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by

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**TURQUIE**

## **Experimental And Theoretical Work On The Dynamic Characteristics Of A Continuous-Flow Agitated Tank Cooled By Jacket (II)**

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### **SUMMARY**

The dynamic properties of the continuous-flow agitated tank were investigated with the step change which was given to the input variables. In this part of the research, different concentration of the glycerin-water solutions were used as feed and the dynamic properties of the tank in these conditions were investigated. Theoretical results obtained from solution of related mathematical models with Laplace transform were compared with the experimental data.

### **INTRODUCTION**

Alpbaz and Erdoğan (1,2), have developed mathematical model for dynamic characteristics of a continuous-flow agitated tank. Firstly they have linearized this developed models and then solved with Laplace transform. Theoretical results were compared with the experimental data.

### **MATHEMATICAL MODEL**

The unsteady-state energy balance for inside of the tank having water and glycerin inputs.

$$Q + M_b C_{pb} T_{bg}^o + M'_b C_{pb}' T_{bg}'^o = (M_b + M_b') C_{pg} T_{gc} \\ + UA \left( T_{gc} - \frac{T_{sg}^o + T_{sc}}{2} \right) + M_T C_{pg} \frac{dT_{gc}}{dt} \quad (1)$$

The unsteady-state energy balance for coolant (1)

$$M_s C_{ps} T_{sg}^o = M_s C_{ps} T_{sc} - UA \left( T_{bg} - \frac{T_{sg}^o + T_{sc}}{2} \right) + M_c C_{pc} \frac{dT_{sc}}{dt} \quad (2)$$

The steady-state energy balance for tank and coolant,  
 $Q + M_b' C_{pb} T_{bg}^o + M_b' C_{pb} T_{bg}^o = (M_b^o + M_b'') C_{pg} T_{gc}^o$

$$+ UA \left( T_{gc}^o - \frac{T_{sg}^o + T_{sc}^o}{2} \right) \quad (3)$$

$$M_s' C_{ps} T_{sg}^o = M_s' C_{ps} T_{sc}^o - UA \left( T_{bg}^o - \frac{T_{sg}^o + T_{sc}^o}{2} \right) \quad (4)$$

The continuous flow-agitated tank cooled by jacket having water and glycerin inputs is shown in Fig. 1.

Necessary assumptions for developing this models have been given in the previous work of Alpbaz and Erdoğan (1)

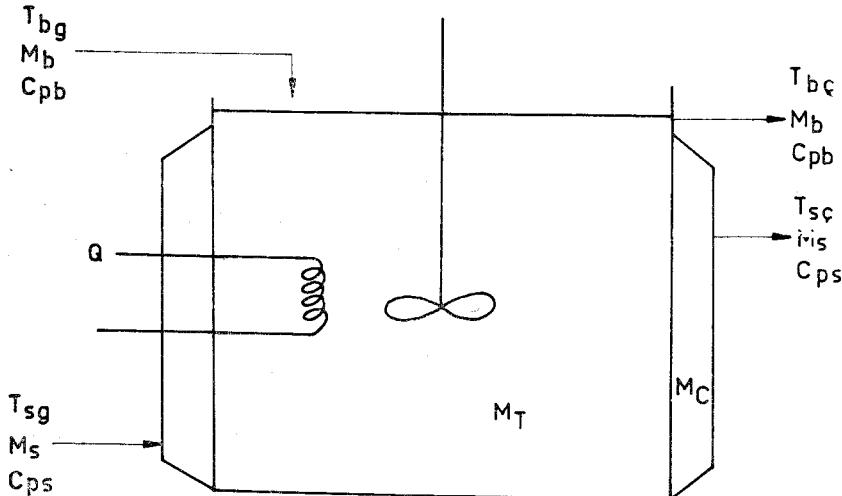


Fig. 1. The continuous flow-agitated tank cooled by jacket having water and glycerin inputs

Similar procedure and calculation can be done as before and by defining perturbation variables, (1)

$$\frac{d\bar{T}_{gc}}{dt} = \left( \frac{C_{pb}T_{bg}^o - C_{pg}T_{gc}^o}{M_T C_{pg}} \right) \bar{M}_b + \left( \frac{C_{pb}'T_{bg}'^o - C_{pg}T_{gc}^o}{M_T C_{pg}} \right) \bar{M}'_b - \left( \frac{M_g C_{pg} + UA}{M_T C_{pg}} \right) \bar{T}_{gc} + \frac{UA}{2M_T C_{pg}} \bar{T}_{sc}$$

and than

$$\frac{d\bar{T}_{gc}}{dt} = K_7 \bar{M}_b + K_8 \bar{M}'_b - K_9 \bar{T}_g + K_{10} \bar{T}_{sc} \quad (6)$$

For coolant energy balance (1),

$$\begin{aligned} \frac{d\bar{T}_{sc}}{dt} &= \left( \frac{T_{sg}^o - T_{sc}^o}{M_C} \right) \bar{M}_s - \left( \frac{M_s C_{ps} + \frac{UA}{2}}{M_C C_{ps}} \right) \bar{T}_{sc} \\ &+ \frac{UA}{M_C C_{ps}} \bar{T}_{bc} \end{aligned} \quad (7)$$

$$\frac{d\bar{T}_{sc}}{dt} = K_4 \bar{M}_s - K_5 \bar{T}_{sc} + K_6 \bar{T}_{bc} \quad (8)$$

