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The Feedback Control Of The Continuous-Flow Agitated Tank Cooled By Jacked

by

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The Feedback Control Of The Continuous-Flow Agitated Tank Cooled By Jacket

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SUMMARY

Three actions of feedback control system was included to the tank for the control work. The approaches of the measured output temperature to the desired value was calculated with the Laplace transform and digital computer using linearized models. Also for the same porpuse, models were solved without linearization on the digital computer.

INTRODUCTION

Alpbaz and Erdoğan (1,2) have investigated the dynamic proporties of a well mixed tank. They have developed related mathematical models and solved linearized models with Laplace Transfrom

Alpbaz, Erdoğan and Koçkar (3) have solved mathematical models with Runge Kutta integration method and also linearized models were solved with matrisis. Comparisons have been done between calculated and experimental results

Alpbaz and Koçkar (4) have applied the Routh stability test to the same system which was under the effect of the feedback control. Suitable control parameters were calculated with this method.

MATHEMATICAL MODEL

The control work has been done when the glyserin was given to the system as an input. The related models were given in the work of Alpbaz, Erdoğan and Koçkar (3).

Energy balance for mixing tank,

Unsteady-state energy balance for tank and coolant,

$$Q+M_pC_pT^o_{pi} = M_pC_pT_{p0} + UA\left[T_{p0} - \left(\frac{T_{c0} + T^o_{ci}}{2}\right)\right] + M_vC_p\frac{dT_{p0}}{dt} \quad (1)$$

$$M_cC_cT^o_{ci} = M_cC_cT_{c0} - UA\left[T_{p0} - \left(\frac{T_{c0} + T^o_{ci}}{2}\right)\right] + M_jC_c\frac{dT_{c0}}{dt} \quad (2)$$

Steady-state energy balance.

$$Q + M^o_pC_pT^o_{pi} = M^o_pC_pT^o_{p0} + UA\left[T^o_{p0} - \left(\frac{T^o_{c0} + T^o_{ci}}{2}\right)\right] \quad (3)$$

$$M^o_cC_cT^o_{ci} = M^o_cC_cT^o_{c0} - UA\left[T^o_{p0} - \left(\frac{T^o_{c0} + T^o_{ci}}{2}\right)\right] \quad (4)$$

Linearized unsteady-state energy balance (1,2) with perturbation variables,

$$\frac{dT'_{p0}}{dt} = \left(\frac{T^o_{pi} - T^o_{p0}}{M_v}\right)M'_p - \left(\frac{UA + M_pC_p}{M_vC_p}\right)T'_{p0} + \left(\frac{UA}{2M_vC_p}\right)T'_{c0} \quad (5)$$

$$\frac{dT'_{c0}}{dt} = \left(\frac{T^o_{ci} - T^o_{c0}}{M_jC_c}\right)M_c' - \left(\frac{M_cC_c + \frac{UA}{2}}{M_jC_c}\right)T'_{c0} + \left(\frac{UA}{M_jC_c}\right)T'_{p0} \quad (6)$$

CONTROL SOLUTIONS

The linearized mathematical model having feedback control system were solved with Laplace transform and state variable approach. Two different feedback control systems were used in the present work Figs. 1,2. The control solution with Laplace transform has been shown below.

When the system was in the steady-state conditions shown in Table. 1. the step change was given to the feed flow rate ($M^o{}_p = 8.20$ g/sec, $M_p = 5.22$ g/sec) and than time response of the output temperature was calculated. The numerical values and Laplace transform of equations (5,6) were given below.

$$\bar{T}'_{c0} = -\left(\frac{3.292x10^{-3}}{s + 4.062x10^{-3}}\right) \bar{M}'_c + \left(\frac{1.218x10^{-3}}{s + 4.062x10^{-3}}\right) \bar{T}'_{p0} \quad (7)$$

$$\bar{T}'_{p0} = -\left(\frac{2.297x10^{-4}}{s + 4.392x10^{-4}}\right) \bar{M}'_p + \left(\frac{1.394x10^{-4}}{s + 4.392x10^{-4}}\right) \bar{T}'_{c0} \quad (8)$$

Using three term feedback controller, the control parameters were chosen according to the stability analysis (4).

