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Stability Analysis For Well Stirred Tank Cooled By Jacked

by

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SUMMARY

In this work, Routh stability analysis was applied to choose the suitable control parameters for the feedback control of the well stirred tank cooled by jacked. For this purposes the related mathematical models (1) was converted to the Laplace Transfrom and the values of parameters were calculated with stability test.

INTRODUCTION

A stable system can be defined as one for which the output response is bounded for all bounded inputs. A system exhibiting an unbounded response to a bounded input is unstable.

The stability criterion can be used to ascertain the stability of control system of the form shown in Fig. 1. From the block diagram of the control system, the related equation can be obtained.

$$C = \frac{G_1G_2}{1 + G_1G_2H} R + \frac{G_2}{1 + G_1G_2H} U \quad (1)$$

$$G = G_1G_2H \quad (2)$$

We call, G, the open-loop transfer fonctions. A linear control system is unstable, if any roots of its characteristic equation, $1+G_1G_2H$, are on the imaginary axis. Otherwise the system is stable. The Routh test for stability is a purely algebraic method for determining how many roots of the characteristic equation have positive reel parts; from this

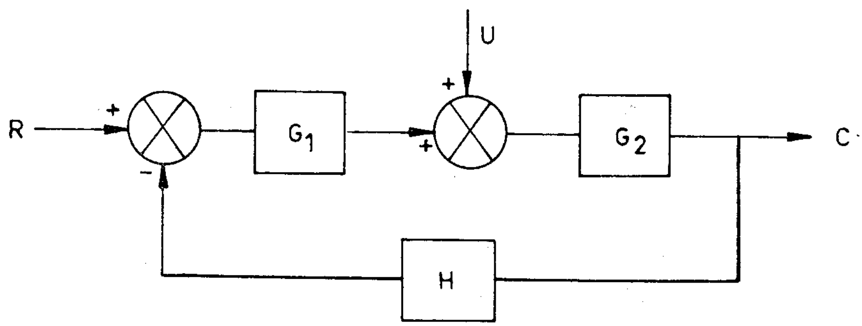

Figure. 1. Simple feedback control system.

it can also be determined if the system is stable, for if there are no roots with positive real parts, the system is stable. The test is limited to system which have polynomial characteristic equations. This stability criterion can be found in many text book (2)

In present work, Routh stability test were applied to the well mixed tank cooled by jacked.

MATHEMATICAL MODELS AND STABILITY ANALSIS

The related mathematical models dealing with well mixed tank cooled by jacked were drived in the work of Alpbaz and Erdoğan (3,4). The linearized models are shown below;

$$\frac{dT'_{po}}{dt} = \left(\frac{T^o_{pi} - T^o_{po}}{M_v}\right) M'_p - \left(\frac{UA + M_pC_p}{M_vC_p}\right) T'_{po} + \left(\frac{UA}{2M_vC_p}\right) T'_{co} \quad (3)$$

$$\frac{dT'_{co}}{dt} = \left(\frac{T^o_{ci} - T^o_{co}}{M_jC_c}\right) M'_c - \left(\frac{M_cC_c + \frac{UA}{2}}{M_jC_c}\right) T'_{co} + \left(\frac{UA}{M_jC_c}\right) T'_{po} \quad (4)$$

The range of parameters and the solution of related models with Laplace transform and digital computer solutions are found in the work of Alpbaz, Erdoğan and Koçkar (5).