The Laplace transform of the equations (6,7) were evaluated and than putting in order they become,

$$\bar{T}_{gc}(s) = \frac{K_7}{s + K_9} \bar{M}_b(s) + \frac{K_8}{s + K_9} \bar{M}'_b(s) + \frac{K_{10}}{s + K_9} \bar{T}_{sc}(s) \quad (9)$$

$$\bar{T}_{sc}(s) = \frac{K_4}{s + K_5} \bar{M}_s(s) + \frac{K_6}{s + K_5} \bar{T}_{bc}(s) \quad (10)$$

For obtaining  $\bar{T}_{bc}$ , equation (10) is put into equation (9) and arranging,

$$\begin{aligned} \bar{T}_{gc}(s) &= \frac{K_7(s + K_5)}{s^2 + (K_5 + K_9)s + (K_5 K_9 - K_6 K_{10})} \bar{M}_b(s) \\ &+ \frac{K_8(s + K_5)}{s^2 + (K_5 + K_9)s + (K_5 K_9 - K_6 K_{10})} \bar{M}'_b(s) \end{aligned} \quad (11)$$

$$+ \frac{K_4 K_{10}}{s^2 + (K_5 + K_9)s + (K_5 K_9 - K_6 K_{10})} \bar{M}_s(s)$$

For coolant temperature,  $\bar{T}_{sc}$

$$\begin{aligned} \bar{T}_{sc}(s) &= \frac{K_4(s + K_9)}{s^2 + (K_5 + K_9)s + (K_5 K_9 - K_6 K_{10})} \bar{M}_s(s) \\ &+ \frac{K_6 K_7}{s^2 + (K_5 + K_9)s + (K_5 K_9 - K_6 K_{10})} \bar{M}_b(s) \\ &+ \frac{K_6 K_8}{s^2 + (K_5 + K_9)s + (K_5 K_9 - K_6 K_{10})} \bar{M}_b'(s) \end{aligned} \quad (12)$$

If the step change was given to the feed and coolant flow rates,

$$\bar{M}_B(s) = \frac{A}{s}$$

$$\bar{M}_B'(s) = \frac{A'}{s} \quad (13)$$

$$\bar{M}_s(s) = \frac{B}{s}$$

Equation (13) can be put into equations (11,12) and than solved with the Laplace transform.

The numerical solution was obtained for output and coolant temperature with specific conditions taken from experimental work. The numerical example is given below.

For this calculation, the input and operating condition given in Table. 1. were chosen as typical example. Step change was given to the coolant flow rate and 80 % glycerin concentration in mixture was used as an input.

If related parameters are put in to equation (5),  
 $M_T = 31535 \text{ g}$   
 $M_C = 4922.8 \text{ g}$

and than,

Table 1. The operating and steady-state conditions for experimental work.

$C_1 (\%)$	$C_o (\%)$	$M'^o_b \left( \frac{g}{sec} \right)$	$M'^o_b \left( \frac{g}{sec} \right)$	$T'^o_{bg} = T'^o_{bg} ({}^{\circ}C)$	$T'^o_{bg} ({}^{\circ}C)$	$UA \left( \frac{Cal}{{}^{\circ}C sec} \right)$	$M_b \left( \frac{g}{sec} \right)$	$M'_b \left( \frac{g}{sec} \right)$	$M_s \left( \frac{g}{sec} \right)$
Step change in feed flow rate									
95	95	8.61	—	17	60	18	5.75	17.22	—
80	95	6.95	1.30	17	60	18	8.40	13.90	2.60
60	80	6.22	1.95	17	60	18	10.85	12.44	3.88
50	60	6.44	1.30	17	60	18	12.48	12.88	2.60
25	35	5.20	2.09	17	60	18	14.58	10.40	4.18
Step change in cooling flow rate									
95	95	8.61	—	17	60	18	5.57	8.61	—
80	95	6.95	1.30	17	60	18	8.40	6.95	1.30
60	80	6.22	1.94	17	60	18	10.85	6.22	1.94
50	60	6.44	1.30	17	60	18	12.48	6.44	1.30
25	35	5.20	2.09	17	60	18	14.58	5.20	2.09

$$\frac{d\bar{T}_{ge}}{dt} = \frac{0.66x60 - 0.66x65}{31535x0.66} \bar{M}_b + \frac{1x60 - 0.66x65}{31535x0.66} \bar{M}'_b - \frac{14x1.19x60x0.66 + 8.4x60}{31535x0.66} \bar{T}_{ge} + \frac{8.4x60}{2x31535x0.66} \bar{T}_{sc} \quad (14)$$

$$\frac{d\bar{T}_{sc}}{dt} = \frac{18-37}{4922.8} \bar{M}_s + \frac{73x60 + \frac{8.4x60}{2}}{4922.8x1} \bar{T}_{sc} + \frac{8.4x60}{4922.8} \bar{T}_{ge} \quad (15)$$

Step change for feed and cooling flow rates

$$\bar{M}_b = \frac{417.24}{s}$$

$$\bar{M}'_b = \frac{78}{s} \quad (16)$$

$$M_s = \frac{4320}{s}$$

The solution for output temperature with inverse Laplace transform,

$$\begin{aligned} \bar{T}_{ge}(t) &= -0.019 (18.48 - 0.0024 e^{-0.943t} - 18.48 e^{-0.054t}) \\ &\quad - 0.202 (19.64 + 1.1900 e^{-0.943t} - 20.83 e^{-0.054t}) \\ &= -4.318 (1.00 + 0.061 e^{-0.943t} - 1.108 e^{-0.054t}) \end{aligned} \quad (17)$$

For coolant temperature,

$$\begin{aligned} \bar{T}_{sc}(t) &= -16.680 (1.100 - 1.060 e^{-0.943t} - 0.042 e^{-0.054t}) \\ &\quad - 0.0133 (19.700 + 1.220 e^{-0.943t} - 20.900 e^{-0.054t}) \\ &= -18.610 (1.000 - 0.940 e^{-0.943t} - 0.051 e^{-0.054t}) \end{aligned} \quad (18)$$

These results were compared with experimental results obtained from same conditions.

