MANAS Journal of Engineering
ISSN 1694-7398 | e-ISSN 1694-7398

Volume 10, Issue 1, (2022) Pages 42-51
https://doi.org/10.51354/mjen. 1000248

Launching to an orbit with a chemical propellant staged rocket systems

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ABSTRACT

There is one way to explore space by using the space launch vehicles, which is known as rockets, and it can carry useful load named simply as payload of satelite from Earth into Space. In this study, performance predictions of the multi rocket motors are discussed and compared with single rocket motor with the same amount of propellant used for space travel.
In this article in serial or tandem staging schemes, the boosting stage is usually the largest, the second stage and subsequent upper stages are above it, usually decreasing in size are used. In boosting stage parallel staging schemes solid or liquid rocket boosters are used to assist with launch. At low level starting to high altitude higher density fuel solid fuels, kerogen and cryogenic hydrogen(-250°C) are used as fuel. In solid propellants oxidizer is generally ammonium per chloride is used but in cryogenic liquid propellants oxygen(183.3°C) are used.
In the first stage, both liquid propellant in a booster and five solid rocket propellent are used to reach about a certain altitude and velocity. In second stage, after reducing the weight by ejecting the five solid rocket propellent and only liquid propellant is used only to reach the an extra altitudes and velocities at low earth orbit (LEO). Drag and gravity effects are successfully used in all of the calculations.
The added total result of velocities and altidudes found by these staged rockets are higher than the first single staged case. The advantage of multistage rockets, having same amount of propellant in staged rockets where total velocity will be increased by separating and removing waste from the system weight out of the system. Use of staged rocket system are usefull for increasing the amount of payload and decreasing the cost per unit weight as well.

ARTICLE INFO

Research article

Received: 24.09.2022
Accepted: 3.02.2022

Keywords:
Staged and single stage rocket,
useful load,
payload,
low earth orbit (LEO),
apogee,
perigee

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1 Introduction

Most modern, high-performance rocket, particularly those used in space application are the multi-stage rocket systems. Multi-staging is an excellent way to attain higher velocity and higher altitude or more payload by using staged model. Staging is a concept where one engine's velocity is added to an already existing velocity of another, engine one engine's altitude is added to an already existing altitude of another engine. If you keep the velocity same then you can increase your payload for making more economical flight. Mathematical calculation of this case study is presented in three parts as staged rocket calculations.

Tsiolkovsky's Rocket Equation states that the rocket velocity is the function of exhaust velocity and the propellant to total mass change while the propellant is consumed

$$\mathrm{V} = \mathrm{V_e} \mathrm{Ln} \frac{\mathrm{m_0}}{\mathrm{m}} \tag{1.1}$$

where ,
V, required velocity change,
V_e, is exhaust velocity,
m_0 the initial mass and
m the mass of the rocket at each time

$$V_e = I_{SP} g_0 \tag{1.2}$$

I_{SP} is the specific impulse,
$g_0 = 9.80665 \frac{m}{s^2}$, is the gravitational acceleration at sea level.

Specific impulse (usually abbreviated Isp) is a measure of how efficiently a reaction mass engine (a rocket using propellant or a jet engine using fuel) creates thrust. For engines whose reaction mass is only the fuel they carry, specific impulse is exactly proportional to exhaust gas velocity. Specific impulse which may be expressed as total impulse namely thrust time per unit weight of propellant is a fuel property and for solid propellants 180-250s, liquid propellant 300-475s have been used.

Table 1. *Practical Isp values for common rocket propellants[1]*

Propellant type	***Isp [sec]***
Solid	180 - 270
*N2O4/MMH **N2O4/UDMH	260 - 310
Kerogene/LO2	300 - 350
***LH2/LO2	455

*(N2O4 Dinitrogen Tetraoxide) (MMH Monomethylhydrazine) (UDMH
**Unsymmetrical Dimethylhydrazine ((CH3)2NNH2))
***(LH2/LO2 Liquid H2/O2)

2. Theory

In this article, two fundamental types of rocket stages, five solid propellant boosters (SRB) with liquid propellant center core booster (CCB) are at the same time are fired in parallel Fig (1) and Fig (2). A popular method for producing large first stage has been to cluster several SRB rockets together to provide greater combined thrust without actually having to build the larger rocket. Number of solid rocket boosters may be change starting from two to seven. In this article taken as five and when those SRB rockets rocket motors are completely consumed, ejected away in the flight. Until the end of CCB liquid booster continues to burn as booster. After ejection of SRB system since the total weight is reduced CCB may be referred as vertically staging sustainer rocket.

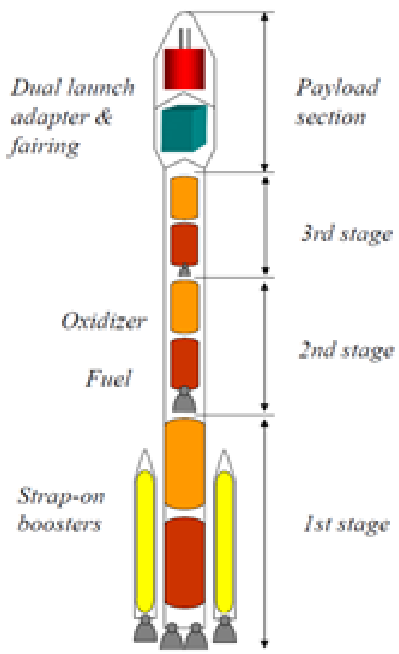

Figure 1. *Three staged Rocket System [2]*

The total rocket system mass is composed of n units of SRB, one CCB with solid and liquid propellant

$$m_T = n m_{SRB} + m_{LCCB} + m_{L1S} + m_{L2S} \qquad (1.3)$$

n , *number solid rocket booster*
m_{SRB}, *solid rocket booster mass*
m_{SRBP}, *solid rocket booster propellant mass*
m_{LCCB}, *total center core mass*
m_{LRBP}, *center core propellant mass*
m_{LB} , total center core mass of the first stage
m_{L1S} , mass of first stage rocket motor
m_{L2S} , mass of second stage rocket motor

