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Research Article

Identification and Stochastic Optimizing the UAV Motion Control in Turbulent Atmosphere

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Abstract

In a class of diffusion Markov processes, we formulate a problem of identification of nonlinear stochastic dynamic systems with random parameters, multiplicative and additive noises, control functions, and the state vector at a final time moment. For such systems, the identifiability conditions are being studied, and necessary conditions are formulated in terms of the general theory of extreme problems. The developed engineering methods for identification and optimizing nonlinear stochastic systems are presented as well as their application for unmanned aerial vehicles under wind disturbances caused by atmospheric turbulence, namely, for optimizing the autopilot parameters during a rotary maneuver of an unmanned aerial vehicle in translational motion, taking into account the identification of its angular velocities.

Keywords

Unmanned Aerial Vehicle
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1. Introduction

Stochastic differential and discrete models are applied in the study of complex controlled systems under conditions of random parametric, structural, and external disturbances.

The mathematical foundations for such researches are presented in well-known monographs by Bulinsky and Shiryaev, 2005; Evlanov and Konstantinov, 1976; Fleming and Rishel, 1975; Gikhman and Skorokhod, 1977; Kazakov, 1977; Oksendal, 2000; Solodov and Solodov, 1988 et al. . Here and further, we apply for the alphabetical citation order.

Strict mathematical methods for optimizing nonlinear systems are also known; for example, Dubovitskii and Milyutin, 1965; Girsanov, 1970; Ioffe and Tikhomirov, 1974; Kazakov and Artemyev, 1980; Kolosov, 1984 et al..

In the applied theory of optimizing nonlinear stochastic systems, approximate methods based on parametric or functional approximation of the a posteriori probability distribution density are used. Parametric approximation methods are applied to determine the characteristics of stochastic processes, namely, a posteriori moments or cumulants, which are usually called semi-invariants, see the monographs by Bodner et al., 1987; Chernetsky, 1968; Denisov and Rodnishchev, 2017; Dostupov, 1970; Kozhevnikov, 1978; Malakhov, 1978; Potseluyev, 1984; Pugachev and Sinitsyn, 1985 et al..

Our theoretical results on this topic were presented in research papers (Rodnishchev, 2001a,b; Rodnishchev and Khairullin, 2010). These results were implemented in the Russian aerospace industry, namely in control

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systems of spacecraft (Rodnishchev et al., 2019), including mini-satellites for communication (Fig. 1) and the Earth-observing (Fig. 2), space robot-manipulators (SRMs), Fig. 3 (Somov et al. l., 2019), as well as Russian passenger airliners, Fig. 4 (Rodnishchev and Somova, 2019), in control systems of turboprop engines for various aviation equipment (Bodner et al., 1987; Kozhevnikov et al., 1989).

In this paper, methods for identification in stochastic control systems and study the problem of optimizing parameters of unmanned aerial vehicle (UAV) autopilot during its turning maneuver in translational movement under stochastic atmospheric turbulence are briefly presented.

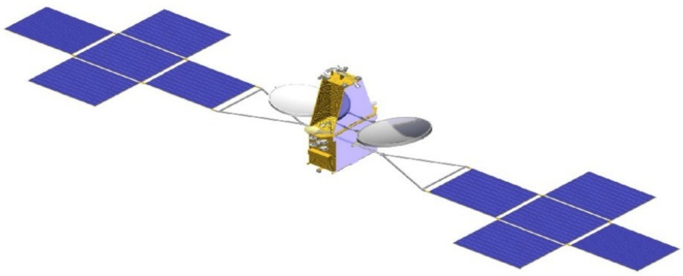

Fig. 1. The communication mini-satellite

Fig. 2. The Earth-surveying mini-satellite

2. Models and the Problem Statement

In this section, the problem of identifying vectors of parameters $\mathbf{a} \equiv \{a_i\} \in \mathrm{R}^m$ and control $\mathbf{u} \equiv \{u_i\} \in \mathrm{R}^r$ is studied for nonlinear stochastic system

$$dy_i = (\Sigma C_{ij}(t,\mathbf{b})\,dt + dw_{ij}(t))\,\phi_{ij}(t,\mathbf{y},\mathbf{b},\mathbf{u},\mathbf{a})) + \Sigma\sigma_{ij}(t,\mathbf{y},\mathbf{b})\,d\eta_j(t),\ i \in \mathrm{N}_1^n,\ \ t \in [t_\mathrm{i}, t_\mathrm{f}] \tag{1}$$

