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by

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# Three Dimensional Vortex Flow In The Liquid Cyclone* †

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## ABSTRACT

Flow patterns in the liquid cyclone consist of the circulation and translation of a film, or of a formed vortex. The Navier Stokes equations for the flow of a viscous incompressible fluid in a liquid cyclone are considered. The equations are reduced to two ordinary differential equations in terms of stream and circulation functions. These equations contain two dimensionless numbers: the Reynolds and Rossby numbers and it is assumed that they may be applied to flow in a liquid cyclone with Re interpreted as a constant turbulent Reynolds number. The differential equations have no analytical solution but may be integrated numerically. This has been done under conditions which correspond to the experiments of Kelsall and satisfactory agreement with the experimental results was obtained.

## INTRODUCTION

Kelsall (1) has made an extensive determination of the streamlines in the liquid cyclone due to the combination of vertical and radial velocities. With some special assumptions Rietema(2) derived the tangential velocity theoretically. Rietema was able to derive an expression for the tangential velocity profile by making the following assumptions.

a. When time averages are used the turbulent fluctuations can be neglected.

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b. The quadratic terms of these fluctuations can be represented by introducing the term turbulent kinematic viscosity, which must be added to the ordinary kinematic viscosity.

The present treatment is an attempt to improve on Rietema's analysis and to obtain stream and circulation functions. We have found that it is possible to obtain not only the tangential velocity component but also the stream function (combination of vertical and radial velocity components) and the distribution of the circulation function from the Navier Stokes equations making slightly less severe assumptions than Rietema.

The maintenance of laminer flow conditions can not be assumed to hold during the normal operation of a liquid cyclone. The velocity and the pressure terms may be taken to be time smoothed and the viscosity is the sum of dynamic and eddy viscosities.

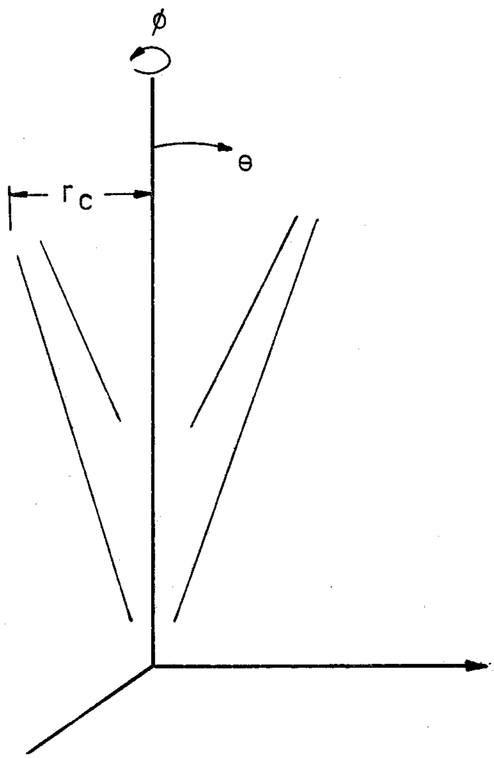


Fig – 1. Flow in liquid - cyclones in spherical coordinates

Thus for all operating conditions it is assumed that the streamlines are parallel(5).

It should be noted that whereas the theory is developed for the flow of an incompressible, single phase fluid a liquid cyclone handles suspensions. This should not lead to serious errors; generally analysis of the flow of a liquid of a given density is also relevant to suspensions.

Numerical integration which is eventually necessary, has been carried out for an initial operating value corresponding to Kelsall's experiments.