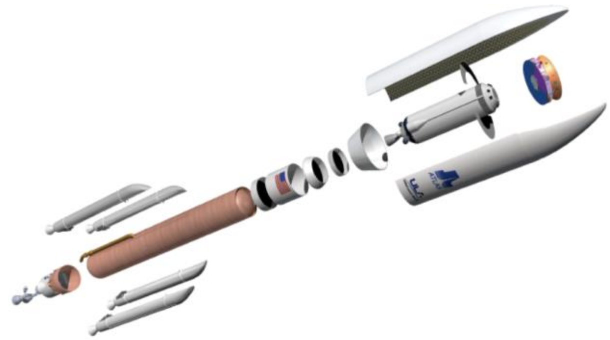

Figure 2. *Multiple Rocket System Configurations [3]*

In the calculation of the altitude of any potential rocket, one must take into account the weights of the rockets and the forces produced by both the thrust of the engine and the gravitational pull of the Earth. A simplified approach can be developed for estimating altitude performance of model rocket vehicles.

Table 2. *U.S. Standard Atmosphere Air Properties - SI Units[4]*

Geo potential Altitude above Sea Level - h - (m)	Temperature - t - (°C)	Acceleration of Gravity - g - (m/s²)	Absolute Pressure - p - (10^4 N/m²)	Density - ρ - (10^{-1} kg/m³)	Dynamic Viscosity - μ - (10^{-5} N.s/m²)
-1000	21.50	9.810	11.39	13.47	1.821
0	15.00	9.807	10.13	12.25	1.789
1000	8.50	9.804	8.988	11.12	1.758
2000	2.00	9.801	7.950	10.07	1.726
3000	-4.49	9.797	7.012	9.093	1.694
4000	-10.98	9.794	6.166	8.194	1.661
5000	-17.47	9.791	5.405	7.364	1.628
6000	-23.96	9.788	4.722	6.601	1.595
7000	-30.45	9.785	4.111	5.900	1.561
8000	-36.94	9.782	3.565	5.258	1.527
9000	-43.42	9.779	3.080	4.671	1.493
10000	-49.90	9.776	2.650	4.135	1.458
15000	-56.50	9.761	1.211	1.948	1.422
20000	-56.50	9.745	0.5529	0.8891	1.422
25000	-51.60	9.730	0.2549	0.4008	1.448
30000	-46.64	9.715	0.1197	0.1841	1.475
40000	-22.80	9.684	0.0287	0.03996	1.601
50000	-25	9.654	0.007978	0.01027	1.704
60000	-26.13	9.624	0.002196	0.003097	1.584
70000	-53.57	9.594	0.00052	0.0008283	1.438
80000	-74.51	9.564	0.00011	0.0001846	1.321

2.1. First Boosting Stage Five Rockets with Solid and Liquid Propellant in CCB

To calculate time and velocity at burn out solid rockets and half of the liquid used the Newton's second law;

$$m_{SLB} = m_T - \frac{5m_{SRBP}}{2} - \frac{m_{LRBP1}}{2} \quad (1.4)$$

$\boldsymbol{m_{SLB}}$, average mass of the rocket system .
m_{LRBP1}, center core first liquid propellant mass of the first stage

Where, the liquid propellant is used with 5 solid propellant rocket is fired then the rest of liquid propellant continue to burn in CCB.

In the configuration design, n unit d_{SB}, diameter of solid rocket boosters are added symmetrically to peripheral of central core booster with diameter d_{CCB}. The aerodynamic cross section areas A_{SLB} are calculated below.

$$A_{SLB} = \frac{\pi}{4}[\mathrm{d}^2_{\mathrm{CCB}} + 5\mathrm{d}^2_{\mathrm{SRB}}] \quad (1.5)$$

2.1.1. Burn out time at boosting stage

$$F = m_{SLB}.a$$

$$\boldsymbol{F = m_{SLB}.\frac{\partial v}{\partial t}}$$

$$T_{SB} = n.T_{SRB} \quad (1.6)$$

$\boldsymbol{T_{LB}}$ ***, Thrust Force of Liquid Booster***

$\boldsymbol{T_{SRB}}$***, Thrust Force of SRB***

$$T_{SLB} = n.T_{SRB} + \mathrm{T_{LB}} \quad (1.7)$$

The drag force, $F_D = \frac{1}{2}\rho_{AIR} v^2 C_D A_{SLB}$ (1.8)

Where, ρ is the air density, and V is the instantaneous speed of the rocket.
The drag coefficient C_D is related to the geometry of the rocket and the quality of flow (laminar, turbulent, etc.) over the surface of the rocket. The quantity A is a reference area to indicate rocket size." k" is air drag coefficient in kg/m and defined as [4,5,6];

$$k_{SLB} = \frac{1}{2}\rho_{AIR}.\sigma_{SLB} C_D.A_{SLB} \quad (1.9)$$

Air density, ρ_{AIR} = 1.223 kg/m³

Drag coefficient, C_D = 0.75

$$m_{SLB}.\frac{\partial v}{\partial t} = T_{SLB} - m_{SLB}.g - k_{SLB}.v^2$$

$$\partial t = \frac{m_{SLB}.\partial v}{k_{SLB}\frac{T_{SB}-m_{SLB}.g}{k_{SLB}} - k_{SLB}.v^2}$$