$$\bar{M}'_c = \left[5 + 0.02\, s + 0.02 \frac{1}{s}\right] \bar{T}'_{p0} \quad (9)$$

$$\bar{T}'_{c0} = -\left(\frac{3.292x10^{-3}}{s + 4.062x10^{-3}}\right) \left[5 + 0.025 + 0.02 \frac{1}{s}\right] \bar{T}'_{p0} \quad (10)$$

$$+ \left(\frac{1.218x10^{-3}}{s + 4.062x10^{-3}}\right) \bar{T}'_{p0}$$

Step change in feed flow rate,

$$\bar{M}'_p = -\frac{2.98}{s} \quad (11)$$

and than taking inverse of the Laplace transform,

$$T'_{p0} = -1.276x10^{-3}\, e^{-4.09}\, e^{-3t} + 1.28x10^{-3}\, e^{-2.035}\, x10^{-4t} \quad (12)$$

$$\cos 1.492x10^{-3}\, t + 0.459\, e^{-2.035x10^{-4t}} \sin 1.492x10^{-3t}$$

For the analysis and design of linear control systems, they can be represented in state variable from as (3),

$$\dot{X}(t) = \underline{A}\, \underline{X}(t) + \underline{B}\, U(t) \quad (13)$$

$$U(t) = K\, [\, r(t) - \underline{k}\, \underline{X}(t)\,] \quad (14)$$

$$Y(t) = \underline{C}\, \underline{X}(t)$$

If related mathematical models (5,6) are described as state variable form,

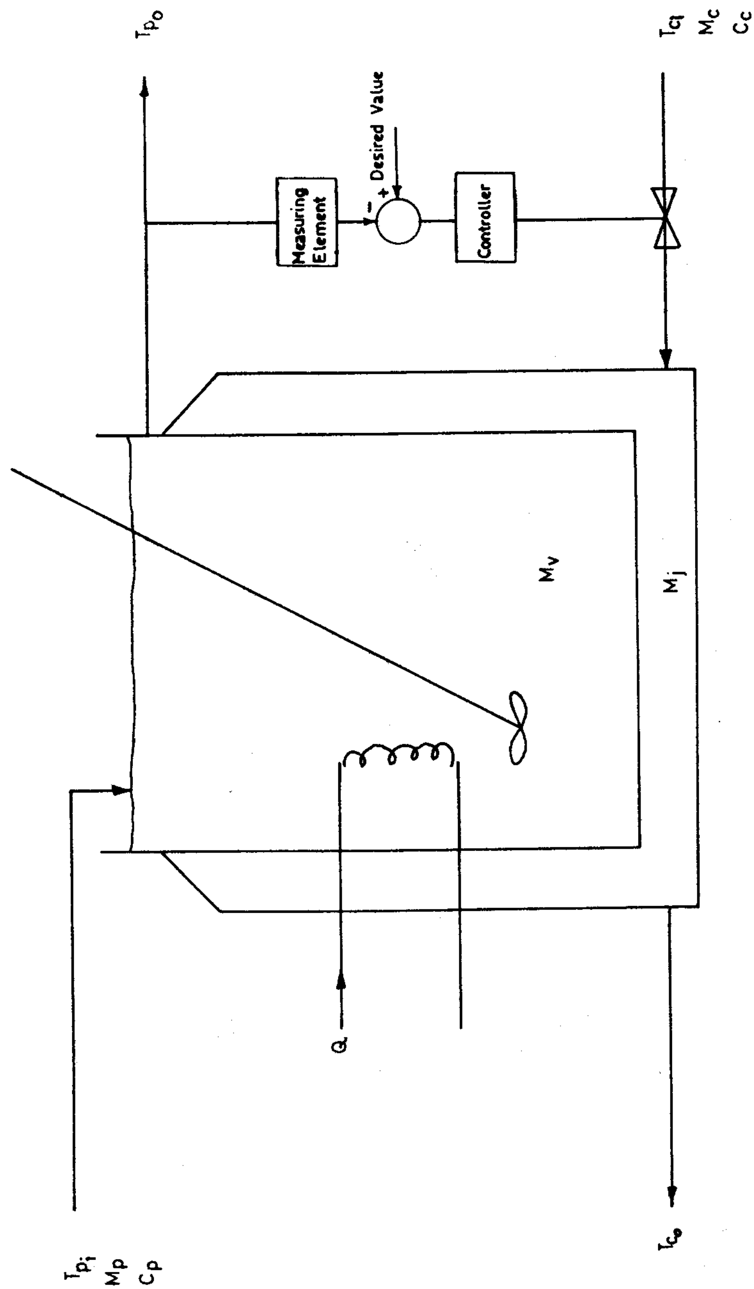

Figure.1. Feedback control of the tank for which step change was given to the feed flow rate