#### COMPARISON OF THE EXPERIMENTAL RESPONSE WITH THEORETICAL RESULTS

Description of equipment and experimental work for this research were explained in the previous work of Alpbaz and Erdogan (1). But

additional data for this work was necessary. The physical properties of mixture of glycerin and water can be found in the literature (2). The variation of heat transfer coefficient, UA, with concentration of glycerin is given in Table. 2, and it is shown in Fig. 2. Because the heat transfer coefficient changes with operating conditions, related values for UA are given separately for each experimental work.

Table. 2. The variation of UA with glycerin concentration

% C	UA $\left( \frac{\text{Cal}}{\text{oC sec}} \right)$
95	5.57
80	8.40
60	10.85
50	12.48
40	13.77
30	15.17
25	14.58
0	18.19

In this part of this work, water and glycerin were given separately to the tank to get the mixture of them. Also, the aim of this work is to investigate the effect of the viscosity change on the transient behavior of the tank. In this research, the concentration for glycerin was taken between the values of 95 % and 25 %. Related steady-state and transient conditions are given in Table. 1.

The concentrations of 80 % and 35 % glycerin were taken as an typical example for investigation of transient response of the tank.

When the system was in the steady-state conditions shown in Table. 1, the step change was given to the feed flow rate ( $V_b^0 = 7 \text{ ml/sec}$ ,  $V_b = 14 \text{ ml/sec}$ ). The change of output variables as a results of experimental response and theoretical calculation are given in Figs. 3-7.

Similarly when the system was in the steady-state, the step change was given to the coolant flow rate ( $M_s^0 = 17 \text{ g/sec}$ ,  $M_s = 89 \text{ g/sec}$ ). The transient response of the tank and coolant temperature are given in Fig. 7-11.

On the dynamic works related with the step change given to the feed flow rate, using concentration of glycerin between the values of 95 % - 80 %, the temperature of the tank and coolant came to the sec-

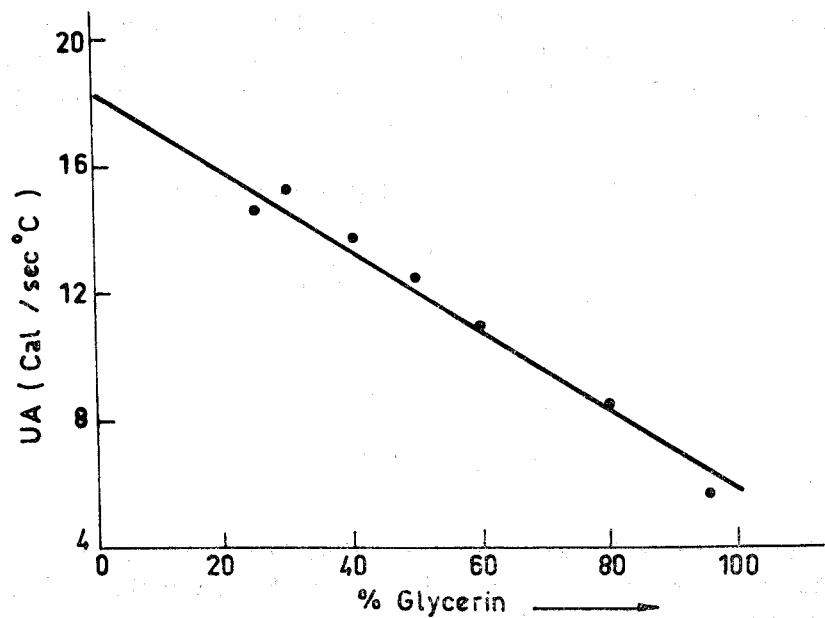


Fig. 2. The variation of UA with glycerin concentration

ond steady-state lower than first one. Beside of them, in lower concentration of glycerin the related temperatures came to the second steady state higher than first one. From these results it was observed that the change of viscosity affected the heat transfer coefficient and than dynamic response of the tank. Firstly UA affected the first and second steady-state and than affected the transient period of the vessel. But this effect was not observed when the step change was given to the coolant flow rate.

In Figs. 2-11, there is little descripincies between the experimental and theoretical results. It was concluded that the tank was well agitated and there was no temperature and concentration distributions.