The terminal velocity of solid and liquid propellant booster phase is given below;

$$\boldsymbol{q_{SLB} = \sqrt{\frac{T_{SLB}-m_{SLB}.g}{k_{SLB}}}} \quad (1.10)$$

$$\boldsymbol{\partial t = \frac{m_{SLB}.\partial v}{k_{SLB}.q^2 - k_{SLB}.v^2}}$$

$$\int \partial t = \frac{m_{SLB}}{k_{SLB}}\int_0^v \frac{\partial v}{q_{SLB}{}^2 - v^2}$$

The burnout time of solid and liquid boosters

$$\boldsymbol{t_{SLB} = \frac{m_{SLB}}{k_{SLB}}\frac{1}{2q_{SLB}}\ln\frac{q_{SLB}+v}{q_{SLB}-v}} \quad (1.11)$$

2.1.2 Burn out velocity at boosting stage

$$\frac{2.q_{SB}.k_{SLB}}{m_{SLB}}t = \ln\frac{q_{SLB}+v}{q_{SLB}-v}$$

Let $x_{SB} = \frac{2.q_{SLB}.k_{SLB}}{m_{SLB}}$
So that

$$-x_{SLB}t = \ln\frac{q_{SLB}-v_{SLB}}{q_{SLB}+v_{SLB}}$$

$$e^{-x_{SLB}t} = \frac{q_{SLB}-v_{SLB}}{q_{SLB}+v_{SLB}}$$

$$e^{-x_{SLB}.t}.q_{SLB} + e^{-x_{SLB}.t}.v_{SLB} = q_{SLB} - v_{SLB}$$

The burnout velocity solid and liquid boosters are expressed as

$$v_{SLB} = q_{SLB}\frac{1+e^{-x_{SLB}t}}{1-e^{-x_{SLB}t}} \quad (1.12)$$

2.1.3. Burn out altitude at boosting stage

For finding burnout altitude similarly Newtons second law is used.

$$F = m_{SLB}\frac{\partial v}{\partial h}\frac{\partial h}{\partial t} = m_{SLB}\frac{\partial v}{\partial h}v$$

$$T_{SLB} - m_{SLB}.g - k_{SLB}v^2 = m_{SLB}.v\ .\frac{\partial v}{\partial h}$$

$$\partial h = \frac{m_{SLB}.}{T_{SLB}-m_{SLB}.g-k_{SLB}v^2} v\, \partial v$$

$$\int \partial h = \frac{m_{SLB}.}{2k_{SLB}} \int \frac{1}{T_{SLB}-m_{SLB}.g-k_{SLB}v^2} \partial v$$

The burnout altitude solid and liquid booster is found as

$$h_{SLB} = \frac{m_{SLB}.}{2.k_{SLB}} ln \frac{T_{SLB}-m_{SLB}.g}{T_{SLB}-m_{SLB}.g-k_{SLB}.v_{SLB}{}^2} \quad (1.13)$$

Velocity and Total Altitude of the Rocket System.

$$v_{BT1} = v_{SLB} \quad (1.14)$$
$$h_{BT1} = h_{SLB} \quad (1.15)$$

2.2 Second Boosting Stage with only Liquid Propellant in CCB

After n solid rockets ejected from the system, new calculations are made for the liquid stage of center core booster with reduced weight of the system. To calculate time and velocity at burnout stage used the Newton's second law;

$$m_{LB} = m_T - n.m_{SRB} - m_{LRBP1} - \frac{m_{LRBP2}}{2}$$

m_{LB} , average mass of system after ejected five rockets
m_{LRBP2}, center core second liquid propellant mass of the first stage

After ejections of n unit d_{SRB} diameter solid rockets, the cross-section area central core booster with diameter d_{CCB} is calculated.

$$A_{LB} = \frac{\pi}{4}[d_{CCB}^2] \quad (1.16)$$

2.2.1. Burnout time at boosting stage

$$F = m_{LB}.a$$
$$\boldsymbol{F = m_{LB}.\frac{\partial v}{\partial t}}$$

$$F = T_{LB} - m_{LB}.g - k_{LB}.v^2$$

"k" is air drag coefficient in kg/m and defined as;

$$k_{LB} = \frac{1}{2}\rho_{AIR}.C_D.A_{LB} \quad (1.17)$$

Air density ,ρ_{AIR} = 1.223 kg/m³

Drag coefficient , C_D = 0.75

Mean altitude value, σ_{BL}

$$m_{LB}.\frac{\partial v}{\partial t} = T_{LB} - m_{LB}.g - k_{LB}.v^2$$

$$\partial t = \frac{m_{LB}.\partial v}{k_{LB}\frac{T_{LB}-m_{LB}.g}{k_{LB}} - k_{LB}.v^2}$$

$$\boldsymbol{q_{LB}{}^2 = \frac{T_{LB}-m_{LB}.g}{k_{LB}}}$$

$$\boldsymbol{q_{LB} = \sqrt{\frac{T_{LB}-m_{LB}.g}{k_{LB}}}} \quad (1.18)$$

Above liquid propellant booster phase terminal velocity is found.

