



Tracking control of a ball on plate system using PID controller and Lead/Lag compensator with a double loop feedback scheme

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Abstract

Nowadays, technology is in continuous evolution, movement and balance control are in full expansion (humanoid robots, drones and parallel robots). The difficulty that robotics software developers are often facing is maintaining stability and balance, a robot that loses its balance is an instant threat to its environment, therefore, the ball on plate system is the best method to test the performance of a controller to ensure the balance of a given system.

This platform is an upgraded version of the ball and beam system, it is a multivariable and a nonlinear system which has an underactuated feature that makes it one of the most complicated systems in terms of control, requiring reliable, efficient and fast controllers to meet the end goal of this task. In this work, two types of controllers for the ball stabilization, classical PID controller and Lead / Lag compensator in a double loop feedback scheme, were presented in order to achieve a fast and precise response with a minimal tracking error. Finally, a comparison of the results obtained by these two control techniques was made which revealed the superiority of the Lead / Lag compensator in dealing with this kind of system.

Keywords: Nonlinear & underactuated system, PID, Lead/Lag compensator, Ball stabilization.

PID denetleyicisi ve çift döngülü geri bildirim şemasına sahip Öncü/Gecikme dengeleyici kullanılarak bir plaka sistemi üzerinde bir topun izleme kontrolü

Öz

Günümüzde teknoloji sürekli evrim halindedir, hareket ve denge kontrolü tamamen genişlemektedir (insansı robotlar, dronlar ve paralel robotlar). Robotik yazılım geliştiricilerinin sıklıkla karşılaştığı zorluk, istikrarı ve dengeyi korumaktır, dengesini kaybeden bir robot, çevresi için anında bir tehdit oluşturur, bu nedenle, plaka üzerinde top sistemi, bir denetleyicinin performansını test etmek için en iyi yöntemdir. Belirli bir sistemin dengesi.

Bu platform, bilya ve kiriş sisteminin yükseltilmiş bir versiyonudur, çok değişkenli ve doğrusal olmayan bir sistemdir, bu sistem onu kontrol açısından en karmaşık sistemlerden biri haline getiren, gereksinimleri karşılamak için güvenilir, verimli ve hızlı kontrolörler gerektiren bir underactuated özelliğine sahiptir. Bu görevin nihai hedefi. Bu çalışmada, minimum izleme hatası ile hızlı ve kesin bir yanıt elde etmek için top stabilizasyonu için iki tip kontrolör, klasik PID kontrolör ve çift döngülü geri besleme şemasında Önde/Gecikme dengeleyici sunulmuştur. Son olarak, bu iki kontrol tekniği ile elde edilen sonuçların bir karşılaştırması yapıldı ve bu tür bir sistemle uğraşırken Öncü / Gecikme kompensatörünün üstünlüğünü ortaya koydu.

Anahtar Kelimeler: Doğrusal olmayan ve yetersiz çalıştırılmış sistem, PID, Öncü/Gecikme dengeleyici, Top stabilizasyonu

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1. Introduction

Maintaining balance is a challenging task and can be used effectively to illustrate the role of control system, especially in mechatronics application [1] The Ball-and-Plate (B&P) system can be described as an enhanced version of a Ball-and-Beam system, whereby the ball positioning is controlled in dual directions. The B&P system finds practical applications in several dynamic systems, such as robotics, rocket systems, and unmanned aerial vehicles. The enumerated systems are often expected to follow a time parameterized reference path [2]. Due to the BP complexity and its non-linearity, the mathematical model of the BPS presents uncertainties which increase the difficulty of designing a suitable controller.

For more effective control, PID controller and lead & lag compensator are tested based on double loop feedback structure, where the inner loop provides the necessary effort for the rotation of the servo motor, while the outer loop comes up with the necessary servo motor angle which allows reaching the desired position.