with a state vector $\mathbf{y} = \{y_i\} \in \mathrm{R}^n$ at initial condition $\mathbf{y}_\mathrm{i} = \{y_{\mathrm{i}i}\}$ where $\mathrm{N}_1^n \equiv [1,2,\ldots n]$ and observations

$$z_k = \Sigma c_{k\nu} y_\nu + \dot{w}_k(t),\ k \in \mathrm{N}_1^p. \tag{2}$$

Identification efficiency is evaluated by functional minimum

$$I_0(\mathbf{u},\mathbf{a}) = \int_{t_\mathrm{i}}^{t_\mathrm{f}} E[\Sigma\alpha_k(z_k - \Sigma c_{k\nu} y_\nu)^2]\,dt, \tag{3}$$

and control objectives, technical and operational requirements to the system are determined by constraints on the final system status of the equality type

$$I_s(\mathbf{u},\mathbf{a}) = E[f_s(\mathbf{y}_\mathrm{f})] - c_s = 0, \quad s \in \mathrm{N}_1^q. \tag{4}$$

Here $E[\cdot]$ is the expectation operator, t_i and t_f are the initial and final points of a time interval and $\mathbf{y}_\mathrm{f} = \{y_{\mathrm{f}i}\} = \mathbf{y}(t_\mathrm{f})$.

As a control vector $\mathbf{u}(\cdot)$, we study a program control $\mathbf{u}(t)$ or feedback control $\mathbf{u}(t,\mathbf{y})$. Control $\mathbf{u}(t)$ is determined on the set $\mathrm{S} = \{\mathbf{u}(t) \in L_2[t_\mathrm{i},t_\mathrm{f}] : \mathbf{u}(t)\} \in U, t \in [t_\mathrm{i},t_\mathrm{f}]\}$, where $L_2[t_\mathrm{i},t_\mathrm{f}]$ is the space of measurable functions with quadratic metrics and $U \subset \mathrm{R}^r$ is a convex set.

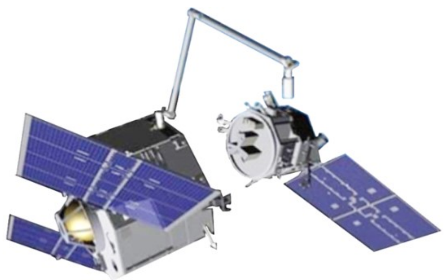

Fig. 3. The SRM is preparing to capture a failed satellite

Fig. 4. Russian passenger airliner IL 96-300 on landing

The feedback control $\mathbf{u}(t,\mathbf{y})$ is considered as Borelian function as either a random element in L_2 or a non-anticipating process relative to Wiener processes w_{ij}, η_j with values in U.

Next, column $\mathbf{a}$ determinate the controlling parameters; $\mathbf{b} \equiv \{b_i\} \in \mathrm{R}^l$ is a random vector; $w_{ij}(t) = W_{ij}(t)\,dt$ and $\eta_j(t) = \mathrm{N}_j(t)\,dt$ are Stratonovich stochastic differentials of Wiener processes $w_{ij}(t)$ and $\eta_j(t)$, moreover, the process $w_{ij}(t)$ describes multiplicative noises affecting the system, the process $\eta_j(t)$ describes additive noises, $W_{ij}(t)$ are white Gaussian noises that describes random internal perturbations common for the system, $\mathrm{N}_j(t)$ are the white Gaussian noises; $C_{ij}(t,\mathbf{b})$, $\phi_{ij}(t,\mathbf{y},\mathbf{b},\mathbf{u},\mathbf{a})$ and $\sigma_{ij}(t,\mathbf{y},\mathbf{b})$ are given nonrandom functions satisfying given requirements of solution existence (1), and $\mathbf{z}\equiv\{z_k\}\in\mathrm{R}^p$ is observed vector of tester coordinates $z_k, k\in\mathrm{N}_1^p, p\le n$. The matrix component $(c_{k\nu})$ determines the selection of observed system coordinates (1), $\dot{w}_k(t)$ is derivative of Wiener process, $w_k(t)$ is component $R_k(t)$ of tester additive white noise.