$$\boldsymbol{\partial t = \frac{m_{LB}.\partial v}{k_{LB}.q^2 - k_{LB}.v^2}}$$

$$\boldsymbol{\int \partial t = \frac{m_{LB}}{k_{LB}} \int_0^v \frac{\partial v}{q_{LB}{}^2 - v^2}}$$

The burnout time half of only CCB

$$\boldsymbol{t_{LB} = \frac{m_{LB}}{k_{LB}} \frac{1}{2q_{LB}} ln \frac{q_{LB}+v}{q_{LB}-v}} \quad \mathbf{(1.19)}$$

2.2.2. Burn out velocity at boosting stage

$$\frac{2.q_{LB}.k_{LB}}{m_{LB}} t = ln \frac{q_{LB}+v}{q_{LB}-v}$$

$$Let \quad x_{LB} = \frac{2.q_{LB}.k_{LB}}{m_{LB}} \quad (1.20)$$

$$x_{LB}.t = ln \frac{q_{BL}+v}{q_{BL}-v}$$

$$-x_{LB}t = ln \frac{q_{LB}-v}{q_{LB}+v}$$

$$e^{-x_{LB}t} = \frac{q_{LB}-v}{q_{LB}+v}$$

$$e^{-x_{LB}.t}.q_{LB} + e^{-x_{LB}.t}.v = q_{LB} - v$$

$$s_{LB} = \frac{q+v_{SLB}}{q-v_{SLB}} \quad (1.21)$$

$$v_{LB} = q_{LB} \frac{s_{LB}-e^{-x_{LB}t}}{s_{LB}+e^{-x_{LB}t}} \quad (1.22)$$

2.2.3. Burn out altitude at boosting stage

For finding burnout altitude similarly Newtons second law is used.

$$F = m_{LB}\frac{\partial v}{\partial h}\frac{\partial h}{\partial t} = m_{LB}\frac{\partial v}{\partial h} v = m_{LB} v \frac{\partial v}{\partial h}$$

$$T_{LB} - m_{LB}.g - k_{LB}v^2 = m_{LB}.v.\frac{\partial v}{\partial h}$$

$$\partial h = \frac{m_{LB}\cdot}{k_{LB}q^2 - k_{LB}v^2}\partial v$$

$$\int \partial h = \frac{m_{LB}\cdot}{k_{LB}}\int \frac{v}{q_{LB}{}^2 - v^2}\partial v$$

$$h_{LB} = \frac{m_{LB}\cdot}{2.k_{LB}}\, ln \frac{T_{LB} - m_{LB}.g - k_{LB}v_{SB}{}^2}{T_{LB} - m_{LB}.g - k_{LB}v_{LB}{}^2} \quad (1.23)$$

Velocity and Total Altitude of the Rocket System.

$$v_{BT2} = v_{SLB} + v_{LB} \quad (1.24)$$
$$h_{BT2} = h_{SLB} + h_{LB} \quad (1.25)$$

2.3. Calculation Of Time, Velocity And Altitude At Stage 1

Average mass of system after the ejection of center core booster, at first stage

$$m_{L1} = m_T - 5m_{SRB} - m_{LB} - \frac{m_{L1P}}{2} \quad (1.26)$$

m_{L1} , avarage mass of system after the ejection of center core booster, at first stage
m_{L1S}, 1. stage liquid propellant rocket mass with its structural mass
m_{L1SP}, 1. stage liquid propellant

The cross section of first stage with a diameter of d_{L1}

$$A_{L1} = \frac{\pi}{4}[d_{L1}^2] \quad (1.27)$$

2.3.1 Burn out time at first stage

$$F = m_{L1}.a$$

$$\boldsymbol{F = m_{L1}.\frac{\partial v}{\partial t}}$$

$$F = T_{L1} - m_{L1}.g - k_{L1}.v^2$$

T_{L1} , thrust of first stage

" k_{L1}" is air drag coefficient in kg/m and defined as;

$$k_{L1} = \frac{1}{2}\rho_{AIR}.\sigma_{L1}C_D.A_{L1} \quad (1.28)$$

Air density ,ρ_{AIR} = 1.223 kg/m³

Drag coefficient , C_D = 0.75

$$m_{L1}.\frac{\partial v}{\partial t} = T_{L1} - m_{L1}.g - k_{L1}.v^2$$

$$\partial t = \frac{m_{L1}.\partial v}{k_{L1}\frac{T_{L1} - m_{L1}.g}{k_{L1}} - k_{L1}.v^2}$$

$$q_{L1}{}^2 = \frac{T_{L1} - m_{L1}.g}{k_{L1}}$$

$$q_{L1} = \sqrt{\frac{T_{L1} - m_{L1}.g}{k_{L1}}} \quad (1.29)$$

Above liquid propellant sustainer phase terminal velocity is found.

$$\boldsymbol{\partial t = \frac{m_{L1}.\partial v}{k_{L1}.q_{L1}{}^2 - k_{L1}.v^2}}$$

$$\boldsymbol{\int \partial t = \frac{m_{L1}}{k_{L1}}\int_0^v \frac{\partial v}{q^2 - v^2}}$$

$$t_{L1} = \frac{m_{L1}}{k_{L1}}\frac{1}{2q_{L1}}\ln\frac{q_{L1}+v}{q_{L1}-v} \quad (1.30)$$

