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by

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Faculté des Sciences de l'Université d'Ankara
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# Experimental And Theoretical Work On The Dynamic Characteristics Of A Continuous - Flow Agitated Tank Cooled By Jacked (I)

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**SUMMARY**

In this work, the dynamic characteristics of a continuous-flow agitated tank, showing a lump-parameter system proporties were studied both experimentally and theoretically.

In this research, the mathematical models of a continuous-flow agitated tank cooled by jacked were developed. This developed models were linearised and then solved with Laplace transform. The dynamic proporties of the continuous-flow agitated tank were investigated with the step change which was given to the input variables. In the first part of this research, for the case in which water was fed to the agitated tank, the dynamic proporties of the tank were investigated and also the best operating conditions for the experimental work were found.

## INTRODUCTION

Alpbaz and Erdoğan (1) have investigated the dynamic properties of a well stirred tank cooled by jacked experimentally. They also have developed the mathematical models for this vessel and solved linearised models with the aid of Laplace Transform.

Özdemir (2) has presented a set of analytical solutions for a well stirred tank cooled by jacked dynamics and control. He performed to provide guidance in the analysis of the results obtained from the computer tests.

## MATHEMATICAL MODEL

The unsteady-state energy balance for inside of the tank,

$$Q + M_b C_{pb} T_{bg}^{o} = M_b C_{pb} T_{bç} + UA \left( T_{bç} - \frac{T_{sg}^{o} + T_{sç}}{2} \right) + M_T C_{pb} \frac{dT_{bç}}{dt} \qquad (1)$$

The unsteady-state energy balance for coolant,

$$M_s C_{ps} T_{sg}^{o} = M_s C_{ps} T_{sç}^{o} - UA \left( T_{bç} - \frac{T_{sg}^{o} + T_{sç}}{2} \right) + M_c C_{pc} \frac{dT_{sç}}{dt} \qquad (2)$$

The steady-state energy balance for tank and coolant,

$$Q + M_b^{o} C_{pb} T_{bg}^{o} = M_b^{o} C_{pb} T_{bç}^{o} + UA \left( T_{bç}^{o} - \frac{T_{sg}^{o} + T_{sç}^{o}}{2} \right) \qquad (3)$$

$$M_s^{o} C_{ps} T_{sg}^{o} = M_s^{o} C_{ps} T_{sç}^{o} - UA \left( T_{bç}^{o} - \frac{T_{sg}^{o} + T_{sç}^{o}}{2} \right) \qquad (4)$$

The related continuous-flow agitated tank cooled by jacked having water input is shown in Fig. 1.

Assumptions for developing the mathematical models were shown below,

1– The value of heat transfer coefficient is constant during the transient time

2– The physical properties for the tank content are constant

3– While there was perfect mixing in the tank and jacked, there was not temperature and concentration profile

4– There was negligible heat loss to the surroundings and the vessel shell

## THE ANALITICAL SOLUTION OF MATHEMATICAL MODEL

The Laplace transform was used to solve mathematical model.

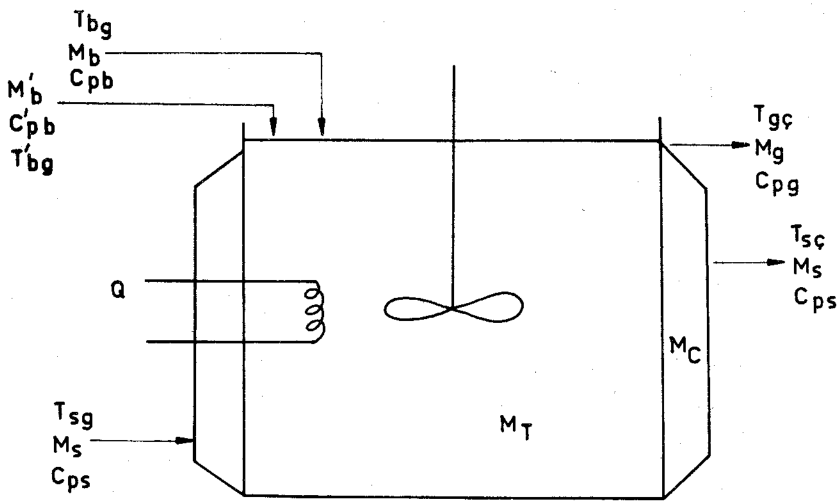


Fig. 1. A continuous-flow agitated tank cooled by jacked having water input.