At last, $I_s(\mathbf{u},\mathbf{a})$ are continuous and continuously differentiable functionals on a set of variables, and $I_0(\mathbf{u},\mathbf{a})$ is bounded functionally differentiable on the set of variables and α_k are the weight numbers.

At accepted conditions concerning the right parts, solution (1) exists, and it is unambiguous. However, this solution does not need to be a Markov process. That is why, to make (1) describe the Markov process, we introduce an extended state vector $\mathbf{x}\equiv\{x_i\}=\{\mathbf{y},\mathbf{b}\}$. Relative to the state vector $\mathbf{x}$ equations (1) reduce themselves to an equivalent system of diffusion stochastic differential equations

$$
\begin{aligned}
dx_i &= (\Sigma C_{ij}(t,\mathbf{x})\,dt + dw_{ij}(t))\,\phi_{ij}(t,\mathbf{x},\mathbf{u},\mathbf{a})) \\
&\quad + \Sigma\sigma_{ij}(t,\mathbf{x})\,d\eta_j(t),\;\; x_i(t_\mathrm{i}) = y_{\mathrm{i}i}\;\; \forall i\in\mathrm{N}_1^n; \\
dx_i &= 0, \qquad\qquad x_i(t_\mathrm{i}) = b_i \quad \forall i\in\mathrm{N}_{n+1}^{n+l}.
\end{aligned} \tag{5}
$$

These equations have a single solution and describe the diffusion Markov process $\forall t\in[t_\mathrm{i},t_\mathrm{f}]$; the posterior density of state probability $p(t,\mathbf{x}\,|\,\bar{\mathbf{z}})$ satisfies the Stratonovich-Kushner equation (a generalization of the well-known Fokker-Planck- Kolmogorov equation)

$$\partial p(\cdot)/\partial t = L(\cdot)p(\cdot) + (F(\cdot) - \textstyle\int F(\cdot)p(\cdot)\,d\,\mathbf{x})\;p(\cdot) \tag{6}$$

with $\mathbf{x}\in\Omega\subset\mathrm{R}^n$ and initial condition $p(t,\mathbf{x}\,|\,\bar{\mathbf{z}})|_{t=t_\mathrm{i}} = p(t_\mathrm{i},\mathbf{x})$. Here $F(\cdot)=F(\mathbf{x},\mathbf{z})$ and $L(\cdot)p(\cdot)=L(t,\mathbf{x},\mathbf{u},\mathbf{a})\,p(t,\mathbf{x}\,|\,\bar{\mathbf{z}})$ is an elliptic operator determined by the formula

$$L(\cdot)\,p(\cdot) = -\Sigma\partial[A_i\;p(\cdot)]/\partial x_i + (1/2)\Sigma\partial^2[B_{ip}\;p(\cdot)]/\partial x_i\partial x_p \tag{7}$$

with coefficients of the drift $A_i = A_i(t,\mathbf{x},\mathbf{u},\mathbf{a})$ and diffusion $B_{ip}=B_{ip}(t,\mathbf{x},\mathbf{u},\mathbf{a})$. In the density function, $p(t,\mathbf{x}\,|\,\bar{\mathbf{z}})$ the vector $\bar{\mathbf{z}}$ means that the whole output signal realization of the tester observed on the time interval $t\in[t_\mathrm{i},t_\mathrm{f}]$ is applied.

Assume that G_{pk} are the mutual forces of tester noises and $F(\mathbf{x},\mathbf{z})=\Sigma(c_{pq}x_q/G_{pk})(z_k-(1/2)\Sigma c_{k\nu}x_\nu)$ scalar function characterize the tester properties. According to a theory of Markov processes, original identification problem (1) – (3) concerning the extended vector of system state $\bar{\mathbf{x}}=\{\mathbf{x},x_{n+1}\}$ reduces itself to an equivalent terminal problem with distributed parameters relative to a posterior density function $p(t,\bar{\mathbf{x}}\,|\,\bar{\mathbf{z}})$ in the form

$$I_0(\mathbf{u},\mathbf{a}) = \textstyle\int x_{n+1}\,p(t_\mathrm{f},\bar{\mathbf{x}}\,|\,\bar{\mathbf{z}})\,d\,\bar{\mathbf{x}} \;\Rightarrow\; \min; \tag{8}$$