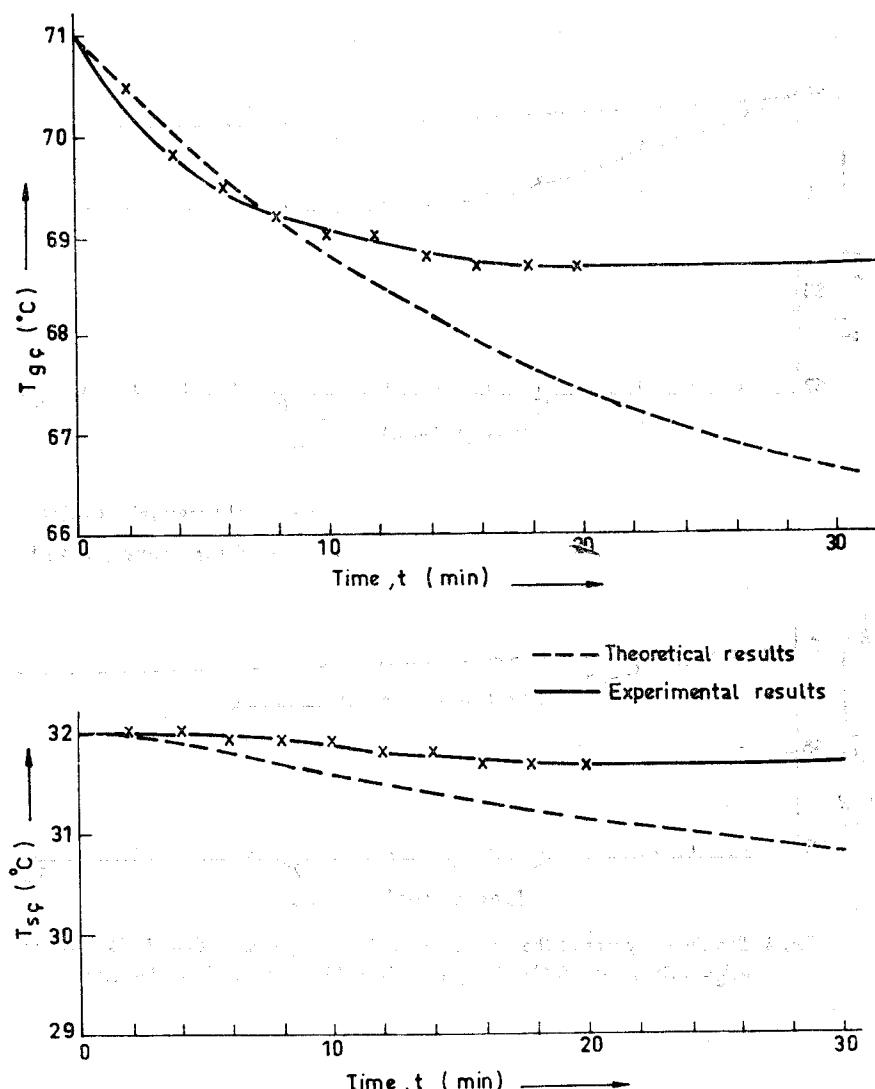


Fig. 3. The time response of the output and coolant temperature  
 $(C_1 = 95\% \text{ Glycerin}, M_b^0 = 8.61 \text{ g/sec}, M_b = 17.22 \text{ g/sec})$

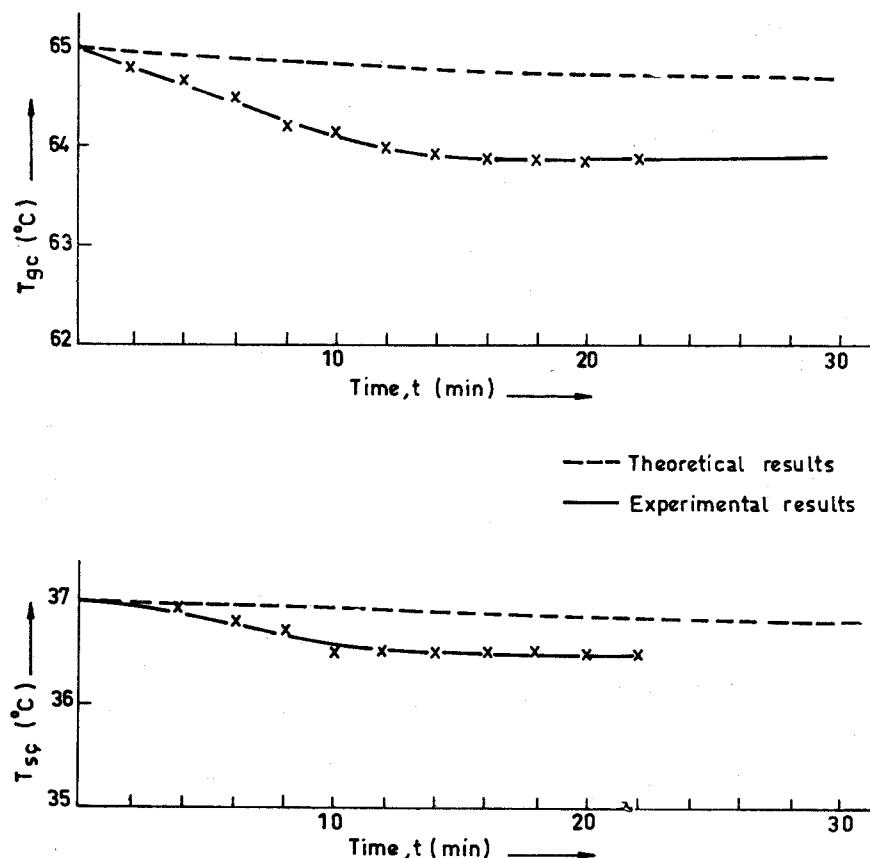


Fig. 4. The time response of the output and coolant temperature ( $C_1 = 80\% \text{ Glycerin}$ ,  $M_b^0 = 6.95 \text{ g/sec}$ ,  $M_b^{''0} = 1.3 \text{ g/sec}$ ,  $M_b = 13.90 \text{ g/sec}$ ,  $M_b' = 2.6 \text{ g/sec}$ )