2.3.2 Burn out velocity at boosting stage

$$\frac{2.q_{L1}.k_{L1S}}{m_{L1}}t = ln\frac{q_{L1}+v}{q_{L1}-v}\ , \text{ Let } \quad x_{L1} = \frac{2.q_{L1}.k_{L1}}{m_{L1}} \quad (1.29)$$

So that $x_{L1}.t = ln\frac{q_{L1}+v}{q_{L1}-v}$

$$-x_{L1}t = ln\frac{q_{L1}+v}{q_{L1}-v}$$
$$e^{-x_{L1}t} = \frac{q_{L1}-v}{q_{L1}+v}$$
$$e^{-x_{L1}.t}.q_{L1} + e^{-x_{L1}.t}.v = q_{L1} - v$$
$$\boldsymbol{s_{L1} = \frac{1+v_{BT2}}{1-v_{BT2}}} \quad (1.31)$$

the burnout velocity half of only CCB ($t = t_{L1}$)

$$\boldsymbol{v_{L1} = q_{L1}\frac{s_{L1} - e^{-x_{L1}t}}{s_{L1} + e^{-x_{L1}t}}} \quad (1.32)$$

2.3.3 Burnout altitude at boosting stage

For finding burnout altitude similarly Newtons second law is used.

$$F = m_{L1}\frac{\partial v}{\partial h}\frac{\partial h}{\partial t} = m_{L1}\frac{\partial v}{\partial h}v = m_{L1}v\frac{\partial v}{\partial h}$$
$$T_{L1} - m_{L1}.g - k_{L1}v^2 = m_{L1}.v.\frac{\partial v}{\partial h}$$
$$\partial h = \frac{m_{L1}.g}{k_{L1}q_{L1}{}^2 - k_{L1}v^2}\partial v$$
$$\int \partial h = \frac{m_{L1}.g}{k_{L1}}\int \frac{v}{q_{L1}{}^2 - v^2}\partial v$$

The burnout altitude at the second half of only CCB:

$$h_{L1} = \frac{m_{L1}.g}{2.k_{L1}}.\, ln\frac{T_{L1} - m_{L1}.g - k_{L1}v_{BT2}{}^2}{T_{L1} - m_{L1}.g - k_{L1}v_{L1}{}^2} \quad (1.33)$$

Velocity and Total Altitude of the Rocket System.

$$v_{TS2} = v_{SLB} + v_{LB} + v_{L1} \quad (1.34)$$
$$h_{TS2} = h_{SLB} + h_{LB} + h_{L1} \quad (1.35)$$

2.4. Calculation of time, velocity and altitude at stage 2

$$\mathrm{m_{L2}} = \mathrm{m_T} - 5\mathrm{m_{SRB}} - 2\mathrm{m_{LB}} - \mathrm{m_{CCBS}} - \mathrm{m_{L1P}} - \mathrm{m_{1S}} - \frac{\mathrm{m_{L2P}}}{2} \quad (1.36)$$

m_{L2} , avarage mass of system during the second stage.
m_{L2} 2. Stage liquid rocket mass
m_{L2P} 2. Stage liquid rocket propellant mass

The cross section of second stage with a same diameter of d_{L2}

$$A_{L2} = \frac{\pi}{4}[d_{L2}^2] \quad (1.37)$$

$$k_{L2} = \frac{1}{2}\rho_{AIR}.C_D.A_{L2} \quad (1.38)$$

$\mathrm{T_{L2}}$ = Thrust of first stage rocket after booster

Burn out time at first stage,
Terminal velocity of the first stage after booster:

$$q_{L2} = \sqrt{\frac{T_{L2} - m_{L2}.g}{k_{L2}}} \quad (1.39)$$

$$\mathrm{x_{L2}} = \frac{2.\mathrm{q_{L2}}.\mathrm{k_{L2}}}{\mathrm{m_{L2}}} \quad (1.40)$$

$$\mathrm{s_{L2}} = \frac{\mathrm{q_{L2}} + \mathrm{v_{T1}}}{\mathrm{q_{L2}} - \mathrm{v_{T1}}} \quad (1.41)$$

$$\mathrm{v_{L2}} = \mathrm{q_{L2}}\frac{\mathrm{s_{L1}} - e^{-\mathrm{x_{L1}}t}}{\mathrm{s_{L1S}} + e^{-\mathrm{x_{L1}}t}} \quad (1.42)$$

$$\mathrm{h_{L2}} = \frac{\mathrm{m_{LB}}}{2.\mathrm{k_{L2}}}\ln\frac{\mathrm{T_{CCB}} - \mathrm{m_{L2}}.g - \mathrm{k_{L2}}{\mathrm{v_{T1}}}^2}{\mathrm{T_{CCB}} - \mathrm{m_{L2}}.g - \mathrm{k_{L2}}{\mathrm{v_{L2}}}^2} \quad (1.43)$$

All velocity and altitude calculations were made for booster and sustainer rockets before are added to find out Total Velocity and Total Altitude of the Rocket System:

$$v_{T2} = v_{SLB} + v_{BL} + v_{L1} + v_{L2} \quad (1.44)$$

$$h_{T2} = h_{SLB} + h_{LB} + h_{L1} + h_{L2} \quad (1.45)$$

In the remaining system of the two-stage booster and two stage sustainer rockets with some constructional mass, completes its mission with orbit engines running with some chemical fuel and of course with some payload.