$$\partial p(\cdot)/\partial t = \bar{L}(\cdot)p(\cdot) + (F(\cdot) - \textstyle\int F(\cdot)p(\cdot)\,d\,\bar{\mathbf{x}})\;p(\cdot) \tag{9}$$

with $F(\cdot)=F(\bar{\mathbf{x}},\mathbf{z})$ and the condition $p(t,\bar{\mathbf{x}}\,|\,\bar{\mathbf{z}})|_{t=t_\mathrm{i}} = p(t_\mathrm{i},\bar{\mathbf{x}})$;

$$I_s(\mathbf{u},\mathbf{a}) = \textstyle\int f_s(\mathbf{x})\,p(t_\mathrm{f},\bar{\mathbf{x}}\,|\,\bar{\mathbf{z}})\,d\,\bar{\mathbf{x}} - c_s = 0, \quad s\in\mathrm{N}_1^q. \tag{10}$$

Here the component $x_{n+1}(t)$ is determined by equation

$$d\,x_{n+1} = \Sigma\alpha_k(z_k - \Sigma c_{k\nu}x_\nu)^2\,dt \equiv g(\mathbf{x})\,dt \tag{11}$$

with initial condition $x_{n+1}(t_\mathrm{i})=0$ and the operator

$$\bar{L}(\cdot)p(\cdot) \equiv L(\cdot)p(\cdot) - \partial[g(\mathbf{x})p(t,\bar{\mathbf{x}}\,|\,\bar{\mathbf{z}})]/\partial x_{n+1}.$$

This model is identifiable if the system (5), (11) is controllable (Rodnishchev, 2001a; Denisov and Rodnishchev, 2017).

The main objectives of this paper are a brief presentation of our general approach to the considered problem and its practical application to optimizing the UAV autopilot parameters at its route turning under the turbulent wind perturbations.

3. Method and the Approach

For obtaining the necessary identification conditions in terms of conjugate cones in the general theory of extreme problems (Dubovitskii and Milyutin, 1965; Ioffe and Tikhomirov, 1974), the method by Girsanov (1970) is applied.

In this case, the solution to the equation (6) is needed as well as the coupled Bellman parabolic equation. However, as it is known, analytic solutions to linear and special cases of nonlinear stochastic systems only can be obtained. Here our approach (Rodnishchev, 2001) is applied, which is based on employing mathematical statistics.

The identification problem with respect to a posteriori semi invariant is reduced to the equivalent extreme problem of the estimation parameters, control, and components of the state vector in the form

$$
\begin{aligned}
&\omega_1^{n+1}(t_\mathrm{f}) \Rightarrow \min, \\
&\dot{\omega}_1^{n+1} = E[\Sigma\alpha_k(z_k - \Sigma c_{k\nu}x_\nu)^2], \quad \omega_1^{n+1}(t_\mathrm{i}) = 0;
\end{aligned} \tag{12}
$$

$$\dot\omega_1^j = E[A_j(\cdot)] + E[x_j F(\mathbf{x},\mathbf{z})] - \omega_1^j E[F(\mathbf{x},\mathbf{z})] \equiv g_1^j,$$
$$\dot\omega_{11}^{jp} = E[(x_j - \omega_1^j)A_p + (x_p - \omega_1^p)A_j + E[B_{jp}(\cdot)]$$
$$+ E[(x_j - \omega_1^j)(x_p - \omega_1^p)F(\mathbf{x},\mathbf{z})] - \omega_{11}^{jp} E[F(\mathbf{x},\mathbf{z})],$$
$$\dot\omega_2^j = 2E[(x_j - \omega_1^j)A_j(\cdot)] + E[B_{jj}(\cdot)]$$
$$+ E[(x_j - \omega_1^j)^2 F(\mathbf{x},\mathbf{z})] - \omega_2^j E[F(\mathbf{x},\mathbf{z})],$$
$$\ldots\ldots\ldots\ldots$$
$$\dot\omega_{N_j}^j = N_j E[(x_j - \omega_1^j)A_j^{N_j-1}(\cdot)] + (1/2)N_j(N_j-1)$$
$$E[(x_j - \omega_1^j)^{N_j-2} B_{jj}(\cdot)] - \Sigma_{q_j=1}^{N_j-2} C_{N_j}^{q_j} g_{q_j}^j \omega_{N_j-q_j}^j$$
$$+ E[(x_j - \omega_1^j)^{N_j} F(\mathbf{x},\mathbf{z})] - \omega_{N_j}^j E[F(\mathbf{x},\mathbf{z})] \qquad (13)$$