Consider the system differantial equations (3,4) with numerical values, Table. 1.

$$\frac{dT'_{po}}{dt} = \left(\frac{64-71.49}{32595}\right)M'_p - \left(\frac{6+3.45015}{21512.7}\right)T'_{po} + \left(\frac{6}{(2)\,(21512.7)}\right)T'_{co} \tag{5}$$

$$\frac{dT'_{co}}{dt} = \left(\frac{17-33.21}{4922.8}\right)M'_c - \left(\frac{17+\frac{6}{2}}{4922.8}\right)T'_{co} + \left(\frac{6}{4922.8}\right)T'_{po} \tag{6}$$

$$\frac{dT'_{po}}{dt} = -(2.2978x10^{-4})\,M'_p - (4.3928x10^{-4})\,T'_{po} + (1.394x10^{-4})T'_{co} \tag{7}$$

$$\frac{dT'_{co}}{dt} = -(4.062x10^{-3})\,T'_{co} + (1.218x10^{-3})\,T'_{po} - (3.292x10^{-3})\,M'_c \tag{8}$$

Transforming;

$$\bar{T}'_{po} = -\left(\frac{2.2978x10^{-4}}{s+4.3928x10^{-4}}\right)\bar{M}'_p + \left(\frac{1.394x10^{-4}}{s+4.3928x10^{-4}}\right)\bar{T}'_{co} \tag{9}$$

$$\bar{T}'_{co} = \left(\frac{1.218x10^{-3}}{s+4.062x10^{-3}}\right)\bar{T}'_{po} - \left(\frac{3.292x10^{-3}}{s+4.062x10^{-3}}\right)\bar{M}'_c \tag{10}$$

The stability analysis were applied for each control system.

i-Proportional Control

$$\bar{M}'_c = -K_c\bar{T}'_{po} \tag{11}$$

If equation (11) is put into equation (10);

$$\bar{T}'_{co} = \left(\frac{1.218x10^{-3}}{s+4.062x10^{-3}}\right)\bar{T}'_{po} - \left(\frac{3.292x10^{-3}}{s+4.062x10^{-3}}\right)(-K_c\bar{T}'_{po}) \tag{12}$$

$$\bar{T}'_{co} = \left(\frac{3.292x10^{-3}\,K_c + 1.218x10^{-3}}{s+4.062x10^{-3}}\right)\bar{T}'_{po} \tag{13}$$

If equation (13) is put into equation (9);

$$T'_{po} = -\left(\frac{2.2978x10^{-4}}{s+4.392x10^{-4}}\right)\bar{M}'_p + \left(\frac{1.394x10^{-4}}{s+4.392x10^{-4}}\right) \tag{14}$$

$$\left(\frac{3.292x10^{-3}\,K_c + 1.218x10^{-3}}{s+4.062x10^{-3}}\right)\bar{T}'_{po}$$

and than

$$\bar{T}'_{po} = -(2.297x10^{-4}) \left[\frac{s + 4.062x10^{-3}}{s^2 + 4.5012x10^{-3} s + 1.6143x10^{-6} - 4.5892x10^{-7} K_c}\right] \bar{M}'_p \quad (15)$$

The characteristic equations;

$$s^2 + 4.5012x10^{-3} s + (1.6143x10^{-6} - 4.5892x10^{-7}) K_c = 0 \quad (16)$$

For stable time response, the roots should be real;

$$(4.5012x10^{-3})^2 - 4(1.6143x10^{-6} - 4.5892x10^{-7} K_c) > 0 \quad (17)$$

$$1.3803x10^{-5} + 1.8356x10^{-6} K_c > 0 \quad (18)$$

Therefore,

$K_c > -7.5$

For Routh stability test, if any coefficient of the characteristic equation is negative or zero a system is unstable,

1	1	$1.6143x10^{-6} - 4.5892x10^{-7} K_c$
2	$4.5012x10^{-3}$	
3	$1.6143x10^{-6} - 4.5892x10^{-7} K_c$	

Hence,

$K_c < 3.51$

ii– Proportional + Integral control

$$\bar{M}'_c = -K_c \left(1 + \frac{K_i}{K_c} \frac{1}{s}\right) \bar{T}'_{po} \quad (19)$$