## MODEL AND DERIVATION OF EQUATIONS

Spherical coordinates (Fig. 1) are appropriate for the analysis of axisymmetric flow in liquid cyclones. Radial, tangential, vertical velocity components (u, $\omega$, $\upsilon$ respectively) and pressure (p) are functions of r and $\theta$. From the conservation of mass and momentum during steady motion we have the flow model for an incompressible fluid with constant viscosity, assuming axial symmetry

$$(1) \quad \frac{\partial u}{\partial r} + \frac{2u}{r} + \frac{\upsilon}{r} \cdot \cot\theta + \frac{1}{r} \cdot \frac{\partial \upsilon}{\partial \theta} = 0$$

$$(2) \quad u \frac{\partial u}{\partial r} + \frac{\upsilon}{r} \cdot \frac{\partial u}{\partial \theta} - \frac{\upsilon^2}{r} - \frac{\omega^2}{r} = - \frac{1}{\rho} \cdot \frac{\partial p}{\partial r}$$
$$+ \nu \left( \frac{4}{r} \cdot \frac{\partial u}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial u}{\partial \theta} \cdot \cot\theta + \frac{1}{r^2} \cdot \frac{\partial^2 u}{\partial \theta^2} + \frac{2u}{r^2} \right)$$

$$(3) \quad u \frac{\partial \upsilon}{\partial r} + \frac{\upsilon}{r} \cdot \frac{\partial \upsilon}{\partial \theta} + \frac{u\upsilon}{r} \cdot - \frac{\omega^2 \cot\theta}{r} = - \frac{1}{r\rho} \cdot \frac{\partial p}{\partial \theta}$$
$$+ \nu \left( \frac{2}{r} \cdot \frac{\partial \upsilon}{\partial r} + \frac{\partial^2 \upsilon}{\partial r^2} + \frac{1}{r^2} \cdot \frac{\partial \upsilon}{\partial \theta} \cdot \cot\theta \right.$$
$$\left. + \frac{1}{r^2} \cdot \frac{\partial^2 \upsilon}{\partial \theta^2} + \frac{2}{r^2} \cdot \frac{\partial u}{\partial \theta} - \frac{\upsilon}{r^2 \sin^2\theta} \right)$$

$$(4)\quad u\frac{\partial \omega}{\partial r} + \frac{\upsilon}{r} \cdot \frac{\partial \upsilon}{\partial \theta} + \frac{u\omega}{r} + \frac{\omega\upsilon}{r} \cdot \cot\theta$$

$$= \nu \left( \frac{2}{r} \cdot \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r^2} \cdot \frac{\partial \omega}{\partial \theta} \cot\theta \right.$$

$$\left. + \frac{1}{r^2} \cdot \frac{\partial^2 \omega}{\partial \theta^2} - \frac{\omega}{r^2 \sin^2 \theta} \right)$$

where $\nu$ is the total kinematic viscosity. Specifically we retain the quadratic terms but we find it necessary to ignore only the term $\frac{\omega^2 \cot\theta}{r}$ on the left hand side of eq. – 3.

By substituting eq. –1 into eqs. –2, 3, 4 the terms $\frac{\partial \upsilon}{\partial \theta}$ and $\frac{\partial^2 \upsilon}{\partial \theta^2}$ are eliminated. The derivatives of eqs. –2 and 3 are then taken with respect to $\theta$ and r respectively. The order of both equations is therefore increased by one; and by cross differentiation the pressure term may be eliminated.

$$(5)\quad u\frac{\partial^2 u}{\partial r \partial \theta} + \frac{\upsilon}{r} \cdot \frac{\partial^2 u}{\partial \theta^2} - ru \frac{\partial^2 \upsilon}{\partial r^2} + 4\,\upsilon \frac{\partial u}{\partial r}$$

$$+ r\upsilon \frac{\partial^2 u}{\partial r^2} - \frac{2u}{r} \cdot \frac{\partial u}{\partial \theta} - \frac{\upsilon}{r} \cdot \frac{\partial u}{\partial \theta} \cdot \cot\theta$$

$$+ 2\cot\theta \cdot \frac{\partial \upsilon}{\partial r} + 4\frac{u\upsilon}{r} + 2\frac{\upsilon^2}{r} \cdot \cot\theta - \frac{1}{r} \cdot \frac{\partial^2 \omega}{\partial \theta^2}$$

$$= \nu \left( \frac{4}{r} \cdot \frac{\partial^2 u}{\partial r\, \partial \theta} + 2 \cdot \frac{\partial^3 u}{\partial r^2 \partial \theta} + \frac{1}{r^2} \cdot \frac{\partial^2 u}{\partial \theta^2} \cdot \cot\theta \right.$$