for $N_j \geq 3$ with initial conditions

$$\omega_1^j(t_\mathrm{i}) = c_{1\mathrm{i}}^j,\ \omega_{11}^{jp}(t_\mathrm{i}) = b_{11\mathrm{i}}^{jp},\ \omega_2^j(t_\mathrm{i}) = b_{2\mathrm{i}}^j,\ \ldots,\ \omega_{N_j}^j(t_\mathrm{i}) = b_{N_j\mathrm{i}}^j$$

and indexes $j, p \in \mathrm{N}_1^n$, also taking into account (10).

The equations (13) present variations of semi-invariants ω_1^j of the 1-st order by j-th components for the system state vector, coinciding with the mathematical expectations; semi-invariants ω_2^j and ω_{11}^{jp} of 2-nd order by j-th components and the relationship between j-th and p-th components, coinciding with the dispersion and correlation functions of the state vector; at last, semi-invariants $\omega_{N_j}^j$ of the N_j-th order.

For the closure of a shortened system of differential equations (13) and an approximate representation of the higher moments through the lower moments, the method of moment semi-invariants is applied (Dashevskii, 1976).

Assume $N_j = 8$, that is sufficient to solve practical problems. Semi-invariants are not independent, and they are bound by the conditions for the functions $f_\mu, \mu \in \mathrm{N}_1^9$ (Malakhov, 1978):

$$f_1 = 2(\omega_2^j)^2 + \omega_4^j \geq 0; \qquad f_2 = 2\omega_2^j\omega_2^p + \omega_{22}^{jp} \geq 0;$$
$$f_3 = a\,\omega_2^j(\omega_2^p)^2 + \omega_{24}^{jp} \geq 0; \quad f_4 = a(\omega_2^j)^2\omega_2^p + \omega_{42}^{jp} \geq 0;$$
$$f_5 = a(\omega_2^j)^2 + \omega_6^j \geq 0; \qquad f_6 = b(\omega_2^j)^2(\omega_2^p)^2 + \omega_{44}^{jp} \geq 0;$$
$$f_7 = b\,\omega_2^j(\omega_2^p)^3 + \omega_{26}^{jp} \geq 0; \quad f_8 = b(\omega_2^j)^3\omega_2^p + \omega_{62}^{jp} \geq 0;$$
$$f_9 = b(\omega_2^j)^4 + \omega_8^p \geq 0, \text{ where } a = 105 \text{ and } b = 166214.$$

These functional inequalities should be performed in the solution of the optimization problem (12), (13) on the time interval $t \in [t_\mathrm{i}, t_\mathrm{f}]$, so that is the functions f_μ must belong to an integral variety $f_\mu \geq 0$ for the set of differential equations

$$(d f_\mu(t)/dt)|_{(13)} = -\rho f_\mu [1 + \mathrm{sign}\,(f_\mu)]/2, \quad \mu \in \mathrm{N}_1^9 \qquad (14)$$

with $\rho > 0$, $\mathrm{sign}(0) = 0$, $f_\mu(t_\mathrm{f}) \geq 0$ when $f_\mu(t_\mathrm{i}) = 0$ $\forall \mu \in \mathrm{N}_1^9$.

4. Optimizing the UAV Autopilot Parameters

This section solves the problem of stochastic optimization of autopilot parameters for the UAV turn mode, taking into account the identification of UAV angular velocities in atmospheric turbulence.

At an angular velocity $\omega_y^\# = \mathrm{const}$, the UAV lateral angular motion is described in standard notations by the equations

$$\dot\beta = k_\gamma\gamma + k_\beta(\beta + \beta_w) + \alpha_\mathrm{o}\omega_x + \omega_y;$$
$$\dot\omega_x = -L_\beta(\beta + \beta_w) + L_\delta\delta - L_x\omega_x - L_y\omega_y - l_\mathrm{e} i_y^\mathrm{e}\omega_y^\#;$$
$$\dot\omega_y = -N_\beta(\beta + \beta_w) - N_x\omega_x - N_y\omega_y;$$
$$\dot\gamma = \omega_x; \quad \dot\psi = \omega_y; \quad \dot\delta = \omega_y - \omega_y^\#;$$
$$\dot\beta_w = -q_x\beta_w + \sigma_{\beta_w}\sqrt{2q_x}\,N(t), \qquad (15)$$