For this purpose, steady-state energy balance (3) was substracted from the unsteady-state balance (1)

$$(M_b - M_b^o)\ C_{pb}T_{bg}^o = M_bC_{pb}T_{bç} - M_b^oC_{pb}T_{bç} + UA\ (T_{bç} - T_{bç}^o) - UA\left(\frac{T_{sç} - T_{sç}^o}{2}\right) + M_TC_{pb}\frac{dT_{bç}}{dt} \qquad (5)$$

Since the equation (5) has term, $M_bT_{bç}$, it becomes non-linear. This related term, $M_bT_{bç}$, was linearized with Taylor theorem. Suitable results coult not be obtained for the solution of differential equations with the Laplace transform using Taylor linearization theorem. For this reason, different mathematical manipulation was applied to the equation (5). If the term, $M_bC_{pb}T_{bç}^o$, was added to the both side of the equation (5),

$$(M_b - M_b^o)\ C_{pb}T_{bg}^o = M_bC_{pb}T_{bç} - M_b^oC_{pb}T_{bç}^o + UA\ (T_{bç} - T_{bç}^o) - UA\left(\frac{T_{sç}-T_{sç}^o}{2}\right) + M_TC_{pb}\frac{dT_{bç}}{dt} + M_bC_{pb}T_{bç}^o - M_bC_{pb}\ T_{bç}^o \qquad (6)$$

Defining perturbation variables and arranging.

$\overline{M}_b = M_b - M_b^o$, $\overline{T}_{bç} = T_{bç} - T_{bç}^o$, $\overline{T}_{sç} = T_{sç} - T_{sç}^o$ and becomes,

$$\frac{d\overline{T}_{bç}}{dt} = \left(\frac{T_{bg}^o - T_{bç}^o}{2}\right) \overline{M}_b - \left(\frac{M_b C_{pb} + UA}{M_T C_{pb}}\right) \overline{T}_{bç} + \left(\frac{UA}{2 M_T C_{pb}}\right) \overline{T}_{sç} \quad (7)$$

$$\frac{d\overline{T}_{bç}}{dt} = K_1 \overline{M}_b - K_2 \overline{T}_{bç} + K_3 \overline{T}_{sç} \quad (8)$$

The similar procedure was repeated for coolant energy balance, When equation (2) was substracted from equation (4),

$$(M_s - M_s^o)\, C_{ps} T_{sg}^o = M_s C_{ps} T_{sç} - M_s^o C_{ps} T_{sç}^o - UA\,(T_{bç} - T_{bç}^o) + UA\left(\frac{T_{sç} + T_{sg}^o}{2}\right) - UA\left(\frac{T_{sç}^o + T_{sg}^o}{2}\right) + M_C C_{ps} \frac{dT_{sç}}{dt} \quad (9)$$

If the term, $M_s C_{ps} T_{sç}^o$, was added to the both side of equation (9)

$$(M_s - M_s^o)\, C_{ps} T_{sg}^o = M_s C_{ps} T_{sç} - M_s^o C_{ps} T_{sç} - UA\,(T_{bç} - T_{bç}^o) + \frac{UA}{2}(T_{sç} - T_{sç}^o) + M_C C_{ps} \frac{dT_{sç}}{dt} + M_s C_{ps} T_{sç}^o - M_s C_{ps} T_{sç}^o \quad (10)$$

Arranging,

$$(M_s - M_s^o)\, C_{ps} T_{sg}^o = M_s C_{ps}\,(T_{sç} - T^o_{sç}) + T_{sç}^o C_{ps}\,(M_s - M_s^o) - UA\,(T_{bç} - T_{bç}^o) + \frac{UA}{2}(T_{sç} - T_{sç}^o) + M_C C_{ps} \frac{dT_{sç}}{dt} \quad (11)$$