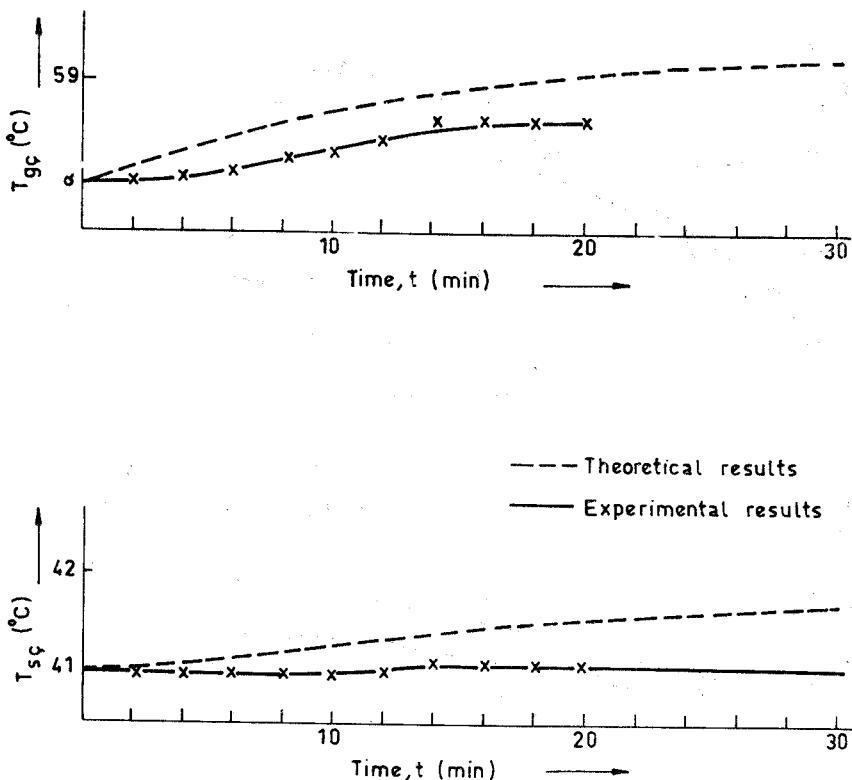


Fig. 5. The time response of the output and coolant temperature ( $C_i = 50\% \text{ Glycerin}$ ,  $M_b^0 = 6.44 \text{ g/sec}$ ,  $M_b' = 1.3 \text{ g/sec}$ ,  $M_b = 12.88 \text{ g/sec}$ ,  $M_b' = 2.6 \text{ g/sec}$ )

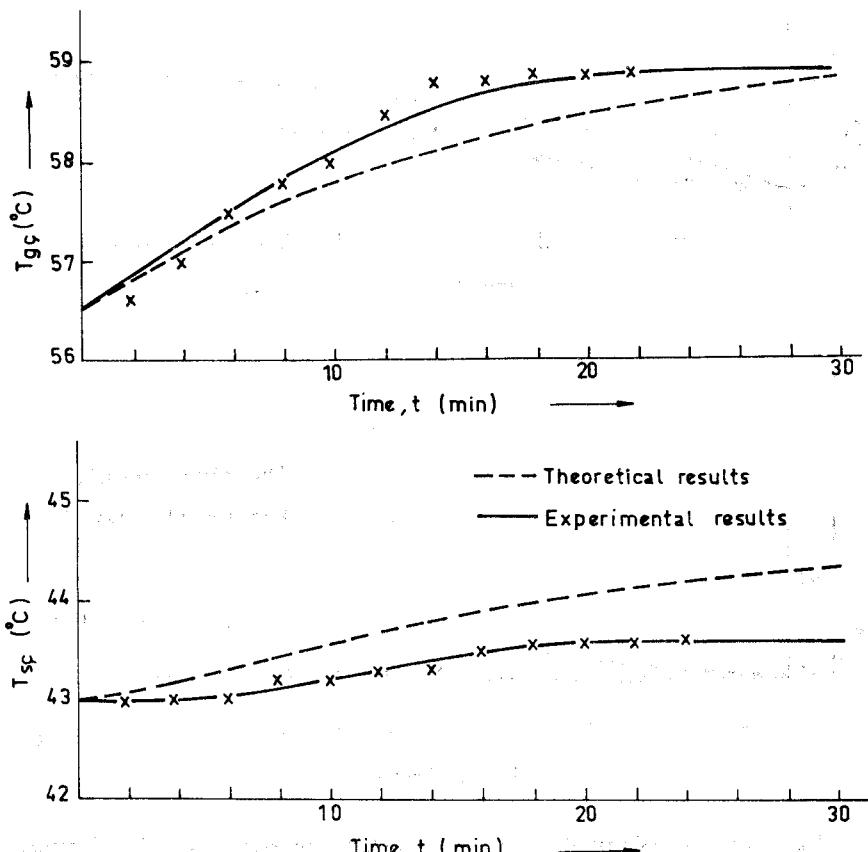


Fig. 6. The time response of the output and coolant temperature ( $C_1 = 25\% \text{ Glycerin}$ ,  $M_b^o = 5.20 \text{ g/sec}$ ,  $M_b'^o = 2.09 \text{ g/sec}$ ,  $M_b = 10.40 \text{ g/sec}$ ,  $M_b' = 4.18 \text{ g/sec}$ )