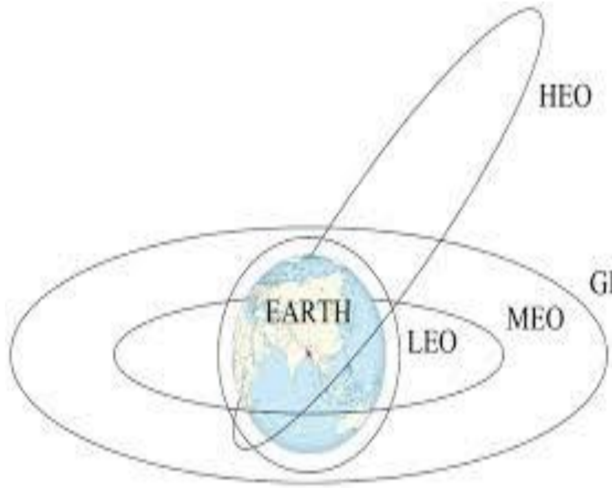

Figure 3. *Orbit altitudes[3] LEO satellites are confined between 500 to 1500 km. altitude. The GEO satellite altitude is of around 36,000 km, MEO satellites altitude is in the range of 10,000 to 15,000 km. and MEO and LEO satellites are referred to as Non-Geostationary Orbit (NGSO) satellites.*

LEO orbits are confined between 500 to 1500 km. altitude. The GEO satellite altitude is of around 36,000 km, MEO satellites altitude are in the range of 10,000 to 15,000 km. and MEO and LEO orbit are referred as Non-Geostationary Orbit (NGSO) Fig (3). In this study designer is to replace the payload to orbit in LEO. Then it is possible to send the payload to GEO or even to the Moon with this infrastructure.

3. Scope of work, results and discussion

In this study, a vehicle with multistage rockets to reach a Low Earth Orbit were designed. The rocket used as solid propellants in five solid rocket booster (SRB) and a liquid propellant in common core booster (CCB) which divided in two tanks.

At the boosting (first stage), the solid propellants in the five solid rocket booster and liquid propellant in CCB is ignited for 90 seconds. After the five solid rockets with pay rejected from the rocket, the other liquid propellant in the boosting stage. The duration of this stage is about 180 second. The table (3) shows all data which are partially taken for design vehicle in boosting phase [7].

Table 3. *Principal boosting phase design data[7]*

Item BOOSTING PHASE	Total
Mass of launch vehicle (kg)	467000
SOLID ROCKET BOOSTERS(SRB)	
Solid Rocket Booster (SRB) no.	5
Fuel mass (Kg)	41000x5=205000
Structure mass (Kg)	5740x5=28700
Burning Time(s)	90
I sp (s)	275
Diameter (m)	1.55
Total Thrust (N)	5x1688400=8442000
COMMON CORE BOOSTERS CCB	
Fuel mass1 (Kg), m_{LRBP} =90s(1030)kg/s=61800kg	92700
Fuel mass2 (Kg), m_{LRBP} = 90s(1030)kg/s=92700kg	92700/185400
Structure mass(Kg)	21173
Burning Time(s)	90/90+
Specific Impulse (Isp) (s)	460
Diameter (m)	3.81
Thrust (n) at sea level	3827000
Thrust (n) at airless(vacuum)	4152000

$$m_T = (nm_{SRB} + m_{LCCB}) + m_{L1S} + m_{L2S}$$

n *number solid rocket booster =5*
m_{SRB} *solid rocket booster mass =47000kg*
m_{SRBP} *solid rocket booster propellant mass =41000kg*

m_{CCBS} *structural mass of the CCB* $= 21173kg$
m_{LBP1} *common core propellant mass* $= 185400kg$
m_{LCCB} *total center core mass =206573kg*

m_{L1S}, 1. *stage liquid propellant with structural mass* $= 22247k$
m_{L1SP}, 1. *stage liquid propellant* $= 20000kg$

m_{L2S}, 2. *stage liquid propellant with structural mass =2018kg*
m_{L2SP}, 2. *stage liquid propellant* $= 1850kg$

3.1. First Boosting Stage Five Rockets with Solid and Liquid Propellant in CCB For 90 Seconds

D= Overall Diameter
D =3.82+1.58=5.4
A = rocket cross-sectional area in m2 A_{SLB} *=22.9m2*
t=90s
$m_{SRBP} = 41000kg$

m_{LRBP} *=1030kg/s(90s)= 92700kg*

$$m_{SLB} = m_T - \frac{5m_{SRBP}}{2} - \frac{m_{LRBP1}}{2} = 308250kg$$

$T_{SRBT} = 5(1680000N)$ *= 8400000N Thrust of Solid Booster*
$T_{LRB} =$ $3827000N$ *=Thrust of Liquid Booster*

Air density, ρ_{AIR} *= 1.223 kg/m³* *Drag coefficient ,* C_D *= 0.75*

Table 4. *First boosting stage five rockets with solid and liquid propellant in ccb for 90 seconds*

BS T*	$k_{SLB}(\frac{kg}{m})$	$q_{SLB}(\frac{m}{s})$	$v_{SLB}(\frac{m}{s})$	$h_{SLB}(m)$	$v_{BT1}(\frac{m}{s})$	$h_{BT1}(m)$
	10.5	931	927	57644	927	57644

3.2 Second boosting stage liquid propellant ın CCB for 90-180 seconds

D= d_{CCB} *= Overall Diameter*
D =3.82+1.58=5.4m
A = rocket cross-sectional area in m2 A = A_{SLB} *=22.9m2*
$m_{CCBS} = 21173kg$
m_{LBP} *=1030kg/s(90s)=92700kg*

$$m_{LB} = m_T - \frac{5m_{SRB}}{2} - m_{LRBP} - \frac{m_{LRBP}}{2} = 467000 - 5(47000) - 92700 - \frac{92700}{2} = 82950kg$$