Some researches have been done on similar systems, a comparison between different methods of control of ball and plate system with 6DOF Stewart platform was developed (Kassem, A., Haddad, H., & Albitar, C. (2015)), PID, LQR, SMC and fuzzy Logic controllers were implemented, The proposed four strategies were applied to 6DOF ball on platesystem, the best controller according to the author is the SMC although it presents chattering, which can cause the servomotor to malfunction. Furthermore, Sliding mode control for the Trajectory of ball and plate system was designed (Liu,H.,& Liang, Y. (2010, March).), this controller proves anovertime the presence of vibrations in the system response. A third comparison was proposed (Betancourt, F. I. R., Alarcon,S. M. B., & Velasquez, L. F. A. (2019, October).), the system's response for both controllers has a considerable settling time and shows the presence of a measurable steady state error.

The aim of this work is to design an optimized controller in terms of performances compared to those already implemented in previous works. These two controllers will be compared to each other with reference to the settling time, overshoot and steady state error.

This paper is divided into three sections, the first describes the mathematical modeling of the ball on plate system, the second addresses the design and the implementation of the controllers, finally, the third part is based on tracking and simulation with comparison of the obtained results.

2. Mathematical Model

The following equations are based on [3] and [4], the Lagrangian approach that can be defined as :

$$L = T - V \tag{1}$$

where (T) is the kinetic energy and (V) is the potential energy.

The Euler-Lagrange equation is written as following :

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{d}{dt} \frac{\partial T}{\partial q_i} + \frac{d}{dt} \frac{\partial V}{\partial q_i} = \partial Q_i \tag{2}$$

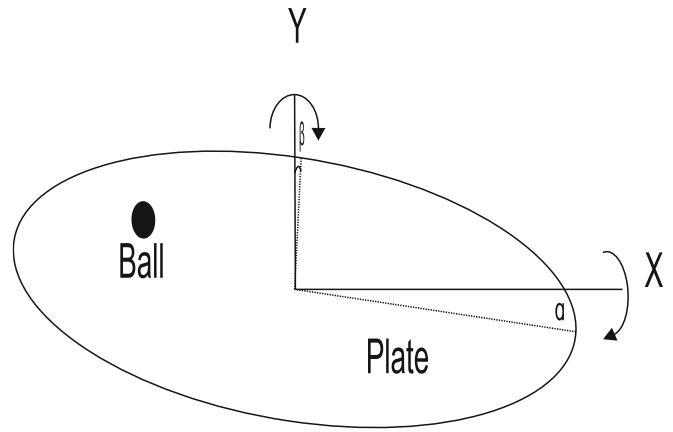


Figure 1. Structure of the ball on plate system

The kinetic energy is given by:

$$T = T_b + T_p \tag{3}$$

The ball kinetic energy can be determined by adding rotational energy to translational energy:

$$T_b = \frac{1}{2} m_b (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I_b (w_x^2 + w_y^2) \tag{4}$$

Where m_b is the translational inertia (mass) and I_b is the rotational inertia; (\dot{x}^2, \dot{y}^2) and (w_x^2, w_y^2) are the ball's linear velocities and the ball's angular velocities along x and y axis respectively. The following relations are confirmed since the ball is rotating without slippage :

$$\dot{x}_b^2 = r_b + w_y, \dot{y}_b^2 = r_b + w_x \tag{5}$$

where r_b is the radius of the ball. By substituting the equations (5) into equation (4) we obtain :

$$T_b = \frac{1}{2} (m_b + \frac{I_b}{r_b^2}) (\dot{x}_b^2 + \dot{y}_b^2) \tag{6}$$

Similarly, the plate mathematical model is derived:

$$T_p = \frac{1}{2} (I_p + I_b) (\dot{\alpha}^2 + \dot{\beta}^2) + \frac{1}{2} m_b (x_b \dot{\alpha} + y_b \dot{\beta})^2 \tag{7}$$

The potential energy expression of the ball V is given by:

$$V = m_b g (x_b \sin \alpha + y_b \sin \beta) \tag{8}$$

Substituting the potential energy equation (8), the ball and plate kinetic energy equation (6) and (7) respectively in the Euler Lagrange equation (1) we obtain:

$$(m_b + \frac{I_b}{r_b^2}) \ddot{x}_b - m_b (x_b \ddot{\alpha}^2 + y_b \ddot{\alpha} \dot{\beta}) + m_b g \sin \alpha = 0 \tag{9}$$

$$(m_b + \frac{I_b}{r_b^2}) \ddot{y}_b - m_b (y_b \ddot{\beta}^2 + x_b \ddot{\alpha} \dot{\beta}) + m_b g \sin \beta = 0 \tag{10}$$