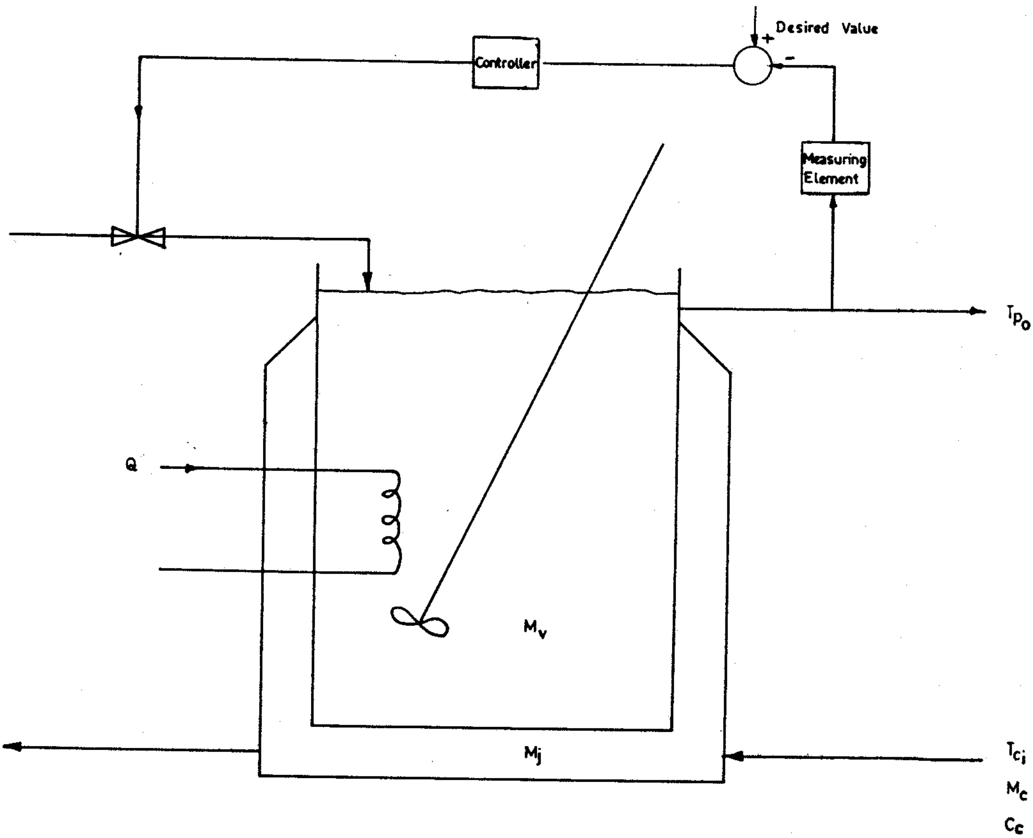

Figure. 2. Feedback control of the tank for which step change was given to the coolant flow rate.

Table. 1. The operating conditions and control parameters for feedback control of the tank

No.	Mp $\left(\frac{g}{sec}\right)$	Mc $\left(\frac{g}{sec}\right)$	Tpi (C°)	Tci (C°)	UA $\left(\frac{Cal}{sec\ °C}\right)$	Kc	Ki	Kd
1	8.20	17.0	54.0	17.0	6.00	-5	—	—
						-5	-0.02	—
	5.22	17.0	54.0	17.0	6.00	-5	-0.02	-0.02
2	8.51	17.0	54.0	17.0	6.06	-5	—	—
	8.61	89.6	54.0	17.0	60.6	-5	-0.02	—
						-5	-0.02	-0.02

$$\begin{bmatrix} \dot{T}'_{p0} \\ \dot{T}'_{c0} \end{bmatrix} = \begin{bmatrix} -\dfrac{UA+M_pC_p}{M_vC_p} & \dfrac{UA}{2M_vC_p} \\ \dfrac{UA}{M_jC_c} & -\dfrac{M_cC_c+\dfrac{UA}{2}}{M_jC_c} \end{bmatrix} \begin{bmatrix} T'_{p0} \\ T'_{c0} \end{bmatrix} + \begin{bmatrix} \dfrac{T^o_{pi}-T^o_{p0}}{M_v} & 0 \\ 0 & \dfrac{T^o_{ci}-T^o_{c0}}{M_jC_c} \end{bmatrix} \begin{bmatrix} M'_p \\ M'_c \end{bmatrix} \quad (16)$$