$$- \frac{1}{r^2 \sin^2 \theta} \cdot \frac{\partial u}{\partial \theta} + \frac{1}{r^2} \cdot \frac{\partial^3 u}{\partial \theta^3}$$

$$\left. + \frac{2}{r^2} \cdot \frac{\partial u}{\partial \theta} - 3\frac{\partial^2 \upsilon}{\partial r^2} - r\frac{\partial^3 \upsilon}{\partial r^3} \right)$$

The number of dependent variables may be reduced once more by defining the usual axisymmetric stream function

$$(6) \quad \frac{\partial\psi}{\partial r} = r\upsilon\sin\theta \; ; \quad -\frac{\partial\psi}{\partial\theta} = r^2 u\sin\theta$$

and the circulation function

$$(7) \quad T = r\omega$$

Substituting these functions in equations 4 and 5

$$(8) \quad \frac{-1}{r^2\sin^2\theta}\cdot\frac{\partial\psi}{\partial\theta}\cdot\frac{\partial\psi}{\partial r} + \frac{\cos\theta}{r^2\sin^2\theta}\cdot T\cdot\frac{\partial\psi}{\partial r}$$

$$-\frac{\cos\theta}{r^2\sin^3\theta}\cdot\left(\frac{\partial\psi}{\partial r}\right)^2 + \frac{1}{r^2\sin^2\theta}\cdot\frac{\partial\psi}{\partial r}\cdot\frac{\partial^2\psi}{\partial r\,\partial\theta}$$

$$= \nu\left(\frac{\partial^2 T}{\partial r^2} + \frac{\cot\theta}{r^2}\cdot\frac{\partial T}{\partial\theta} + \frac{1}{r^2}\cdot\frac{\partial^2 T}{\partial\theta^2} - \frac{T}{r^2\sin^2\theta}\right)$$

$$(9) \quad \frac{4\cos\theta}{r^2\sin^3\theta}\cdot\left(\frac{\partial\psi}{\partial r}\right)^2 + \frac{2\sin^2\theta - 3}{r\sin^2\theta}\cdot\frac{\partial\psi}{\partial r}\cdot\frac{\partial\psi}{\partial\theta}$$

$$-\frac{\cos\theta}{r\sin^3\theta}\cdot\frac{\partial\psi}{\partial\theta}\cdot\frac{\partial^3\psi}{\partial r\partial\theta^2} - \frac{4\cos\theta}{r^2\sin^3\theta}\cdot\frac{\partial\psi}{\partial\theta}\cdot\frac{\partial^2\psi}{\partial\theta^2}$$

$$-\frac{2\,r\cos\theta}{\sin^3\theta}\cdot\frac{\partial\psi}{\partial r}\cdot\frac{\partial^2\psi}{\partial r^2} + \frac{2\cos\theta}{r\sin^3\theta}\cdot\frac{\partial\psi}{\partial r}\cdot\frac{\partial^2\psi}{\partial\theta^2}$$

$$-\frac{2}{\sin^2\theta}\cdot\frac{\partial\psi}{\partial\theta}\cdot\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r\sin^2\theta}\cdot\frac{\partial\psi}{\partial\theta}\cdot\frac{\partial^3\psi}{\partial r\partial\theta^2}$$

$$-\frac{1}{r\sin^2\theta}\cdot\frac{\partial\psi}{\partial r}\cdot\frac{\partial^3\psi}{\partial\theta^3} + \frac{r}{\sin^2\theta}\cdot\frac{\partial\psi}{\partial\theta}\cdot\frac{\partial^3\psi}{\partial r^3}$$

$$-\frac{r}{\sin^2\theta}\cdot\frac{\partial\psi}{\partial r}\cdot\frac{\partial^3\psi}{\partial r^2\,\partial\theta} - \frac{\partial T^2}{\partial\theta}$$