$$tx = (I_p + I_b + m_b x_b^2) \ddot{\alpha} + 2m_b x_b \dot{x}_b \dot{\alpha} + m_b x_b y_b \dot{\beta} + m_b \dot{x}_b y_b \dot{\beta} + m_b x_b \dot{y}_b \dot{\beta} + m_b g x_b \cos \alpha \tag{11}$$

$$ty = (I_p + I_b + m_b y_b^2) \ddot{\beta} + 2m_b y_b \dot{y}_b \dot{\beta} + m_b x_b y_b \ddot{\alpha} + m_b \dot{x}_b y_b \dot{\alpha} + m_b x_b \dot{y}_b \dot{\alpha} + m_b g y_b \cos \beta \tag{12}$$

Where the nonlinear equations of motion (9) and (10) describe the movement of the ball on plate, equations (11) and (12) describe the effect of the servo motor torque on the ball on plate system.

(α, β) are the input angles of the system, ($\dot{\alpha}, \dot{\beta}$) are the angular velocities and ($\ddot{\alpha}, \ddot{\beta}$) are the angular accelerations. Designing a controller for such a complex nonlinear system is almost impossible, thus, to simplify the mathematical model we linearize the system.

2.1. Model linearization :

Due to the complexity of the ball on plate model, linearization is an essential part of the controller design. Since the system inputs are α and β angles, equations (11) and (12) are very hard to work with, equations (9) and (10) are chosen to be linearized around the equilibrium points. Assuming the angular velocities are very small ($\dot{\alpha} \approx 0, \dot{\beta} \approx 0$), also is the plate inclination $[-5^\circ + 5^\circ]$, let us assume: $\sin\alpha \approx \alpha, \sin\beta \approx \beta$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{x}_b + m_b g \alpha = 0 \tag{13}$$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{y}_b + m_b g \beta = 0 \tag{14}$$

Where:

$$I_b = \frac{2}{5} m_b r_b^2$$

As long as the transfer function is the same for both X and Y

Table 1. Servo motor parameters

Parameter	Value	Unit
R_a (Armature Resistance)	1.8	Ohm
L_a (Armature Inductance)	0.24	Henry
B (Frictionl Coefficient of Motor & Load)	0.01	N.m.s
J Moment of Inertia of Motor & Load	0.0006	Kg.m2
K_b Back emf constant	0.24	V.S/rad
K_t Torque constant	0.24	V/(rad/sec)

3. Controller design:

In this section, we will start by giving the global structure of the whole system which is represented in figure (3). This type of structure allows us to use two controllers, each of which has a well-defined function, so the closed loop control will be more efficient, on the contrary, the closed loop with only one signal feedback presents more difficulties especially if we want to obtain good results in terms of stability, robustness and precision. In our case, we will use two types of controllers to test their performances, starting firstly with the PID which will be compared to the lead / lag compensator, the simulation results will be shown in the last section.

axes, in the following sections we will only work with X axis and we will apply the same to the Y axis.

2.2. Servo motor modelling:

The aim of the inner loop is to maintain the plate inclination at the desired point, to do so, we start by deriving the servo motor transfer function.

Figure (2) shows the block diagram of the servo motor transfer function.

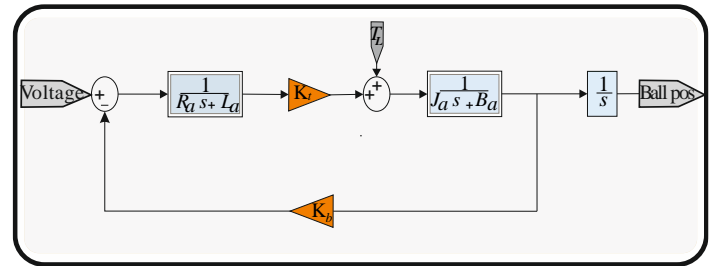


Figure 2. Servo motor transfer function

2.2.1 Servo motor parameter identification:

In the table below, the servo motor parameters are identified using MATLAB toolbox.