For which M_p was given to the glyserin flow rate as an step change,

$$\begin{bmatrix} \dot{T}'_{p0} \\ \dot{T}'_{c0} \\ \dot{M}'_p \end{bmatrix} = \begin{bmatrix} -\dfrac{UA+M_pC_p}{M_vC_p} & \dfrac{UA}{2M_vC_p} & \dfrac{T^o_{pi}-T^o_{p0}}{M_v} \\ \dfrac{UA}{M_jC_c} & -\dfrac{M_cC_c+\dfrac{UA}{2}}{M_jC_c} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{T^o_{ci}-T^o_{c0}}{M_jC_c} \\ 0 \end{bmatrix} M'_c \quad (17)$$

and for proportional control,

$$M'_c = K \left[r(t) - [1\ 0\ 0] \begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \end{bmatrix}\right] \tag{18}$$

$$\underline{Y}(t) = [1\ 1\ 1] \begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \end{bmatrix} \tag{19}$$

With introducing three term feedback control system, the equation (18) become,

$$M'_c = -K \left[T'_{p0} + \frac{K_d}{K} \cdot T'_{p0} + \frac{K_i}{K} T'_i\right] \tag{20}$$

$$T'_c = \int_0^t T'_{p0}\, dt \tag{21}$$

$$T'_c = T'_{p0} \tag{22}$$

and than,

$$M'_c = -K \left[T'_{p0} + \frac{K_d}{K} \left[\left(\frac{T^o_{pi} - T^o_{p0}}{M_v}\right) M'_p - \left(\frac{UA + M_pC_p}{M_vC_p}\right) T'_{p0} + \left(\frac{UA}{2M_vC_p}\right) T'_{c0}\right] + \frac{K_i}{K} T'_i\right] \tag{23}$$

$$M'_c = -K \left[\left(1 - \frac{UA + M_pC_p}{M_vC_p} \frac{K_d}{K}\right) \left(\frac{UA}{2M_vC_p} \frac{K_d}{K}\right) \left(\frac{T^o_{pi} - T^o_{p0}}{M_v} \frac{K_d}{K}\right) \left(\frac{K_i}{K}\right)\right] \begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \\ T'_i \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} \dot{T}'_{p0} \\ \dot{T}'_{c0} \\ \dot{M}'_{p} \\ \dot{T}'_{i} \end{bmatrix} = \begin{bmatrix} -\dfrac{UA+M_pC_p}{M_vC_p} & \dfrac{UA}{2M_vC_p} & \dfrac{T^o_{pi}-T^o_{p0}}{M_v} & 0 \\ \dfrac{UA}{M_jC_c} & -\dfrac{M_cC_c+\dfrac{UA}{2}}{M_jC_c} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_{p} \\ T'_{i} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{T^o_{ci}-T^o_{c0}}{M_jC_c} \\ 0 \\ 0 \end{bmatrix} M'_c \quad (25)$$

$$M'_c = -K\left[1\ 0\ 0\ \frac{K_i}{K}\right]\begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \\ T'_i \end{bmatrix} \tag{26}$$

If the numerical values of the state form are written with the same operating conditions given in Table. 1.

$$\begin{bmatrix} \dot{T}'_{p0} \\ \dot{T}'_{c0} \\ \dot{M}'_p \end{bmatrix} = \begin{bmatrix} -0.000439 & 0.0001394 & -0.000229 \\ 0.0012188 & -0.004062 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}\begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \end{bmatrix} + \begin{bmatrix} 0.0 \\ -0.00329 \\ 0 \end{bmatrix} M'_c \tag{27}$$