$T_{LB} = 3827000N$ *=Thrust of liquid booster*

t=90s

Table 5. *Second boosting stage liquid propellant in CCB for 90-180 seconds*

CCB liquid prop	$k_{SLB}(\frac{kg}{\ })$	$q_{SLB}(\frac{m}{s})$	$v_{SLB}(\frac{m}{s})$	$h_{SLB}(m)$	$v_{BT1}(\frac{m}{s})$	$h_{BT1}(m)$
Results	10.5	535	535	23480	1362	81124

$$v_{BT2} = v_{SLB} + v_{LB} = 927\frac{m}{s} + 535\frac{m}{s} = 1362\frac{m}{s}$$
$$h_{BT2} = h_{SB} + h_{LB} = 57644m + 23480\text{m} = 81124\text{m}$$

In the typical case, the first-stage and booster engines fire to propel the entire rocket upwards. When the boosters run out of fuel, they are detached from the rest of the rocket (usually with some kind of small explosive charge or explosive bolts) and fall away. The first stage then burns to completion and falls off. This leaves a smaller rocket, with the second stage on the bottom, which then fires.

Known in rocketry circles as staging, this process is repeated until the desired final velocity is achieved. In some case with serial staging, the upper stage ignites before the separation—the inter stage ring is designed with this in mind, and the thrust is used to help positively separate the two vehicles.
In this article at start 467ton of three stage rocket system first stage which is called as booster's final altitude is found as 81 km. It is in the range of LEO and vertical velocity is found as 1362 m/s. 205 tons of solid propellant and 185.4 tons liquid propellants are consumed. Additionally,30tons of solid rocket and 21.173tons of used bile load reduced the weight of the system and are creating smaller drag forces on the system at higher altitudes.

3.3. First stage after boosting

The RL10 is a liquid fuel cryogenic rocket engine built in the United States by Aerojet Rocketdyne that burns cryogenic liquid hydrogen and liquid oxygen propellants. Modern versions produce up to 110 kN (24,729 lbf) of thrust per engine in vacuum.

Inert mass: 2,247 kg (4,954 lb) Length: 12.68 m (42 ft
Fuel: Liquid Hydrogen Oxidizer: Liquid oxygen
Fuel and Oxidizer mass: 12000kg
Thrust: 99.2 kN (22,300 lbf)
Burn time: Variable, t=100s (842 s on Atlas V)

$$m_{L1} = m_T - 5m_{SRB} - 2m_{LB} - m_{CCBS} - \frac{m_{L1P}}{2}$$
$$=467000\text{-}5(47000)\text{-}92700 - 92700 - 21173 - \frac{20000}{2} = 5427kg$$

$$A_{L1} = \frac{\pi}{4}[d_{L1}^2] = \frac{\pi}{4}[1.51^2] = 1.79m2$$

$$T_{L1} = 198400\ N$$

Table 6. *First stage after boosting*

Stage ı	$k_{L1}(\frac{kg}{m})$	$q_{L1}(\frac{m}{s})$	$v_{L1}(\frac{m}{s})$	$h_{SLB}(m)$	$v_{TL1}(\frac{m}{s})$	$h_{TL1}(m)$
Results	0.0012	6262	2556	505260	4018	586384

$$v_{TL1}=927\frac{m}{s} + 535\frac{m}{s}+2556=4018\frac{m}{s}$$
$$h_{TL1} = 57644m + 23480m+505260m = 586384m$$

$$V_{OR1} = \sqrt{\frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(5.98\times10^{24}kg)}{6378000\,m + 586000\,m}} = \sqrt{\frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(5.98\times10^{24}kg)}{69.6400}} = 7568m/s$$

At 586 km altitude the orbit velocity 7568m/s which is still higher than 4018 m/s and not able to enter the orbit at 586 km altitude.

3.4. Second stage after booster

Engine Mass,168kg(restartable) Engine Length,2.32m
Engine Diameter,1.51m
Propellant, Hydrazine Attitude Control,4X40N Thrusters

$$m_{L2} = m_T - 5m_{SB} - 2m_{LB} - m_{CCBS} - m_{L1P} - m_{1S}$$
$$m_{L3} = 467000\text{-}\ 5(47000)\text{-}\ 185400 - 21173 - 20000 - 2247kg = 3180kg$$

So far booster and first stage rocket motors has an 586km altitude and 4018 m/s velocity. If its velocity is increased to very close to 8000 m/s, at appropriate altitude the orbit transfer can be completed. According to calculations for this purpose 1847kg of fuel should be added the system for the required fuel to make the velocity around 8000m/s which means we need second stage rocket.

$$m_{LP2} = 3180\left(1 - e^{-\frac{4000}{4600}}\right) = 1847kg$$

In order to reach orbit velocity, required mass=1847kg has been taken as 1850kg

Engine dry-weight: 168 kg

$$m_{L3} = m_{L2} - \frac{1850}{2} = 3180\text{-}925 = 2255kg$$
$$A_{L2} = \frac{\pi}{4}[d_{L2}^2] = \frac{\pi}{4}[1.51^2] = 1.79m2$$
$$T_{L2} = 99200\ N$$
$$t = \frac{m_{L2}}{k_{L2}}\frac{1}{2q_{L2}}\ln\frac{q_{L2}+v}{q_{L2}-v} = \frac{2255}{0.0006(11334)2}\ln\frac{11334+4018}{11334-4018} = 123s$$
$$t_{L3} = 100s$$
$$v_{L2} = q_{L2}\frac{s_{L2}-e^{-x_{L2}t}}{s_{L2}+e^{-x_{L2}t}} = 11334\frac{2.10-0.55}{2.10+0.55} = 6629\frac{m}{s}$$
$$t_{L2} = 80s$$

Table 7. *Second stage after boosting*.