With perturbation variables,

$\overline{M}_s = M_s - M_s^o$, $\overline{T}_s = T_{sç} - T_{sç}^o$, $\overline{T}_{bç} = T_{bç} - T_{bç}^o$

$$\frac{d\overline{T}_{sç}}{dt} = \left(\frac{T_{sg}^o - T_{sç}^o}{M_C}\right) \overline{M}_s - \left(\frac{M_s C_{ps} + \frac{UA}{2}}{M_C C_{ps}}\right) \overline{T}_{sç} + \frac{UA}{M_C C_{ps}} \overline{T}_{bç} \quad (12)$$

$$\frac{d\overline{T}_{sç}}{dt} = K_4\overline{M}_s - K_5\overline{T}_{sç} + K_6\overline{T}_{bç} \tag{13}$$

Laplace transform of the equation (8) was evaluated and than putting in order, it becomes,

$$\overline{T}_{bç}(s) = \frac{K_1}{s + K_2}\,\overline{M}_b(s) + \frac{K_3}{s + K_2}\,\overline{T}_{sç}(s) \tag{14}$$

If similar procedure is repeated for equation (13) and Laplace transform of this equation is become,

$$\overline{T}_{sç}(s) = \frac{K_4}{s + K_5}\,\overline{M}_s(s) + \frac{K_6}{s + K_5}\,\overline{T}_{bç}(s) \tag{15}$$

For obtaining $\overline{T}_{bç}$, equation (15) is put into equation (14) and arrainging,

$$\overline{T}_{bç}(s) = \left[\frac{K_1(s + K_5)}{s^2 + (K_2 + K_5)s + (K_2K_5 - K_3K_6)}\right]\overline{M}_b(s) + \left[\frac{K_3K_4}{s^2 + (K_2 + K_5)s + (K_2K_5 - K_3K_6)}\right]\overline{M}_s(s)$$

If the step change was given to the feed and coolant flow rate

$$\overline{M}_b(s) = \frac{A}{s}$$

$$\overline{M}_s(s) = \frac{B}{s} \tag{16}$$

Equation (16) was put into equation (15) and than taking inverse Laplace transform, the solution of $\overline{T}_{sb}$ can be found,

With similar procedure, the solution of $\overline{T}_{sç}$ can be obtained,

$$\overline{T}_{sç}(s) = \left[\frac{K_4(s + K_2)}{s^2 + (K_2 + K_5)s + (K_2K_5 - K_3K_6)}\right]\overline{M}_s(s) + \left[\frac{K_1K_6}{s^2 + (K_2 + K_5)s + (K_2K_5 - K_3K_6)}\right]\overline{M}_b(s) \tag{17}$$

The numerical solution was obtained for output and coolant temperature with spesific conditions taken from experimental work.

## DESCRIPTION OF EQUIPMENT AND EXPERIMENTAL WORK

The results obtained from the linearized mathematical model solved by Laplace transform were compared with the experimental data of the continuous-flow agitated tank cooled by jacked. For this purpose, a transient response of the tank output and coolent temperatures to a step change given to the feed flow rate was investigated.

i- Desing of agitated tank cooled by jacked,

The related apparatus for this work were shown below,

Agitated tank

The agitated tank has a volume of 0.0265 $m^2$, inside diameter of 30 cm and outside diameter of 31 cm. The material used to construct the tank is iron having a thickness of 0.4 cm. Out side of the vessel was constructed with jacked for cooling purpose. Jacked has a outside diameter of 34 cm and height of 31 cm. Also jacked was isolated to protect from heat losses.

The type of flow in an agitated vessel depends on the type of impeller, the characteristics of the fluid, the size of the tank and agitator. The agitator with propeller used in the vessel mounted centrally and it was made of steel. The agitator turns at speed between 45–2000 rpm. The diameter of propeller is 8.9 cm.

The tank was heated with 30 Ω cupper heater resistance adjusted with variac. The plant components used in this work is shown in Fig. 2.

ii- Experimental Procedure.