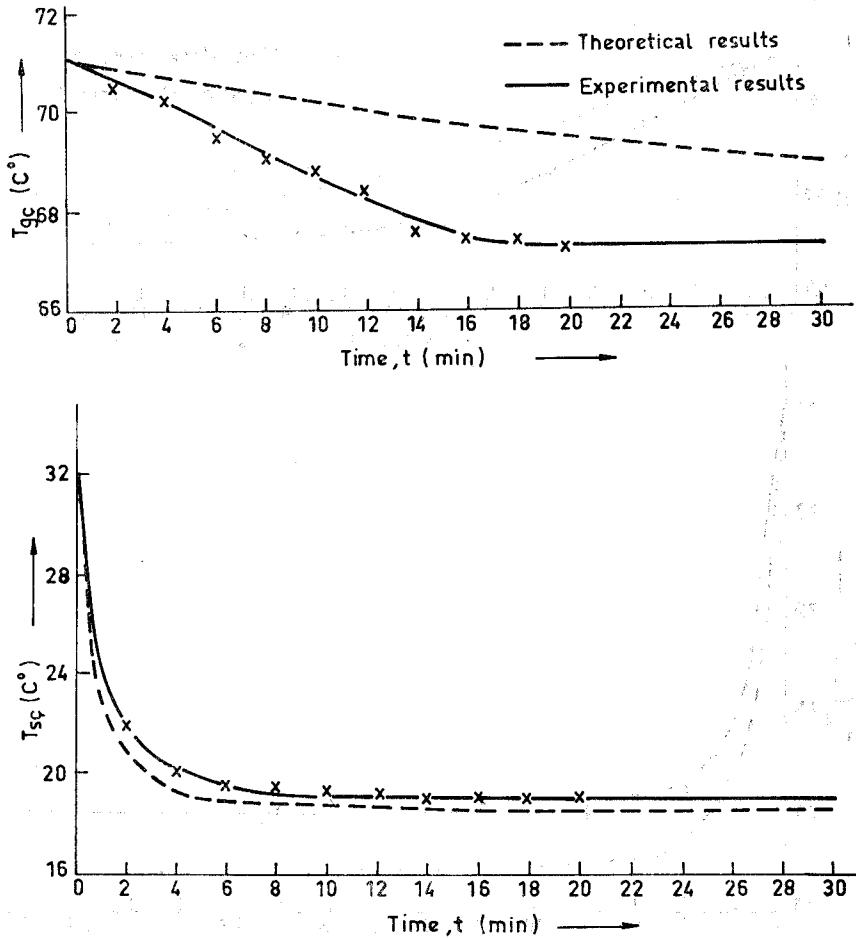


Fig. 7. The time response of the output and coolant temperature  
 $(C_1 = 95\% \text{ Glycerin}, M^0 = 17 \text{ g/sec}, M_s = 89 \text{ g/sec})$

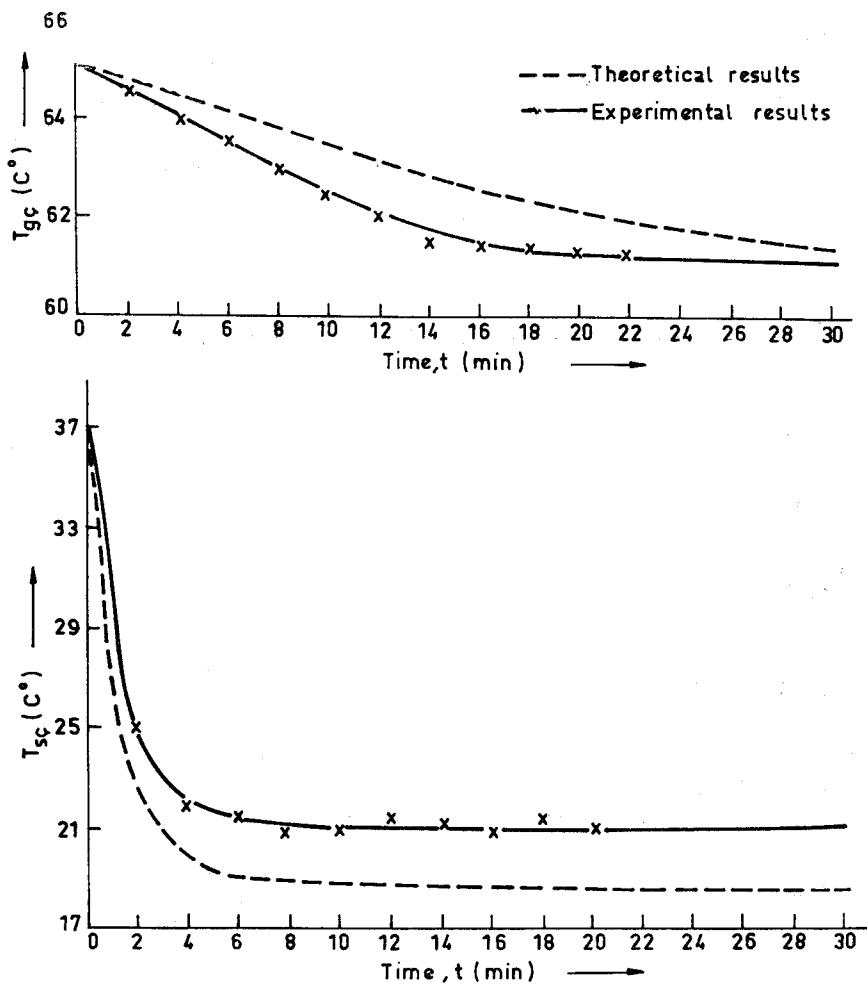


Fig. 8. The time response of the output and coolant temperature  
(C<sub>1</sub> = 80 % Glycerin, M<sub>s</sub><sup>0</sup> = 17 g/sec, M<sub>s</sub> = 89 g/sec)