$$= \nu \left( \frac{4\cos\theta}{r\sin^2\theta} \cdot \frac{\partial\psi}{\partial\theta} + \frac{3\cos\theta}{r\sin^4\theta} \cdot \frac{\partial\psi}{\partial\theta} \right.$$

$$- \frac{5}{r\sin\theta} \cdot \frac{\partial^2\psi}{\partial\theta^2} + \frac{2}{r\sin^3\theta} \cdot \frac{\partial^2\psi}{\partial\theta^2} - \frac{4\cos\theta}{\sin^2\theta} \cdot \frac{\partial^2\psi}{\partial r\partial\theta}$$

$$+ \frac{4}{\sin\theta} \cdot \frac{\partial^3\psi}{\partial r\,\partial\theta^2} + \frac{2r\cos\theta}{\sin^2\theta} \cdot \frac{\partial^3\psi}{\partial r^2\,\partial\theta}$$

$$- \frac{4r}{\sin\theta} \cdot \frac{\partial^4\psi}{\theta r^2\,\partial\theta^2} + \frac{\cos\theta}{r\sin\theta} \cdot \frac{\partial^3\psi}{\partial\theta^3}$$

$$\left. - \frac{1}{r\sin\theta} \cdot \frac{\partial^4\psi}{\partial\theta^4} - \frac{r^3}{\sin\theta} \cdot \frac{\partial^4\psi}{\partial r^4} \right)$$

Whence the four equations (1,2,3,4) have been reduced to two equations (8,9), the number of dependent variables has also been reduced from four (u, ω, υ , p) to two (ψ, T).

For convenience, eqs. —8 and 9 can be somewhat simplified by introducing the following dimensionless quantities

$$(10)\ \eta = \frac{r}{r_c}\ ;\ \Omega = \frac{T}{T_c}\ ;\ \varnothing = \frac{\psi}{Q\sin\theta}$$

where $\eta$ is the normalized radius, and $\Omega$ and $\varnothing$ are normalized circulation and stream functions, respectively. Here $r_c$ is the characteristic radius of the liquid cyclone, $2\pi\ T_c$ is the circulation at the cyclone inlet region, and $2\pi Q$ is the total volume flow through the liquid cyclone. Thus the resulting equations are the two following partial differential equations where all the quantities are dimensionless:

$$(11) - \frac{\partial\Omega}{\partial\eta} \cdot \frac{\partial\varnothing}{\partial\theta} + \Omega\ \frac{\partial\varnothing}{\partial\eta} .\cot\theta - \frac{Q}{r_cT_c} . \left(\frac{\partial\varnothing}{\partial\eta}\right)^2 \cot\theta$$

$$+ \frac{Q}{r_cT_c} \cdot \frac{\partial\varnothing}{\partial\eta} \cdot \frac{\partial^2\varnothing}{\partial\eta\partial\theta} = \frac{\nu r_c}{Q} \left( \eta^2\ \frac{\partial^2\Omega}{\partial\eta^2} \right.$$

$$+ \frac{\partial \Omega}{\partial \theta} \cot\theta + \frac{\partial^2 \Omega}{\partial \theta^2} - \frac{\Omega}{\sin^2\theta} \Big)$$

$$(12)\ \Omega \frac{\partial \Omega}{\partial \theta} = \frac{Q^2}{2r_c T_c^2} \Big[ \frac{4\cos\theta}{\eta^2 \sin\theta} \cdot \left( \frac{\partial \varnothing}{\partial \theta} \right)^2 - \frac{2}{\eta} \cdot \frac{\partial \varnothing}{\partial \eta} \cdot \frac{\partial \varnothing}{\partial \theta}$$

$$- \frac{3\cos^2\theta}{\eta^2 \sin^2\theta} \cdot \frac{\partial \varnothing}{\partial \theta} \cdot \frac{\partial \varnothing}{\partial \eta} + \frac{2\cos\theta}{\eta \sin\theta} \cdot \frac{\partial \varnothing}{\partial \eta} \cdot \frac{\partial^2 \varnothing}{\partial \theta^2}$$