Introducing T'_i,

$$\begin{bmatrix} \dot{T}'_{p0} \\ \dot{T}'_{c0} \\ \dot{M}'_p \\ \dot{T}'_i \end{bmatrix} = \begin{bmatrix} -0.000439 & 0.0001394 & -0.000229 & 0.0 \\ 0.0012188 & -0.004062 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}\begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \\ T'_i \end{bmatrix} + \begin{bmatrix} 0.0 \\ -0.00329 \\ 0.0 \\ 0.0 \end{bmatrix} M'_c \tag{28}$$

and also,

$$k = [1.0 \quad 0.0 \quad 0.0 \quad 0.004] \tag{29}$$

With three term feedback controller,

$$M'_c = -K\left[\left(1-0.0004392\ \frac{K_d}{K}\right)\ \left(0.0001394\ \frac{K_d}{K}\right)\ \left(0.0002297\ \frac{K_d}{K}\right)\ \left(\frac{K_i}{K}\right)\right]\begin{bmatrix} T'_{p0} \\ T'_{c0} \\ M'_p \\ T'_i \end{bmatrix} \tag{30}$$

The Rational Time Response (RTRESP) program was used to solve the equation (5,6) describing the mathematical models and control of the mixing tank and calculated the time response of the output temperature. In this program r (t) shows the change of the set point or desired value. In the present work there wasn't set point change and than r (t) = 0.0 was taken, (5).

Also for similar purposes, calculation has been done with 4th order Runge Kutta integration method for solving the nonlinear differantial equations. Feedback, measurement and valve subrotines were introduced to the main digital computer program.

RESULTS

In the work of Alpbaz, Erdoğan, and Koçkar (3), the effect of the step change given to the feed flow rate of the continuous flow agitated tank to the output temperature was investigated. In the present work the feedback control system was introduced to the same tank for control of the output temperature. The time response of the output and coolant temperature were calculated with Laplace Transform, Runge Kutta integration method and multivariable approaches. The stability analysis for this system has been done in the work of Alpbaz and Koçkar (4) and than suitable control parameters have been calculated with Routh stability analysis

In the first part of the work, when the system was in the steady-state having input and operating conditions shown in Table. 1. the step change was given to the feed flow rate and than feedback controler dedected the output temperature and adjusted cooling flow rate. The output temperature could come to the first steady-state value. The time response of the output variables were calculated with three different methods as described before. Related diagrams are shown in Figs. 3–5. with different control parameters.

In the second part of the work, when the system was in the same steady-state condition, step change was given to the cooling flow rate and feedback controller dedected and controlled the output temperature with adjusting feed flow rate. Similar calculations have been done and related diagrams are shown in Figs. 6–8.

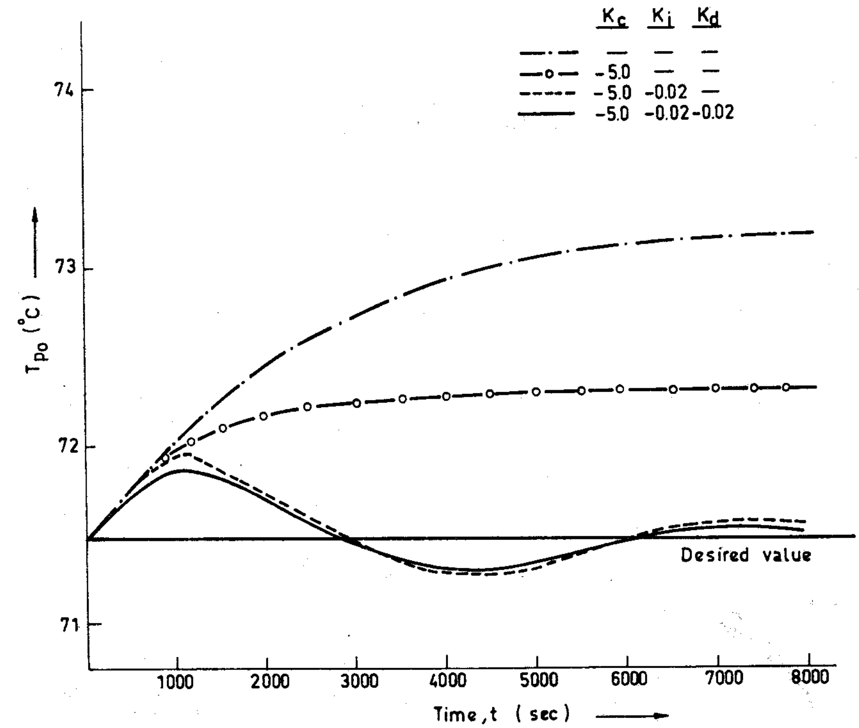

Figure. 3. The feedback control of the output temperature calculated with Runge Kutta integration method (M^o_p = 8.20 g/sec, Mp = 5.22 g/sec)