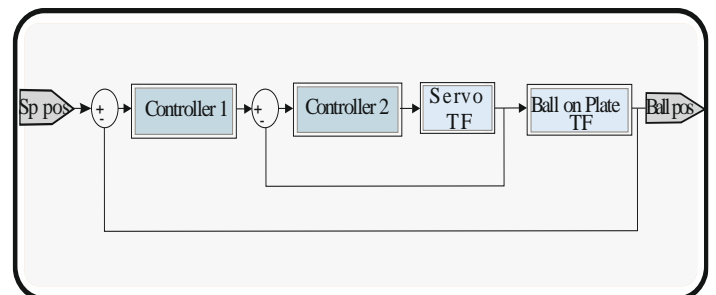


Figure 3. General scheme of ball on plate system

3.1 PID controller:

Firstly, we will use two PID controllers, the aim of the one in the inner loop is to maintain the servo angle at a specified desired input calculated by a second PID controller in the outer loop. The outer controller has as input the difference between the output of the B&P transfer function -ball X coordinate as input; and the desired X coordinate.

Figure (4) displays the double loop feedback structure using PID controller.

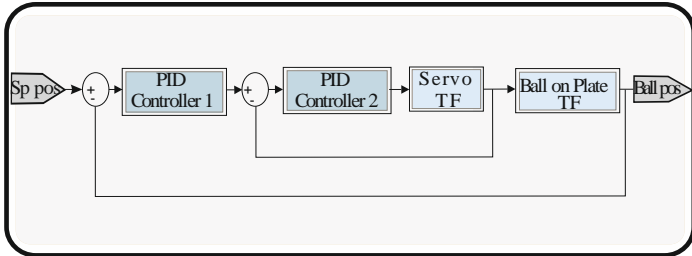


Figure 4. PID controller double loop feedback scheme

3.1 Lead & lag compensator :

Likewise, the lead & lag compensator has the same aim as PID controller, Actually, the implementation of this compensator was a little bit complex, we have chosen to use double lead & lag compensator to get the best response possible. Figure (5) exhibits the double loop feedback structure using lead & lag compensator.

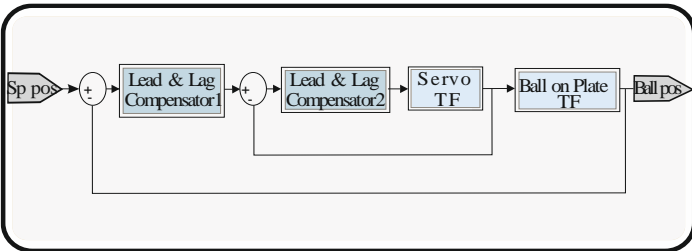


Figure 5. Lead & Lag compensator double loop feedback scheme

4. Simulation results:

4.1 PID controller simulation results:

As it can be seen in Figure (6), the system's response achieves the desired X coordinate after approximately 2.5s with an overshoot of 21%.

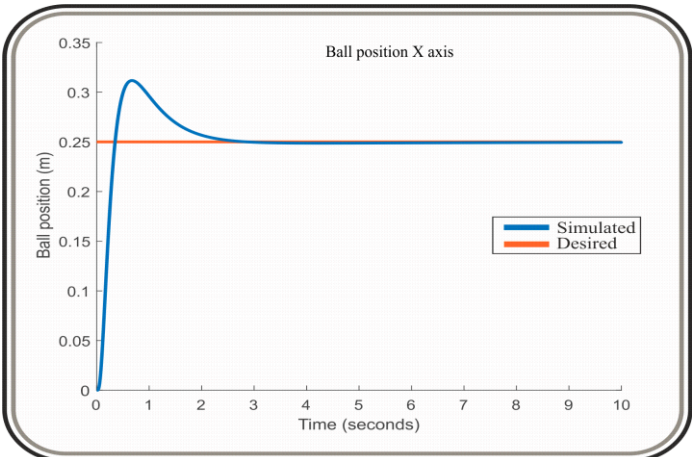


Figure 6. System response using PID controller

We can also notice the presence of a small steady state error, the more we decrease the settling time, the bigger the steady state error is, to solve this problem, we injected the desired input (multiplied by certain value) to the system output and the results are shown in figure (7).