$$- \frac{4}{\eta^2} \cdot \frac{\partial \varnothing}{\partial \theta} \cdot \frac{\partial^2 \varnothing}{\partial \theta^2} - \frac{2\eta \cos\theta}{\sin\theta} \cdot \frac{\partial \varnothing}{\partial \eta} \cdot \frac{\partial^2 \varnothing}{\partial \eta^2} - 2 \frac{\partial \varnothing}{\partial \theta} \cdot \frac{\partial^2 \varnothing}{\partial \eta^2}$$

$$- \frac{\cos\theta}{\eta \sin\theta} \cdot \frac{\partial \varnothing}{\partial \theta} \cdot \frac{\partial^2 \varnothing}{\partial \eta\, \partial \theta} - \frac{1}{\eta} \cdot \frac{\partial \varnothing}{\partial \eta} \cdot \frac{\partial^3 \varnothing}{\partial \theta^3}$$

$$+ \eta \frac{\partial \varnothing}{\partial \theta} \cdot \frac{\partial^3 \varnothing}{\partial \eta^3} + \frac{1}{\eta} \cdot \frac{\partial \varnothing}{\partial \theta} \cdot \frac{\partial^3 \varnothing}{\theta\eta\, \partial \theta^2}$$

$$- \eta \frac{\partial \varnothing}{\partial \eta} \cdot \frac{\partial^3 \varnothing}{\partial \eta^2 \partial \theta} \Big] - \frac{Q^2}{2 r_c^2 T_c^2} \cdot \frac{r_c \nu}{Q} \Big( \frac{6\cos\theta}{\eta \sin\theta} \cdot \frac{\partial \varnothing}{\partial \theta}$$

$$+ \frac{3\cos\theta}{\eta \sin^3\theta} \cdot \frac{\partial \varnothing}{\partial \theta} - \frac{4\cos\theta}{\sin\theta} \cdot \frac{\partial^2 \varnothing}{\partial \eta \partial \theta} + \frac{3}{\eta \sin^2\theta} \cdot \frac{\partial^2 \varnothing}{\partial \theta^2}$$

$$- \frac{4}{\eta} \cdot \frac{\partial^2 \varnothing}{\partial \theta^2} + \frac{\cos^2\theta}{\eta \sin\theta} \cdot \frac{\partial^2 \varnothing}{\partial \theta^2} + 4 \frac{\partial^3 \varnothing}{\partial \eta\, \partial \theta^2}$$

$$+ \frac{\cos\theta}{\eta \sin\theta} \cdot \frac{\partial^3 \varnothing}{\partial \theta^3} + \frac{2\eta \cos\theta}{\sin\theta} \cdot \frac{\partial^3 \varnothing}{\partial \eta^2 \partial \theta} - 2\eta \frac{\partial^4 \varnothing}{\partial \eta^2\, \partial \theta^2}$$

$$- \frac{1}{\eta} \cdot \frac{\partial^4 \varnothing}{\partial \theta^4} - \eta^3 \frac{\partial^4 \varnothing}{\partial \eta^4} \Big)$$

Furthermore, in the above equations,

$$(13) \qquad R_o \equiv \frac{Q}{r_c T_c}, \quad R_e \equiv \frac{Q}{r_c \nu}$$

are Rossby and Reynolds numbers. As indicated above, it is generally accepted that the flow in liquid cyclones is turbulent; it will be assumed below that the above equations may be applied unaltered with Re interpreted as a constant turbulent Reynolds number.