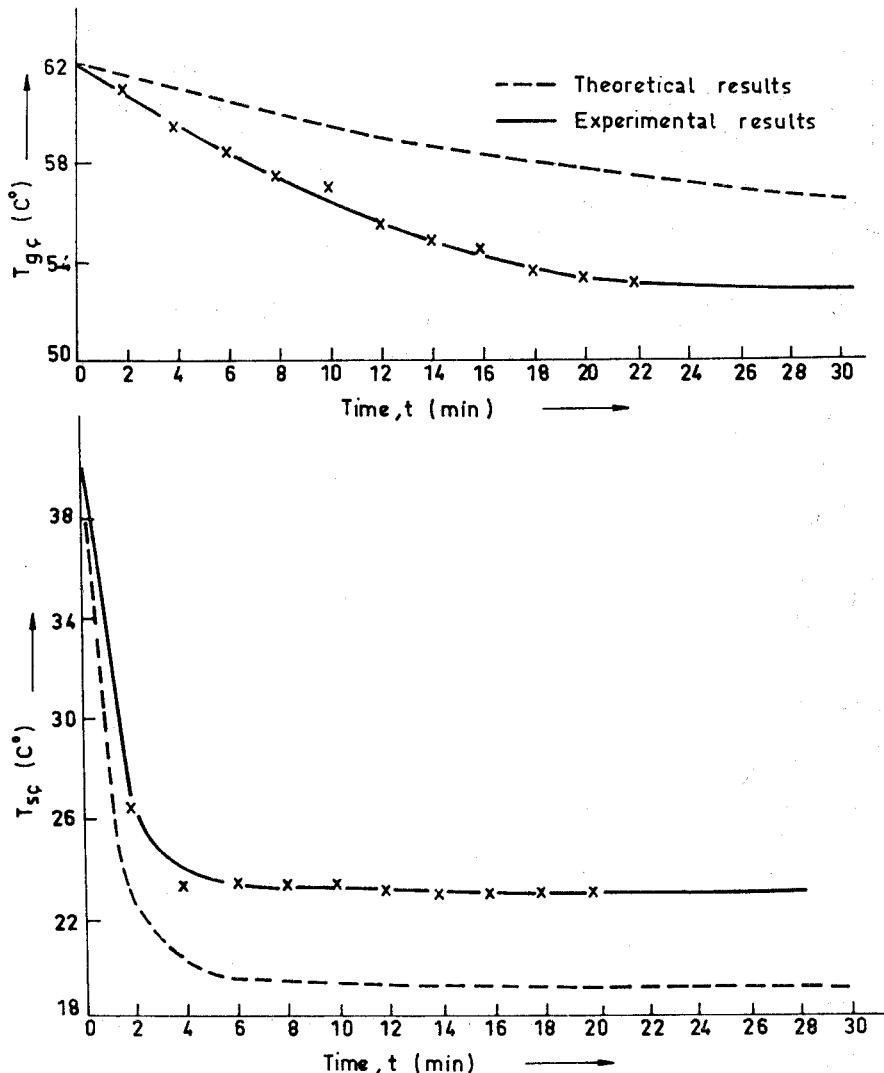


Fig. 9. The time response of the output and coolant temperature  
 $(C_1 = 60\% \text{ Glycerin}, M_s^0 = 17 \text{ g/sec}, M_s = 89 \text{ g/sec})$

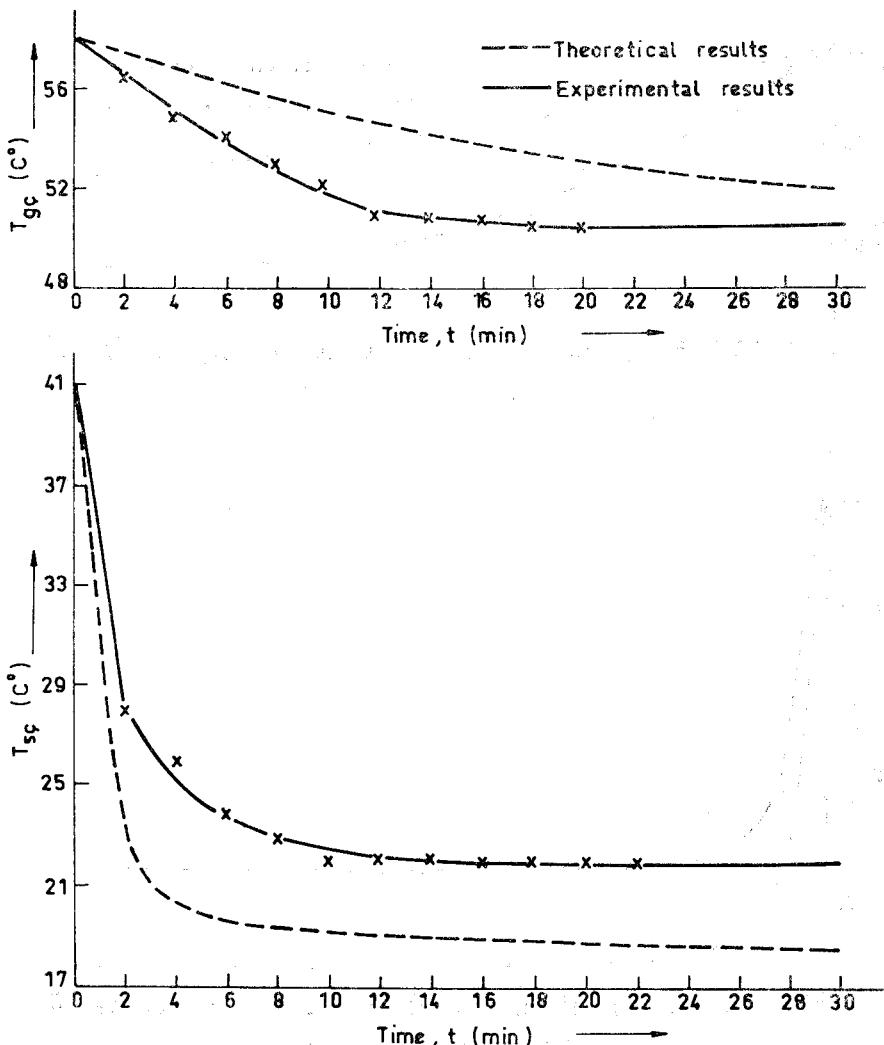


Fig. 10. The time response of the output and coolant temperature  
 $(C_1 = 50\% \text{ Glycerin}, M_s^0 = 17 \text{ g/sec}, M_s = 89 \text{ g/sec})$