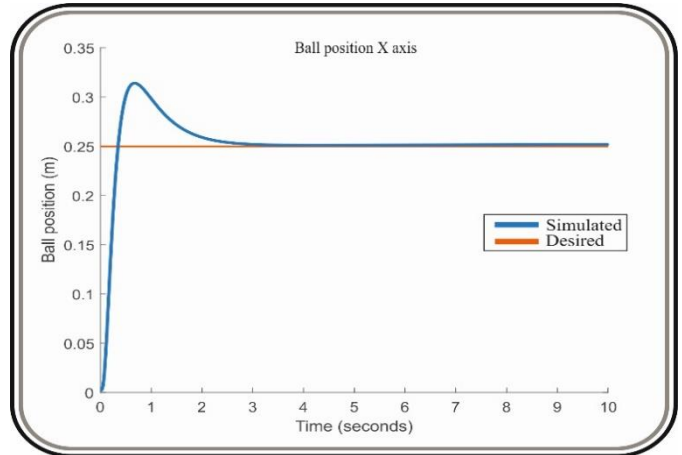


Figure 7. System response using adjusted PID control

4.2 Lead & Lag compensator simulation results:

Similarly, we can notice in figure (8) that the system response using lead & lag compensator achieves the desired X coordinate after approximately 1.9s with approximately the same overshoot value. further more, the steady state error showed is almost zero which makes the system more precise than the first controller.

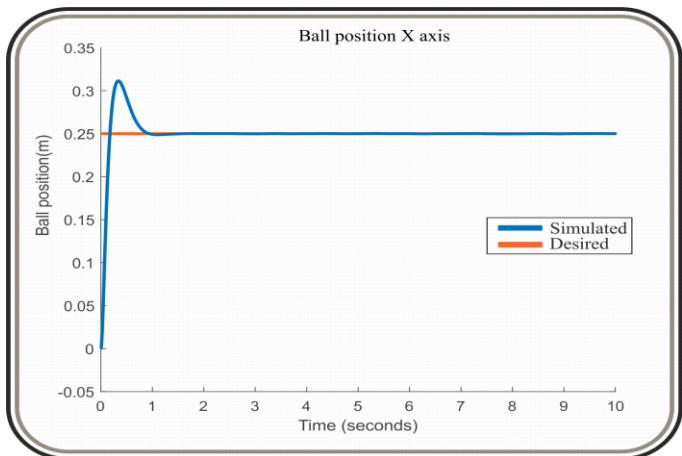


Figure 8. System response using Lead&Lag compensator

Figure (9) represents the controller effort, we can see that both controllers present approximately no chattering effect which makes the servo motor's work safely (chattering has an undesirable effect on the servo motor and actuators in general).

Unlike the SMC controllers presented in the previous research works, both PID & lead & lag compensator have a better reaction to the error variation.

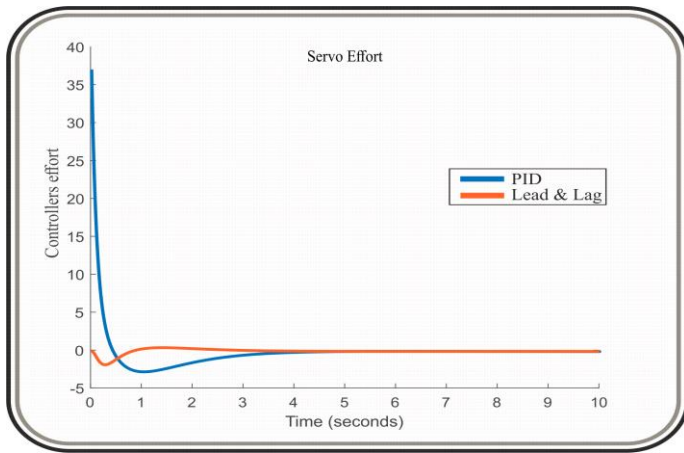


Figure 9. Controllers Effort

4.3 Comparison results:

Figure (10) illustrates the comparison between the controllers used. We can clearly see that lead & lag compensator has better performance.