where $q_x = V_\mathrm{o}/L_{w_x}$ with a nominal true airspeed V_o of the UAV flight and a scale L_{w_x} of turbulence; $\sigma_{\beta_w} = \sigma_{\omega_x}/V_\mathrm{o}$ with the root mean square (RMS) value σ_{ω_x} of the turbulence intensity, $N(t)$ is standard white noise, and α_o is the UAV balancing angle of attack. Here we use the following notations

$$L_\beta = l_\beta + l_\mathrm{e} i_\beta^\mathrm{e};\ L_x = l_{\omega_x} + l_\mathrm{e} i_x^\mathrm{e};\ L_\delta = l_\mathrm{e} q_\mathrm{e};\ L_y = l_{\omega_y} - l_\mathrm{e} i_y^\mathrm{e};$$
$$N_\beta = n_\beta + n_\mathrm{d} i_\beta^\mathrm{d};\quad N_x = n_{\omega_x} + n_\mathrm{d} i_x^\mathrm{d};\quad N_y = n_{\omega_y} - n_\mathrm{d} i_y^\mathrm{d},$$

where $k_\gamma, k_\beta, l_\beta, l_{\omega_x}, l_{\omega_y}, l_\mathrm{e}, n_\beta, n_{\omega_x}, n_{\omega_y}, n_\mathrm{d}$ are the UAV aerodynamic coefficients, and $i_\beta^\mathrm{e}, i_x^\mathrm{e}, i_y^\mathrm{e}, q_\mathrm{e}, i_\beta^\mathrm{d}, i_x^\mathrm{d}, i_y^\mathrm{d}$ are the autopilot gear ratios.

To determine variations of the additional sliding angle β_w, we use a model for the horizontal wind turbulence components, which describes a Gaussian random process with a spectral density $S(\omega) = (2/\pi)\,\sigma_{\omega_x}^2 L_{w_x}/(1 + \omega^2 L_{w_x}^2)$, and when using the formative filter (Pugachev and Sinitsyn, 1985), it is represented by a stochastic differential equation in (15) with the input noise $N(t)$.

The autopilot gear ratios are linearly related to the coefficients $L_x, L_y, L_\delta, N_\beta, N_x, N_y$ of the stochastic system (15), so the definition of the ratios $i_\beta^\mathrm{e}, i_x^\mathrm{e}, i_y^\mathrm{e}, q_\mathrm{e}, i_\beta^\mathrm{d}, i_x^\mathrm{d}, i_y^\mathrm{d}$ is reduced to the optimization of these coefficients with the angular velocity measurements $z_1 = \omega_x + N_{\omega_x}, z_2 = \omega_y + N_{\omega_y}$, where the vector $\mathbf{z} \equiv \{z_1, z_2\}$ N_{ω_x} and N_{ω_y} are standard white noises.

The effectiveness of the linear control law optimization is estimated the functional minimum

$$I_0 = \int_{t_i}^{t_f} E[(\omega_y - \omega_y^{\#})^2]\, dt \Rightarrow \min . \qquad (16)$$

Since wind disturbances caused by horizontal gusts can lead to large deviations when turning the UAV, the parameters of control law are determined so that coefficients $C_i, i \in N_1^5$ of the characteristic polynomial for the closed-loop system (15) provide a consistent choice of parameters from the asymptotic stability region.

With the notation, $q \equiv \alpha_o / k_\gamma$ this region has the boundary defined by the following constraints:

$$L_x^3 - C_1 L_x^2 + C_2 L_x - (C_3 - C_4 q + C_5 q^2) = 0;$$
$$L_x + N_y - (C_1 - k_\beta) = 0;\ k_\gamma L_y N_\beta + (C_4 - C_5) q = 0;$$
$$L_x N_y + N_\beta - (C_2 - k_\beta (C_1 - k_\beta)) = 0;\ k_\gamma L_\delta N_\beta - C_5 = 0.$$

To solve the optimization problem, the additional variable $x_8(t)$ is introduced, which is determined by the solution of the differential equation

$$\dot{x}_8 = (\omega_y - \omega_y^{\#})^2$$

with the initial condition $x_8(t_i) = 0$, and the functional (16) is reduced to the following terminal form

$$I_0 = E[x_8(t_f)] \Rightarrow \min . \qquad (17)$$

Thus, the problem of optimizing the autopilot gear ratios is reduced to determining the vector column

$$\mathbf{a} \equiv \{a_i, i \in N_1^6\} \equiv \{ L_x, L_y, L_\delta, N_\beta, N_x, N_y \}$$

with a providing of the minimum terminal functional (17) and taking into account the need for identification of the angular velocities ω_x and ω_y to the above restrictions on asymptotic stability region.