Stage ıı	$k_{L2}(\frac{kg}{m})$	$q_{L2}(\frac{m}{s})$	$v_{L2}(\frac{m}{s})$	$h_{L2}(m)$	$v_{TL2}(\frac{m}{s})$	$h_{TL2}(m)$
Results	0.0006	11314	4273	879042	8291	1586560

$$v_{TL2}=927\frac{m}{s} + 535\frac{m}{s}+2556+4273\frac{m}{s}=8291\frac{m}{s}$$
$$h_{TL2} = 57644m + 23480m+647534+879042m = 1586560m$$

The staged rocket system either the velocity and altitude calculated above can enter the orbit below the 1500 km which we define as LEO. The corresponding orbit velocity at 1500 km found as

$$V_{OR1500} = \sqrt{\frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(5.98\times10^{24}kg)}{6378000\,m + 1500000\,m}} = \sqrt{\frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(5.98\times10^{24}kg)}{78.78000}} = 7116m/s$$

This means whenever second stage system enters to orbit at any height it may it may go to the higher altitudes by elliptic orbit transfer or even go to the other space regions.

In the literature, artificial satellites are first launched into the desired altitude by conventional liquid/solid propelled rockets after which the satellite may use onboard propulsion systems for orbital station keeping. Once in the desired orbit, they often need some form of attitude control so that they are correctly pointed with respect to the Earth, the Sun, and possibly some astronomical object of interest.[9] They are also subject to drag from the thin atmosphere, so that to stay in orbit for a long period of time some form of propulsion is occasionally necessary to make small corrections (orbital station-keeping).[10] Many satellites need to be moved from one orbit to another from time to time, and this also requires propulsion.[11] A satellite's useful life is usually over once it has exhausted its ability to adjust its orbit.

Table 8. *Satellite orbit engines working with chemical fuel*

Thrust Power (kW)	Battery Weight (kg)	Power Distribution Control Unit (kg)	Power Sub Unit (kg)	Solar Antenna (kg)	Total Weight
				88	163
5	25	20	30	39	114
				35	110
				162	267
10	25	30	50	72	177
				65	170
				250	385
15	25	40	70	111	246*
				100	235

4. Conclusions

In this article a multistage rocket is required to reach orbital speed,7116m/s and successively verified by 3 stage system is the velocity of 8291m/s at 1500km altitude.

Payload is the object or the entity which is being carried by an aircraft or launch vehicle. For a rocket, the payload can be a satellite, space probe, or spacecraft carrying humans, animals or cargo.

Firstly in [8], the travelling to Moon is discussed. By using the data Table (3) and the results of the article, it is possible to travel to Moon. The payload is found as 758 kg which may be considered as a reasonable payload. The vehicle 3180m payload by consuming nearly 407 kg Lunar Transfer Fuel for a hyperbola that encounters the Moon for 6 days, total velocity change has been expected approximately 4000m/s having at least with 7116 m/s launcher velocity. The mass driver starts orbiting at 1500km at this Earth altitude to Lunar Orbit of 100km height of Moon orbit Table (9).

Table 9. *Direct transfers from 185-km circular Earth orbits to 100-km prograde lunar orbits, given in inertial frame[8].*

$m_{L2}(kg)$	m_{LP2}(kg)	Total Lunar Transfer Fuel(kg)	2ndStage Mass(kg)	Payload(kg)
3180	1847	407	168	758

Secondly if this vehicle is separated from main mass body, 168kg of 2nd stage the new capsule is satellite may go to the to Geosynchronously Equatorial Orbit in Space at 35,786m altitude[9].

Table 10. *Direct transfers from 80-km circular Earth orbits to 42164-km GEO satellite orbits, given in inertial frame[10].*

$m_{L2}(kg$	m_{LP2} (kg)	Apogee / Perigee Fuel (kg)	Orbit engine (kg)	2.Stage Mass (kg)	Payload (kg)
3012	184 7	38/246 (284)	246 *Table (10)	168	467

This results are evaluated from perigee to apogee circular orbit transfers consuming approximately 284 kg of fuel and using 15kw power satellite orbit engine Table(10). Finally, reasonable mass of payload is found as 467kg.

In the scope of this article, mathematical modelling of gravity and drag calculations with changing density of the air with altitude are introduced by a successfully. It is found that multi staged chemical rockets are advantageous with respect to single rockets which has almost 90% of system propellant mass where the system is found highly expensive due to limited amount of payload.

In the literature, rather than relying on high temperature and fluid dynamics to accelerate the reaction mass to high speeds, there are a variety of methods that use electrostatic or electromagnetic forces to accelerate the reaction mass directly by a stream of ions such an engine typically uses electric power, first to ionize atoms, and then to create a voltage gradient to accelerate the ions to high exhaust velocities by using solar, nuclear electric rockets. But, power generation adds significant mass to the spacecraft, and ultimately the weight of the power source limits the performance of the vehicle. Ion thrusters, electro thermal thrusters, electromagnetic thrusters and mass drivers for propulsion are the typical examples of electromagnetic methods.

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