If equation (19) is put into equation (10),

$$\bar{T}'_{co} = \left(\frac{1.218x10^{-3}}{s + 4.062x10^{-3}}\right) \bar{T}'_{po} - \left(\frac{3.292x10^{-3}}{s + 4.062x10^{-3}}\right) \left[-K_c \left(1 + \frac{K_i}{K_c} \frac{1}{s}\right)\right] \bar{T}'_{po} \quad (20)$$

$$\bar{T}'_{co} = \left[\frac{(1.218x10^{-3} + 3.292x10^{-3} K_c) s + 3.292x10^{-3} K_i}{s (s + 4.062x10^{-3})}\right] \bar{T}'_{po} \quad (21)$$

Put this equation (21) into equation (9).

$$\bar{T}'_{po} = -\left(\frac{2.297x10^{-4}}{s + 4.392x10^{-4}}\right) \bar{M}'_p + \left(\frac{1.392x10^{-4}}{s + 4.392x10^{-4}}\right)$$

$$\left[\frac{(1.218x10^{-3} + 3.292x10^{-3} K_c)\, s + 3.292x10^{-3} K_i}{s\,(s + 4.062x10^{-3})}\right] \bar{T}'_{po} \quad (22)$$

The characteristic equation,

$$s^3 + 4.5012x10^{-3}s^2 + (1.614x10^{-6} - 4.589x10^{-7}\ K_c)\, s - 4.\ 589x10^{-7}\ K_i = 0 \quad (23)$$

If Routh test is applied,

For stability, $(-4.589x10^{-7}\ K_i)$ and $(1.614x10^{-6} - 4.589x10^{-7}\ K_c + 1.01x10^{-4}\ K_i)$ must be positive. In this case $K_i < 0.0$ and

$T_R = \dfrac{K_c}{K_i}$ being positive and than $K_c < 0.0$

iii– Proportional + Derivative + Integral Control

$$\bar{M}'_c = - K_c\,(1 + \frac{K_d}{K_c}\ s + \frac{K_i}{K_c}\ \frac{1}{s}\ \bar{T}'_{po} \quad (24)$$

Put into equation (10),

$$\bar{T}'_{c_o} = \left(\frac{1.218x10^{-3}}{s + 4.062x10^{-3}}\right) \bar{T}'_{p_o} - \left(\frac{3.292x10^{-3}}{s + 4.062x10^{-3}}\right) \quad (25)$$

$$\left[- K_c \left(1 + \frac{K_d}{K_c}\ s + \frac{K_i}{K_c}\ \frac{1}{s}\right)\right] \bar{T}'_{p_o} \quad (25)$$

$$\bar{T}'_{co} = \left[\frac{(1.218x10^{-3} + 3.292x10^{-3}K_c)\, s + 3.292x10^{-3}K_d\, s^2 + 3.292x10^{-3}K_i}{s\ (s + 4.062x10^{-3})}\right] \quad (26)$$

If equatin (26) is put into equation (9), characteristic equation is become,

$$s^3 + (4.501x10^{-3} - 4.587x10^{-7}\,K_d)\ s^2 + (1.614x10^{-6} - 4.587x10^{-7}\ K_c)\ s - 4.587x10^{-7}\ K_i = 0 \quad (27)$$

Hence $(-4.587x10^{-7}\ K_i)$ requires that $K_i < 0.0$ and than $K_c < 0.0$

and $T_D = \dfrac{K_D}{K_c}$ should be positive and $K_d < 0.0$

RESULTS

In the previous work (5), the time response of the tank output temperature was calculated and it was observed the variation of output temperature with time was stable when tank was under the effect of the step change given to the feed flow rate.