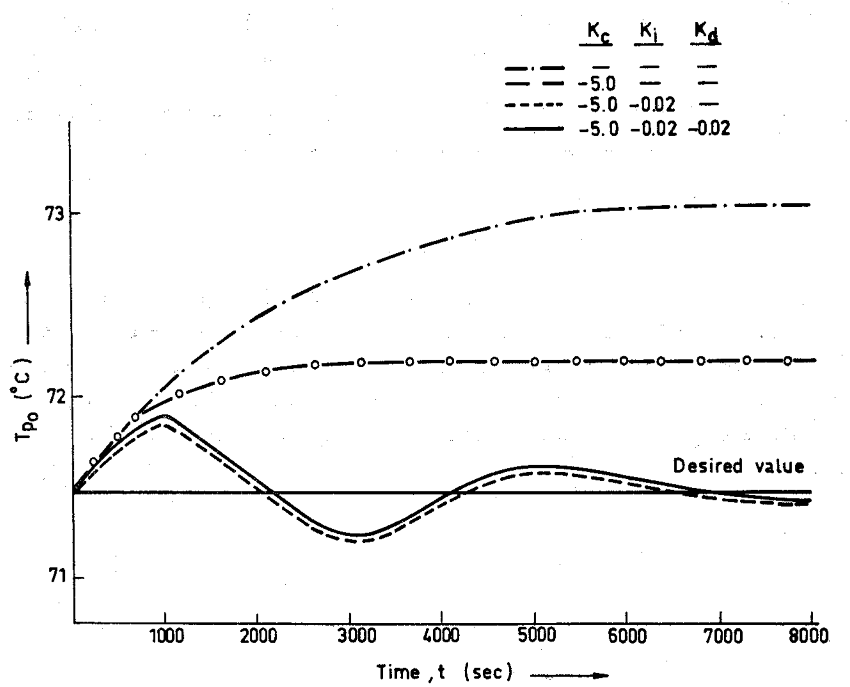

Figure. 4. The feedback control of the output temperature calculated with Laplace transfrom (M^o_p = 8.20 g/sec, Mp = 5.22 g/sec)

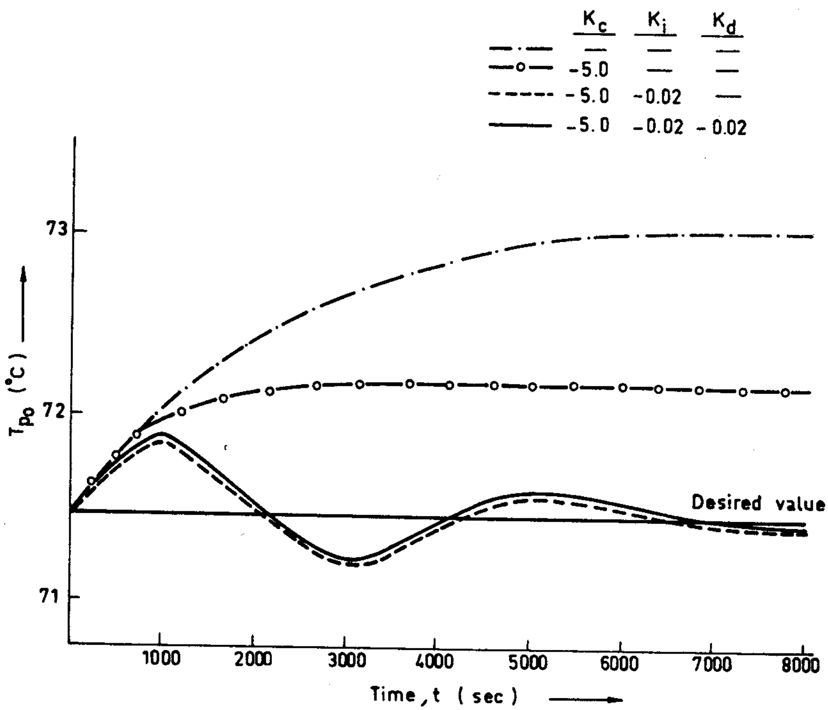

Figure. 5. The feedback control of the output temperature calculated with multivariable approaches (M^o_p = 8.20 g/sec, Mp = 5.22 g/sec)

CONCLUSION

Comparisons have been done between results obtained from the solution of linear and nonlinear models. This comparisons show some deviation. While the results taken from Laplace transformation and multivariable approaches give similar trends, the solution of nolinear equations with Runge Kutta integration method gives some deviation from others.

