$ are soft polygroups.*

The following theorem presents an equivalent definition for SSTSP.

Theorem 28. *Suppose that $\mathbb{F}$ is a soft polygroup over P where Θ is a soft topology on P. Then $(\mathbb{F}, \Theta)$ is an SSTSP over P if and only if the following condition be true:*

For all $a, b\widehat{\in}\mathbb{F}$ and every soft open neighborhoods $\mathbb{F}_{a\circ b}$ of $a \circ b$, there exist an soft open neighborhood $\mathbb{F}_a$ of a and an soft open neighborhood $\mathbb{F}_b$ of b, such that $\mathbb{F}_a\widehat{\circ}\mathbb{F}_b\widehat{\subseteq}\mathbb{F}_{a\circ b}$.

Proof. [$\Rightarrow$] The first assume that $(\mathbb{F}, \Theta)$ is an SSTSP. Then $f : (a, b) \longmapsto a \circ b$ from $(\mathbb{F}\widehat{\times}\mathbb{F}, \Theta\widehat{\times}\Theta)$ to $(\mathbb{F}, \Theta_{\mathbb{F}})$, is soft continuous. Suppose that $a, b\widehat{\in}\mathbb{F}$, and $\mathbb{F}_{a\circ b}$ of an arbitrary soft open neighborhood of $f(a, b) = a \circ b$. Then by soft-continuity in Definition 17, for every $(a, b) \in \mathbb{F}\widehat{\times}\mathbb{F}$ and every soft open neighborhoods $\mathbb{F}_{f(a,b)}$ of $f(a, b)$, there is an soft open neighborhood $\mathbb{F}_{(a,b)}$ of (a, b) such that $a \circ b\widehat{\in}f(\mathbb{F}_{(a,b)})\widehat{\subseteq}\mathbb{F}_{f(a,b)}$.

Now $\mathbb{F}_{(a,b)}$ is a soft open set in $\Theta\widehat{\times}\Theta$, which means there exist $\{\mathbb{F}_{a_i}, \mathbb{F}_{b_i} \in \Theta, i \in I\}$ such that $\mathbb{F}_{(a,b)} = \bigcup_{i\in I} \mathbb{F}_{a_i}\widehat{\times}\mathbb{F}_{b_i}$. That shows there exist $i \in I$ such that $a\widehat{\in}\mathbb{F}_{a_i}$ and $b\widehat{\in}\mathbb{F}_{b_i}$. So, $\mathbb{F}_{a_i}\widehat{\times}\mathbb{F}_{b_i} \in \Theta\widehat{\times}\Theta$ and $\mathbb{F}_{a_i}\widehat{\times}\mathbb{F}_{b_i}\widehat{\subseteq}\mathbb{F}_{(a,b)}$ and

$$\mathbb{F}_{a_i}\widehat{\circ}\mathbb{F}_{b_i} = f(\mathbb{F}_{a_i}\widehat{\times}\mathbb{F}_{b_i})\widehat{\subseteq}f(\mathbb{F}_{(a,b)})\widehat{\subseteq}\mathbb{F}_{f(a,b)}.$$

[$\Leftarrow$] For all $a, b\widehat{\in}\mathbb{F}$ and every soft open neighborhoods $\mathbb{F}_{a\circ b}$ of $a \circ b$, there exist an soft open neighborhood $\mathbb{F}_a$ of a and an soft open neighborhood $\mathbb{F}_b$ of b, such that $\mathbb{F}_a\widehat{\circ}\mathbb{F}_b\widehat{\subseteq}\mathbb{F}_{a\circ b}$.

However, $\mathbb{F}_a\widehat{\circ}\mathbb{F}_b = f(\mathbb{F}_a\widehat{\times}\mathbb{F}_b)$, since $a\widehat{\in}\mathbb{F}_a$ and $b\widehat{\in}\mathbb{F}_b$ and they are soft open neighborhoods in Θ, then $\mathbb{F}_a\widehat{\times}\mathbb{F}_b$ is an soft open neighborhood in $\Theta\widehat{\times}\Theta$ contains (a, b). Therefore, by Definition of soft continuity 17, the mapping f is soft continuous. $\square$

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