In the present work, the feedback control system was added to force the output temperature for reaching to the set point. In control solution, three term control mechanism was introduced to the differantial equations (3,4) which describe the dynamic response of the mixing tank and this equations were solved with Laplace transfrom. When the system was in the steady-state having operating condition shown in Table. 1, the step change was given to the feed flow rate and it was investigated the effect of the control parameters on the output temperature. The effect of the proportional acting factor, K_C, on the output

Table.1. The operating conditions and control parameters for feedback control of the tank

Mp $\left(\frac{g}{sec}\right)$	Mc $\left(\frac{g}{sec}\right)$	Tpi (C°)	Tci (C°)	UA$\left(\frac{Cal}{secC°}\right)$	Kc	Ki	Kd
8.20	17.0	64.0	17.0	6.00	-5	—	—
					-5	-0.02	—
5.22	17.0	64.0	17.0	6.00	-5	-0.02	-0.02

temperature was shown in Fig. 2. When K_C was increasing the output temperature came closer to the setpoint with ossilation. But including two other control parameters, T_R, and, T_D, the time response of the output temperature become unstable, Fig. 3. For this reason Routh stability test was applied to obtain stability on the output variables and than proper control parameters were calculated as it was shown in this work. In Fig. 4. the time response of the output temperature with suitable control parameters was shown. It can be seen that the output variables are stable.

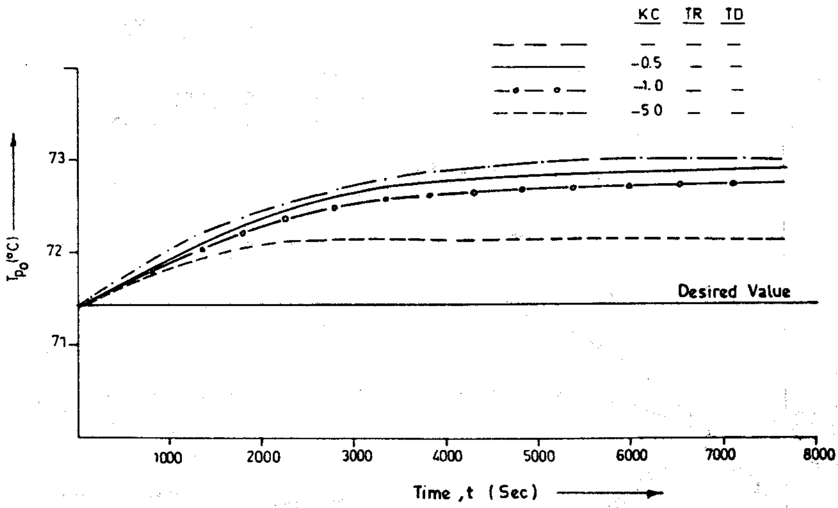

Figure. 2. The effect of the proportional acting factor Kc on the output temperature. ($M^{\circ}_p = 8.20$ g/sec, $M_p = 5.22$ g/sec)

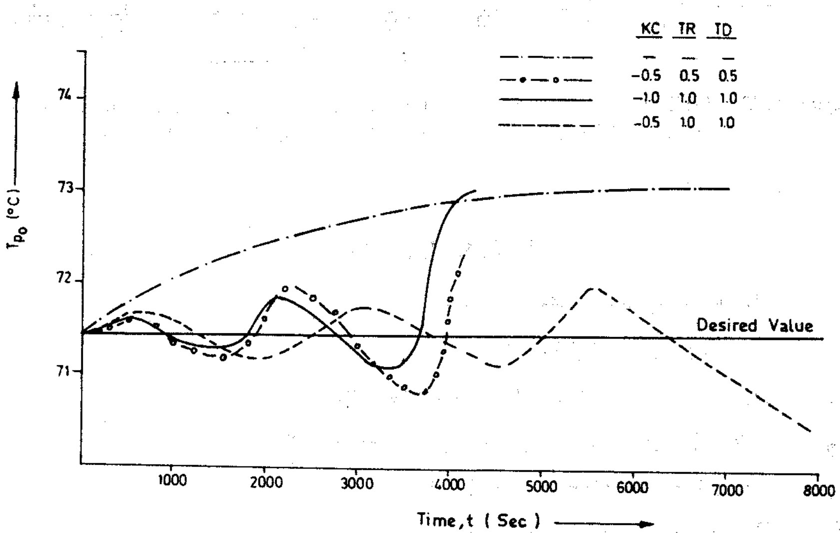

Figure. 3.The effect of the control parameters on the output tank temperature and unstability ($M^{\circ}_p = 8.20$ g/sec, $M_p = 5.22$ g/sec)