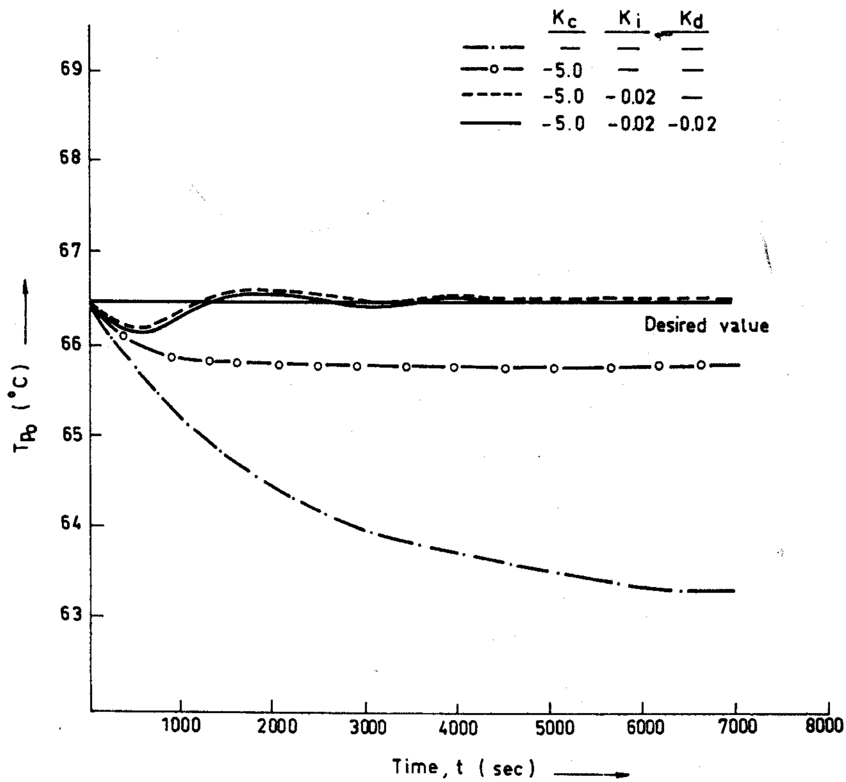

Figure. 6. The feedback control of the output temperature calculated with Runge Kutta interration method (M^o_c = 17.0 g/sec, Mc = 89.6 g/sec)

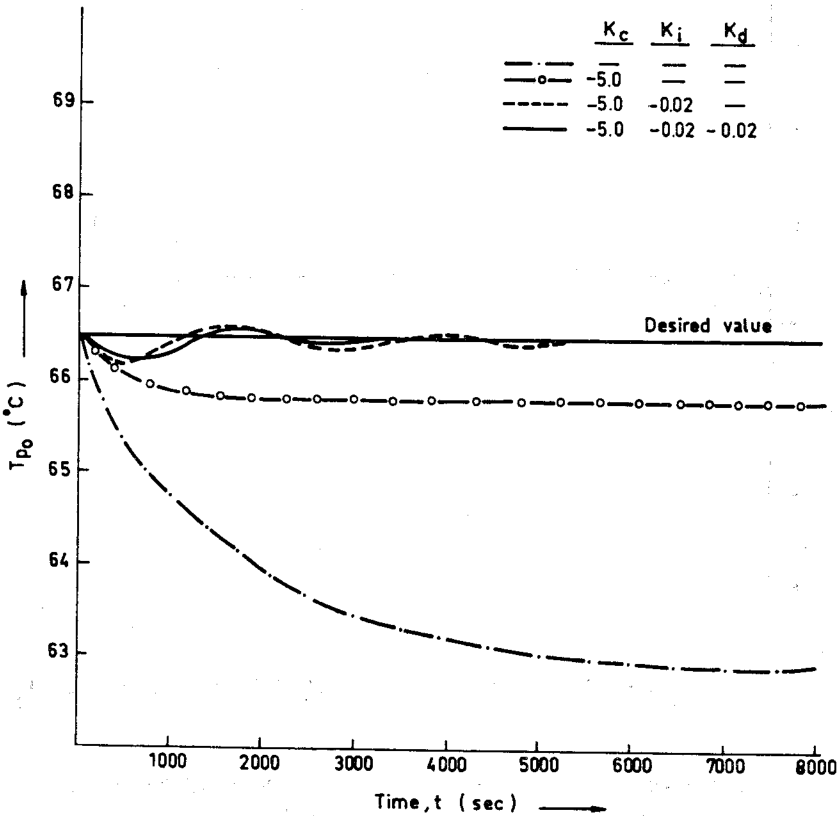

Figure. 7. The feedback control of the output temparature calculated with Laplace transform (M^o_c = 17.0 g/sec, Mc = 89.6 g/sec)