The experimental procedure used in this work is summarized below. Initially the vessel was full with water and than it was heated with cupper heater to rise the average tank temperature. Beside of them, inlet water was agitated in the speed of 300 rpm to get the homogen temperature distribution. After the temperature of the tank reached to a defined constant value, a feed and coolent having defined temperatures and flow

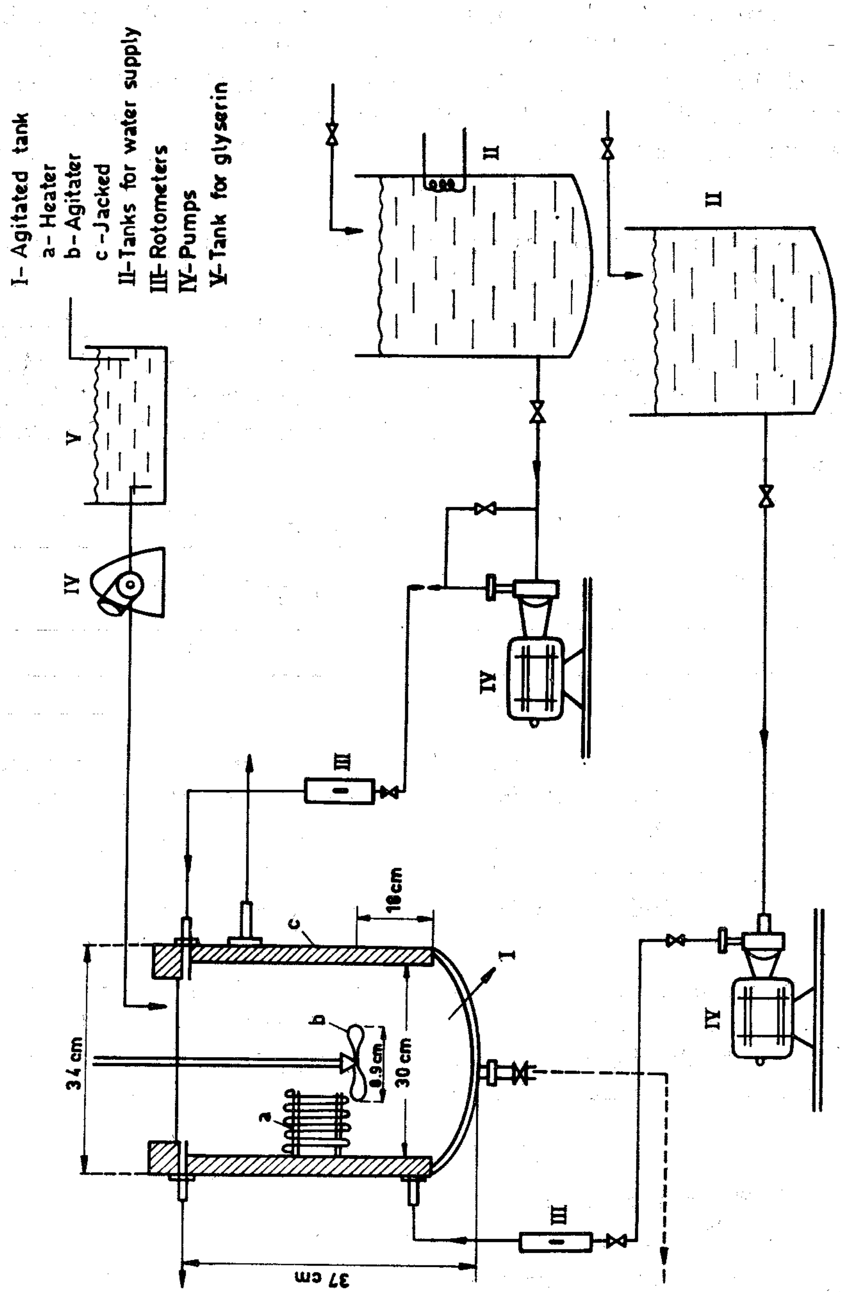


Fig. 2. Overal system showing continuous-flow agitated tank cooled by jacked and related apparatus.

rates were introduced into the tank. The tank was allowed to reach the first steady-state. When the temperatures of the tank and coolant reached the constant values, the tank was considered in the first steady-state. A step change in feed was than introduced and temperature measurements were taken during the transient period and the attainment of a new steady-state conditions.