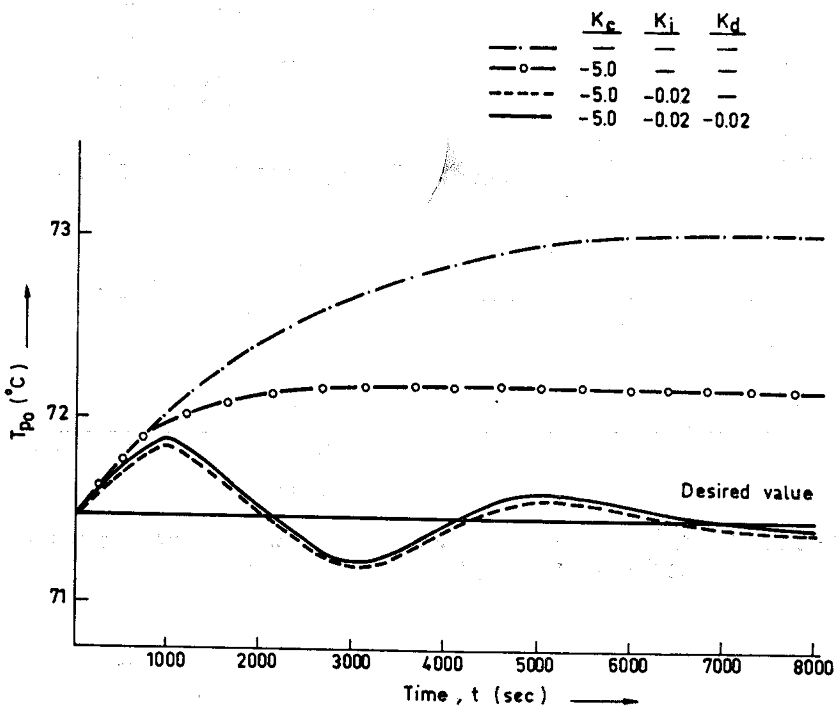

Figure. 4. The feedback control of the output tank temperature when the step change was given to the feed flow rate. ($M°p$ = 8.20 g/sec, Mp = 5.22 g/sec)

NOMENCLATURE

A	Heat transfer surface (cm^2)
C	Controlled variable
C_c	Specific heat of coolant (cal/g °C)
C_p	Specific heat of tank content (cal/g °C)
G_n	Transfer functions of n'th elements
H	Transfer function of measuring element
K_c	Proportionol acting factor.
K_d	Derivative action factor

K_i	Integral acting factor
M_c	Coolant mass flow rate (g /sec)
M_p	Feed mass flow rate (g /sec)
M_v	Mass hold up in the tank (g)
M_j	Mass hold up in the jacked (g)
R	Set point
s	Laplace operature
T_{pi}	Feed temperature (°C)
T_{po}	Output temperature (°C)
T_{co}	Output coolant temperature (°C)
t	Time
U	Overall heat transfer coefficient (cal /cm^2 sec °C), Disturbance
δ	Density (g /cm^3)
μ	Viscosity (g /cm sec)

ÖZET

Bu çalışmada, dışardan ceketle soğutulan tam karıştırmalı akım tankının geri beslemeli kontrolu için kontrol parametrelerinin uygun değerlerinin seçiminde Routh kararlılık testi uygulanmıştır. Bu amaçla ilgili matematiksel modellerin (1) Laplace dönüşümleri alınmış ve kararlılık analizi ile parametrelerin değerleri hesaplanmıştır.

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