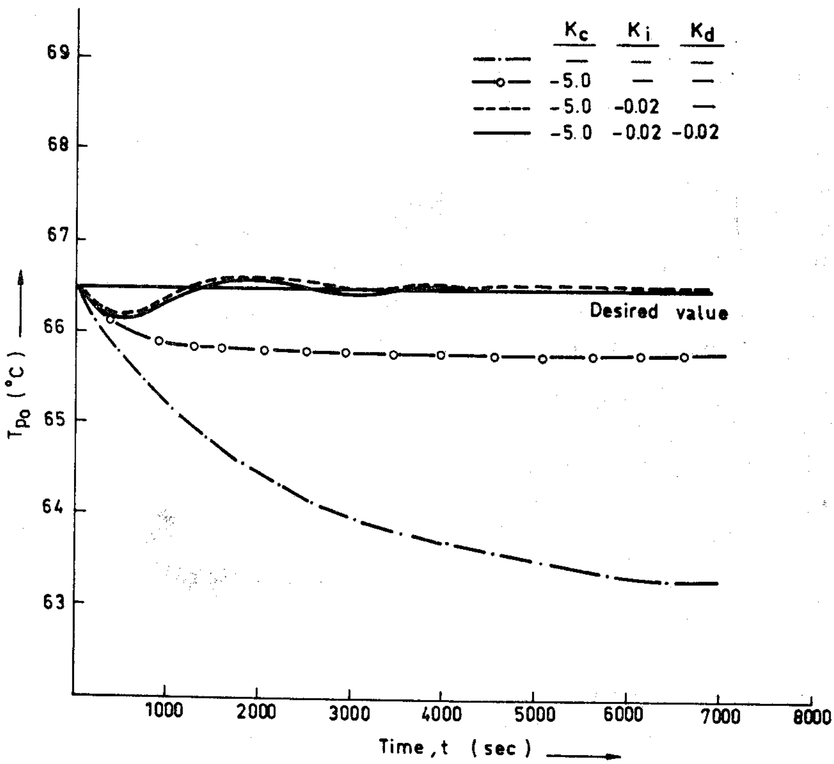

Figure. 8. The feedback control of the output temperature calculated with multivariable approaches(M°_{p} = 17.0 g/sec, Mc = 89.6 g/sec)

NOMENCLATURE

A Heat transfer surface (cm^2)

$\underline{A}$ Plant matrix

$\underline{B}$ Control vector

$\underline{C}$ Output vector

C_c Specific heat of coolant (cal/g °C)

C_p Specific heat of tank content (cal/g °C)

K	The controller gain
K_c	Proportionol acting factor
K_d	Derivative action factor
K_i	Integral acting factor
k	Feedback coefficient
M_c	Coolant mass flow rate (g/sec)
M_p	Feed mass flow rate (g/sec)
M_j	Mass hold up in the jacked (g)
M_v	Mass hold up in the tank (g)
r (t)	Input function
s	Laplace operature
T_{pi}	Feed temperature (°C)
T_{p0}	Output temperature (°C)
T_{c0}	Output coolant temperature (°C)
t	Time
U	Overall heat transfer coefficient (cal/cm^2 sec °C), Controller output
γ	System output
δ	Density (g/cm^3)
μ	Viscosity (g/cm sec)
x	System state variable

ÖZET

Kontrol çalışmaları için, dışardan ceketle soğutulan tam karıştırmalı akım tankına üç terimli geri beslemeli kontrol sistemi ilave edilmiştir. Tankın ölçülen çıkış sıcaklığının istenen değere yaklaşımı doğrusallaştırılan modeller yardımı ile sayısal bilgisayarda matris kullanımı ve Laplace dönüşümü ile hesaplanmış, ayrıca modeller aynı gaye için doğrusallaştırılmadan sayısal bilgisayarda çözülmüştür.

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