## COMPARISON OF THE EXPERIMENTAL RESPONSE WITH THE THEORETICAL RESULT

When the vessel came to the steady-state, related input conditions, the values of output and coolant temperatures are shown in Table 1. When the system was in the steady-state condition given in Table 1. the step changes with different values were given to the feed and coolant flow rate.

Table 1. Steady-state and input conditions

| $M_b^o$(g/sec) | $M_s^o$(g/sec) | $T_{bg}^o$(°C) | $T_{sg}^o$(°C) | UA $\left(\frac{\text{Cal}}{°\text{C sec}}\right)$ | $M_b$(g/sn) | $M_s$(g/sec) |
|---|---|---|---|---|---|---|
| 16.25 | 17.0 | 56.5 | 32 | 15.98 | 29.75 | 17.0 |
| 16.25 | 65.0 | 56.5 | 13 | 16.46 | 16.25 | 138.5 |

In Figs. 3, 4., the experimental temperature response are compared with the calculated results from the solution of the mathematical model of the vessel.

Some discrepancies can be seen between the experimental data and the theoretical results both at the output and cooland temperatures.

It is concluded that the discrepancies came from the heat losess from the vessel and linearization of the mathematical models. Despide of them, the developed mathematical model represent the dynamics of a well agitated tank cooled by jacked.

## NOMENCLATURE

A Heat transfer surface ($cm^2$)

$C_{pb}$ Specific heat of tank content (cal / g°C)

$C_{ps}$ Spesific heat of coolant (cal / g°C)

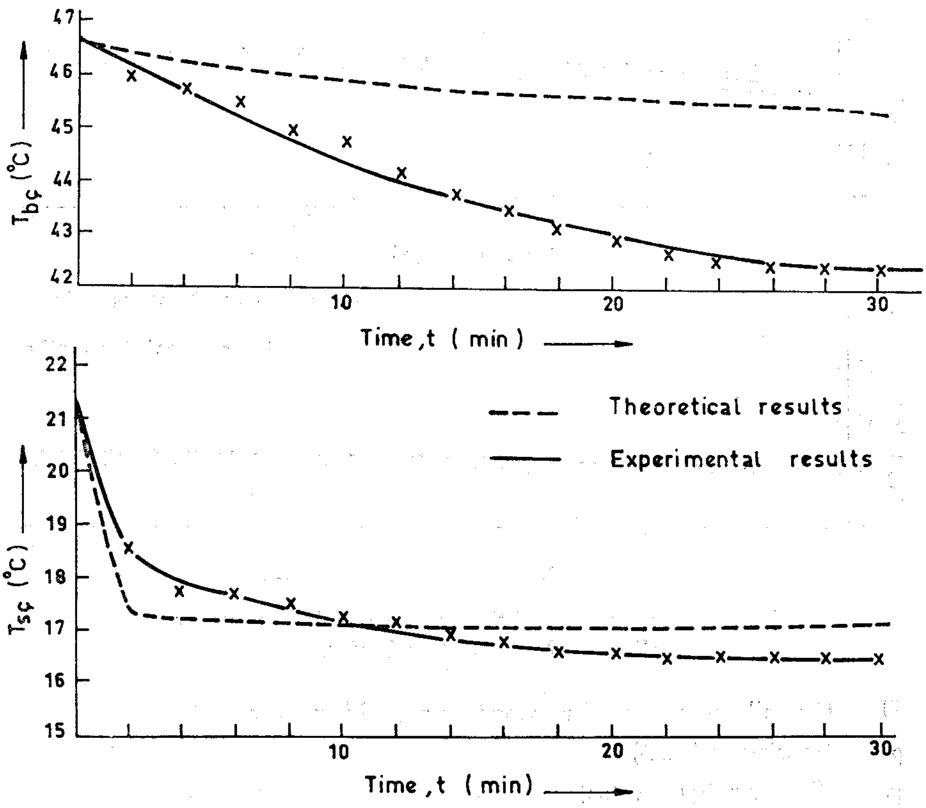


Fig. 3. The time response of the output and coolant temperatures. ($M_s^6$ = 65 g/sec, Ms = 138.5 g/sec)