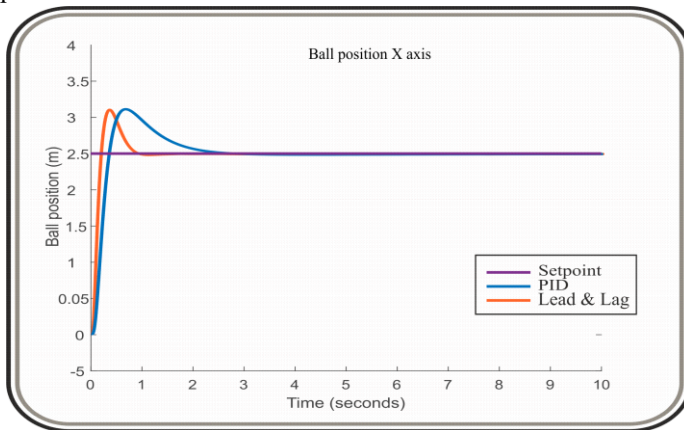


Figure 10. Comparison between the designed controllers.

4.2 Tracking results:

In this section, we will focus on tracking, we have simulated the system response of a sinusoidal function which is shown in figure (11).

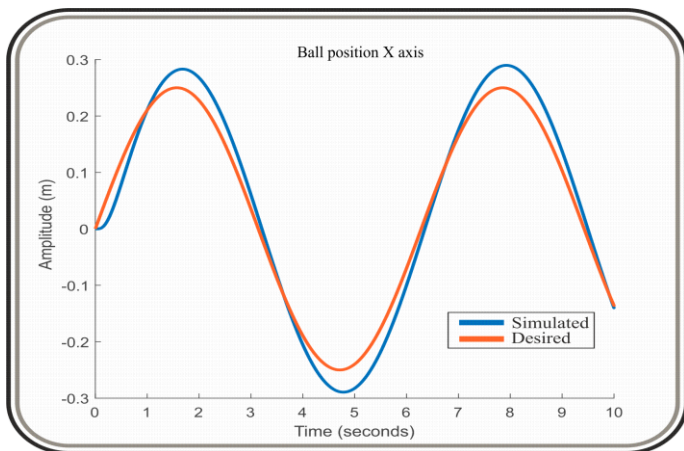


Fig 11. Ball tracking using PID controller

We can see that a small error is present, the system takes a small time to reach the desired value with a small overshoot value.

likewise, figure (12) illustrates the system response due to a sinusoidal desired input.

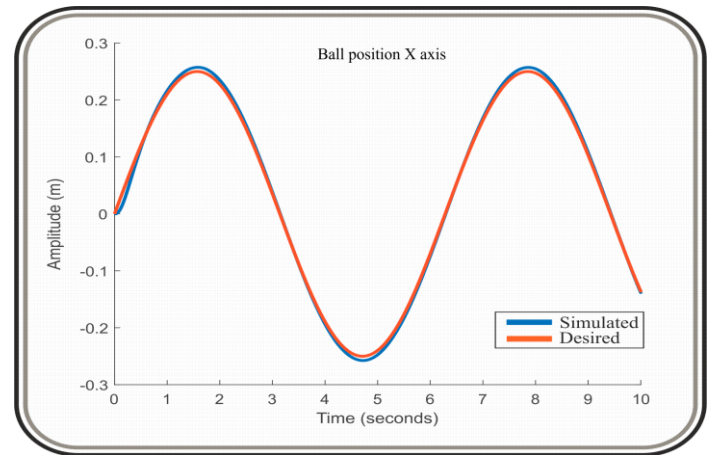


Fig 12. Ball tracking using Lead&Lag compensator

In this case, we see that the response of the system follows the reference with a high precision rate. lead & lag compensator was able to minimize the error while maintaining a small response time.

We can clearly see that lead & lag compensator has better performance in terms of settling time, overshoot and steady state error.

5. Conclusion:

In this research paper, we started with deriving the mathematical model of the BPS. Then, we linearized the mathematical model, after that, using the double feedback loop structure, we were able to implement two different types of controllers, the first one is a PID controller, the second one is a lead & lag compensator, the results showed that the latter offers better performances than PID in terms of response time, overshoot and static error. We can therefore, conclude that lead & lag compensator is better enough for it to be chosen. Further studies can be done by comparing lead & lag compensator with a controller that can handle uncertainties, such as fuzzy logic, mpc algorithms or optimized wavelet NiF with bat algorithm...etc.

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