The system (15) describes a diffusion on Markovian process with the state vector $\mathbf{x} = \{x_i\} \equiv \{\beta, \omega_x, \omega_y, \gamma, \psi, \delta, \beta_w\}$, the coefficients of drift

$$A_1 = -k_\beta (x_1 + x_7) + \alpha_o x_2 + x_3 + k_\gamma x_4;$$
$$A_2 = -a_1 x_2 - a_2 (x_3 - \omega_y^{\#}) + a_3 x_6 - l_{\omega_x} \omega_y^{\#};$$
$$A_3 = -a_4 (x_1 + x_7) - a_5 x_2 - a_6 x_3;\ A_4 = x_2;$$
$$A_5 = x_3;\ A_6 = x_3 - \omega_y^{\#};\ A_7 = -q_x x_7;\ A_8 = (x_3 - \omega_y^{\#})^2$$

and diffusion $B_{77} = 2\sigma_{\omega_x}^2 q_x$. Here, for identification of the above angular velocities, the scalar function

$$F(\mathbf{x}, \mathbf{z}) = x_2 (z_1 - x_2/2)/G_{\omega_x} + x_3 (z_2 - x_3/2)/G_{\omega_y}$$

is applied. So, the stochastic problem is reduced to the deterministic one for the semi-invariants as follows

$$\omega_8^1(t_f) \Rightarrow \min; \qquad (18)$$

$$\dot{\omega}_1^j = E[A_j(\cdot)] + E[x_j F(\mathbf{x}, \mathbf{z})] - \omega_1^j E[F(\mathbf{x}, \mathbf{z})],$$
$$\dot{\omega}_{11}^{jp} = E[(x_j - \omega_1^j) A_p + (x_p - \omega_1^p) A_j$$
$$+ E[(x_j - \omega_1^j)(x_p - \omega_1^p) F(\mathbf{x}, \mathbf{z})] - \omega_{11}^{jp} E[F(\mathbf{x}, \mathbf{z})],$$
$$\dot{\omega}_2^j = 2E[(x_j - \omega_1^j) A_j(\cdot)] + E[B_{jj}(\cdot)]$$
$$+ E[(x_j - \omega_1^j)^2 F(\mathbf{x}, \mathbf{z})] - \omega_2^j E[F(\mathbf{x}, \mathbf{z})] \qquad (19)$$

$\forall j, p \in N_1^7$, taking into account the above restrictions on the asymptotic stability region. Equations (18), (19) are ordinary differential equations of order 36 with respect to semi-invariants ω_8^1 and ω_1^j, ω_2^j, ω_{11}^{jp} with indexes $j, p \in N_1^7$.

5. The Simulation Results and Discussion

The dynamics of the UAV turn with a mass of 320 kg was studied, taking into account wind disturbances caused by atmospheric turbulence. At the flight altitude $H = 1000$ m with airspeed $V_o = 111$ m/s and the balancing angle of attack $\alpha_o = 3.9$ deg, the UAV aerodynamic coefficients have the following values (Romanenko et al., 2012):

$$k_\beta = 0.1946;\ k_\gamma = 0.0833;\ l_\beta = 47.272;$$
$$l_{\omega_x} = 6.776;\ l_{\omega_y} = 1.742;\ l_e = 176.54;$$
$$n_\beta = 13.81;\ n_{\omega_x} = 0.108;\ n_{\omega_y} = 0.859;\ n_d = 7.12.$$