| | |
|---|---|
| $M_P$ | Mass flow rate of feed (g / sec) |
| $M_C$ | Mass hold up in the cooling jacked (g) |
| $M_S$ | Mass flow rate of coolant (g / sec) |
| $M_T$ | Mass hold up in the tank (g) |
| s | Laplace operator |
| Q | Heat output from immertion heaters (cal / sec) |
| $T_{bç}$ | Output temperature (°C) |
| $T_{sç}$ | Feed temperature (°C) |
| $T_{sç}$ | Coolant output temperature (°C) |
| t | Time |

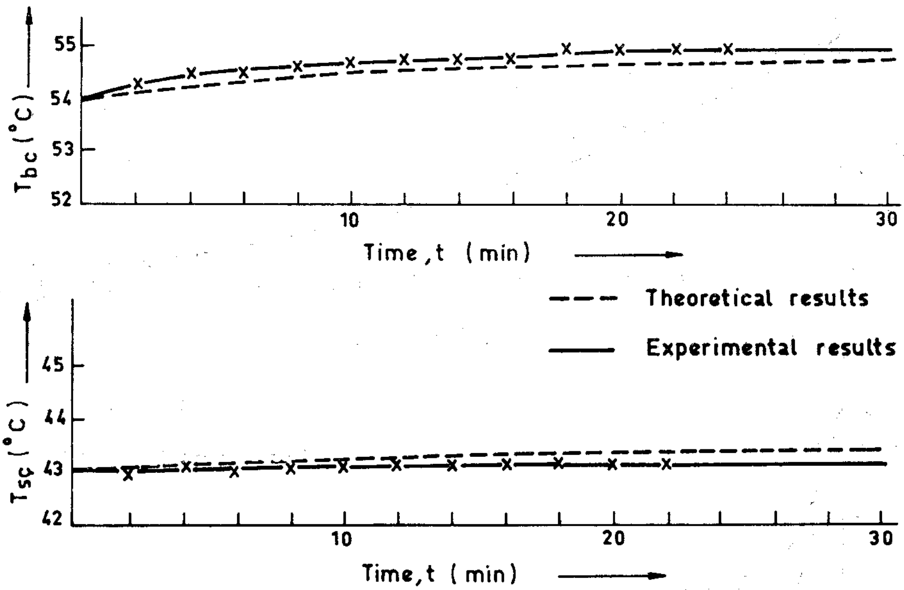


Fig. 4. The time response of the output and coolant temperatures.
($M_s^o$ = 16.25 g / sec, $M_s$ = 29.75 g / sec)

U Overall heat transfer coefficient (cal / cm sec °C)
ρ Density (g / cm²)
μ Viscosity (g / cm sec)

## REFERENCES

1. Erdoğan (Yüceer), S. Ph. D Thesis, Ankara University (1984)
2. Özdemir, E. M.Sc. Thesis, Aston University England (1978).

## ÖZET

Bu çalışmada, bir kademeli-parametreli sistem özelliği gösteren tam karşıtırmalı akım tankının dinamik özellikleri, teorik ve deneysel olarak araştırılmıştır.

Bu araştırmada, dışardan ceketle soğutulan tam karıştırmalı akım tankı için, matematiksel model geliştirilmiştir. Geliştirilen modeller doğrusallaştırılarak Laplace dönüşümü ile çözülmüştür.

Tam karıştırmalı akım tankının dinamik özellikleri, sistemin giriş değişkenlerine kademe değişimi verilmesi ile incelenmiştir. Yapılan çalışmaların birinci kısmında, suyun besleme olarak girdiği hal için dinamik özellikler incelenmiş ve karıştırma tankı için en uygun şartlar saptanmıştır.