We have reduced the above equations of motion to ordinary differential equations by means of the new similarity variable

$$x = \frac{\sin\theta}{\eta} \tag{14}$$

and by defining the new I (x) and f(x) functions,

$$\Omega = \frac{I(x)}{\eta^2} \; ; \; \varnothing = \frac{f(x)}{\cos\theta} \tag{15}$$

With these transformations eqs. — 11 and 12 become

$$x^3 f^{I} I^{I} - x^2 f^{I} I - R_o x^3 f^{I2} + R_o x^3 f^{I} f^{II} = \frac{1}{R_e} (x^2 I^{II} + x I^{I} - I) \tag{16}$$

$$I I^{I} = \frac{R_o^2}{2} . \left( \frac{7}{x} f^{I2} + 5 f^{I} f^{II} \right) - \frac{R_o^2}{2R_e} . \left( \frac{3}{x^3} f^{I} + \frac{4}{x^2} f^{II} + \frac{1}{x} f^{III} - f^{IV} \right) \tag{17}$$

**AN EXAMPLE OF THE NUMERICAL SOLUTION OF THE EQUATIONS OF MOTION**

Equations 16 and 17 have no analytical solution. We discuss their numerical integration for initial values corresponding to the Kelsall's liquid cyclone experiments.

Integration may be achieved by means of fourth order Runge-Kutta formulae in the modification due to Gil(3,4). Accuracy was tested by comparing the results of procedures with single and double increments. Our programme automatically adjusted the increment during the whole computation by halving or by doubling. If more than ten bisections of the increment were necessary to get satisfactory accuracy the subroutine returned to the main programme bearing an error message.

The numerical procedure outlined above was coded for the IBM – 360 system with the initial values below and with Re = 157,000 and $R_o = 0.3$.

$x_o = 0.093$

$I_o = 1.0$ ; $I_o^1 = 33.3$

$f_o = 0.184$ ; $f_o^1 = -11.5$ ; $f_o^{11} = -0.011$ ; $f_o^{111} = 0.0$

where for optimum shape factors

$x \geq 0.093$

These correspond to the conditions used in Kelsall's (1) experiments.

The normalized new stream function f(x) obtained by the numerical integration shows the same shape as that found by Kelsall experimentally (Fig. 2). As expected the normalized new circulation function I(x) increased as x increased or as r decreased towards the apex of the liquid cyclone.

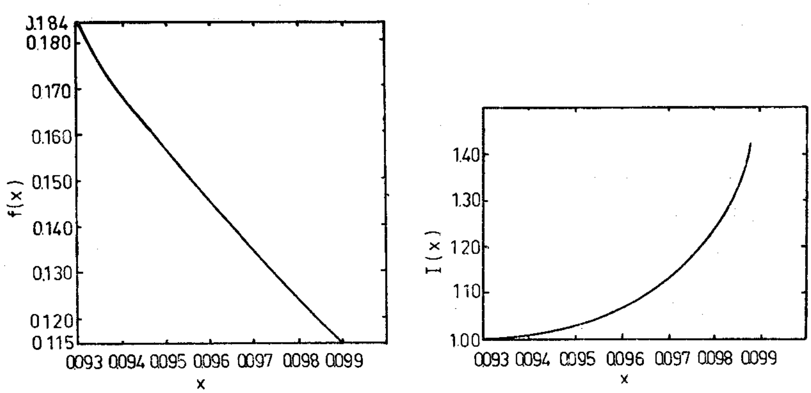


Fig- 2. Distributions of f(x) and I(x) in the liquid-cyclone

## NOTATION

| | | |
|---|---|---|
| $f$ | : | normalized new stream function |
| $I$ | : | normalized new circulation function |
| $p$ | : | pressure |
| $Q$ | : | volumetric flow rate; $2\pi Q$ total volume flow through the liquid cyclone. |
| $r$ | : | radial component |
| $r_c$ | : | characteristic radius of the liquid cyclone |
| $R_e$ | : | Reynolds number |
| $R_o$ | : | Rossby number |
| $u$ | : | radial velocity component |
| $\upsilon$ | : | vertical velocity component |
| $\omega$ | : | tangential velocity component |
| $x$ | : | new independent variable |
| $\eta$ | : | normalized radius |
| $\theta$ | : | vertical component |
| $\nu$ | : | total kinematic viscosity |
| $\rho$ | : | density of the fluid |
| $T$ | : | circulation function; $2\pi T_c$ circulation at the cyclone inlet |
| $\varnothing$ | : | normalized stream function |
| $\psi$ | : | stream function |
| $\Omega$ | : | normalized circulation function |

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