Moreover, coefficients of the characteristic polynomial providing stability have the values

$$C_1 = 14, C_1 = 69.9, C_3 = 213, C_4 = 217, C_5 = 120.$$

In this case, the UAV autopilot ratios are equal to the values

$$i_\beta^e = -0.268;\ i_x^e = 0.0093;\ i_y^e = 0.611;\ q_e = 0.58;$$
$$i_\beta^d = -0.132;\ i_x^d = -0.0152;\ i_y^d = 0.628, \qquad (20)$$

which correspond to the following coefficients $a_i, i \in N_1^6$

$$a_1 = L_x = 8.42;\quad a_2 = L_y = -106.2;$$
$$a_3 = L_\delta = 102.3;\quad a_4 = N_\beta = 12.85;$$
$$a_5 = N_x = 0;\quad a_6 = N_y = 5.39.$$

The problem of optimization (18), (19) with restrictions at the turbulence scale $L_{w_x} = 310$ m is solved with the specified height and the RMS value $\sigma_{\omega_x} = 2.8$ m/s for turbulence of the wind, which corresponds to the "strong" turbulence according to the European airworthiness standards. As a result of solving the optimization problem with identifying the UAV angular velocities by the proposed approach, the coefficients $a_i, i \in \mathrm{N}_1^6$ were obtained with the values

$$a_1 = L_x = 8.91; \quad a_2 = L_y = -109.6;$$
$$a_3 = L_\delta = 99.3; \quad a_4 = N_\beta = 14.125;$$
$$a_5 = N_x = 0; \quad a_6 = N_y = 4.83,$$

which correspond to the autopilot gear ratios with the following values:

$$i_\beta^e = -0.268;\ i_x^e = 0.0121;\ i_y^e = 0.631;\ q_e = 0.56; \quad (21)$$
$$i_\beta^d = 0.045; \quad i_x^d = -0.0152; \quad i_y^d = 0.55.$$

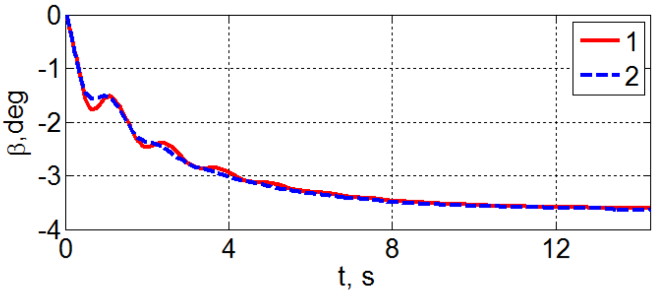

Fig. 5. The changing average values of angle β

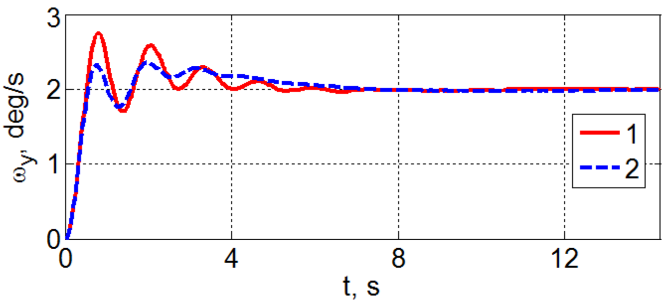

Fig. 6. The changing average values of angular rate

The transients corresponding to the initial values of the autopilot gear ratios (20) are shown in Figs. 5 – 8 in continuous graphs (**1**, red). The transients corresponding to optimized values of the autopilot gear ratios (21), taking into account the wind turbulent disturbances and identification of the angular velocities, ω_x, ω_y are presented in the same figures by dashed graphs (**2**, blue).

The presented data clearly demonstrate that the UAV autopilot parameters synthesized, taking into account wind effects, on average, provide a parry of disturbances caused by atmospheric turbulence, and reduce the amplitude of damped oscillations in the transient processes.

6. Conclusions

Elaborated methods for identification of the parameters and control functions of nonlinear stochastic systems with perturbations, noises, and functional equality type constraints are presented. Important applications relating to optimizing the parameters of the UAV autopilot during its rotational maneuver in translational motion when the turbulent wind disturbances, taking into account the identification of the UAV angular velocities, are briefly represented. The article's main breakthroughs are as follows:

(i) For random controlled processes, a fast calculation of semi-invariants is performed with the necessary accuracy. The results are applied for recurrent parametric optimization on the specified criteria;

(ii) The developed algorithms were implemented in contemporary computer-aided technology of designing UAVs.

Abbreviations

RMS	:	Root Mean Square
SRM		Space Robot-manipulator
UAV	:	Unmanned Aerial Vehicle

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