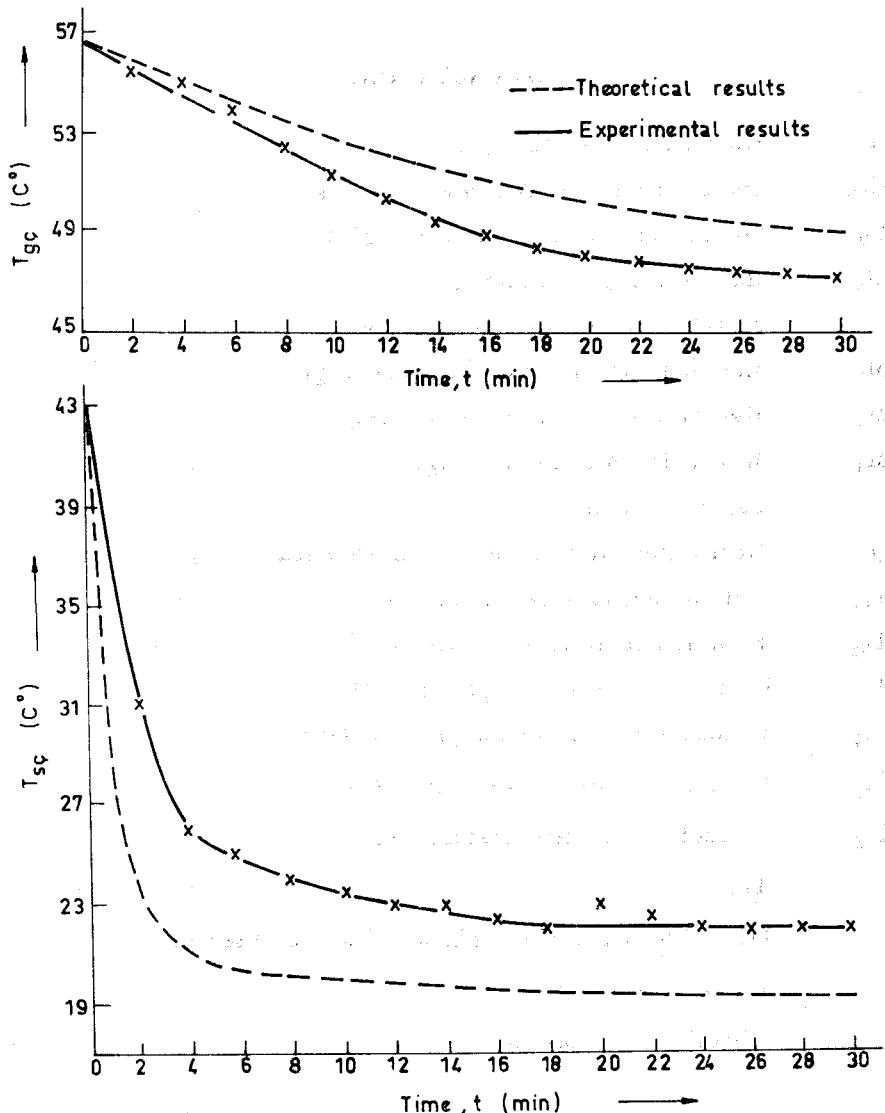


Fig. 11. The time response of the output and coolant temperature  
( $C_1 = 25\%$  Glycerin,  $M_s^0 = 17$  g/sec,  $M_s = 89$  g/sec)

## NOMENCLATURE

A	Heat transfer surface (cm <sup>2</sup> )
C <sub>pb</sub>	Specific heat of tank content (cal / g <sup>o</sup> C)
C <sub>ps</sub>	Specific heat of coolent (cal / g <sup>o</sup> C)
M <sub>b</sub>	Mass flow rate of water (g / sec)
M' <sub>b</sub>	Mass flow rate of glycerin (g / sec)
M <sub>C</sub>	Mass hold up in the cooling jacket (g)
M <sub>s</sub>	Mass flow rate of coolant (g / sec)
M <sub>T</sub>	Mass hold up in the tank (g)
s	Laplace operator
Q	Heat output from immertion heaters (cal / sec)
T <sub>bc</sub>	Output temperature for water (°C)
T <sub>bg</sub>	Feed temperature for water (°C)
T' <sub>bg</sub>	Feed temperature for glycerin (°C)
T <sub>gc</sub>	Output temperature for glycerin (°C)
T <sub>sg</sub>	Coolant output temperature (°C)
T <sub>sg</sub>	Coolant input temperature (°C)
t	Time
U	Overall heat transfer coefficient (cal / cm <sup>2</sup> sec °C)
g	Density (g / cm <sup>3</sup> )
$\mu$	Viscosity (g / cm· sec)

**ÖZET**

Tam karıştırımalı akım tankının dinamik özellikleri, sistemin giriş değişkenlerine kademe değişimi verilmesi ile incelenmiştir. Çalışmanın bu kısmında su ile gliserinin farklı derişimlerinin besleme olarak verildiği hal için, tankın dinamik özellikleri araştırılmıştır. İlgili matematik modellerin Laplace dönüşümleri ile yapılan çözümlerden elde edilen teorik sonuçlar deneySEL ve riler ile karşılaştırılmışlardır.

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