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CONSTRUCTAL STRUCTURES FOR SELF-COOLING: MICROVASCULAR WAVY AND STRAIGHT CHANNELS

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ABSTRACT

This paper shows that a conductive domain which is subjected to heating from its bottom can be cooled with embedded microvascular cooling channels in it. The volume of the domain and the coolant are fixed. The actively cooled domain is mimicked from the human skin (which regulates temperature with microvascular blood vessels). The effect of the shape of cooling channels (sinusoidal or straight) and their locations in the direction perpendicular to the bottom surface on the peak and average temperatures are studied. In addition, the effect of pressure difference in between the inlet and outlet is varied. The pressure drop in the sinusoidal channel configurations is greater than the straight channel configurations for a fixed cooling channel volume. The peak and average temperatures are the smallest with straight cooling channels located at y = 0.7 mm. Furthermore, how the cooling channel configuration should change when the heat is generated throughout the volume is studied. The peak and average temperatures are smaller with straight channels than the sinusoidal ones when the pressure drop is less than 420 Pa, and they become smaller with sinusoidal channel configurations when the pressure drop is greater than 420 Pa. In addition, the peak and average temperatures are the smallest with sinusoidal channels for a fixed flow rate. Furthermore, the peak temperatures for multiple cooling channels is documented, and the multiple channel configurations promise to the smallest peak temperature for a fixed pressure drop value. This paper uncovers that there is no optimal cooling channel design for any condition, but there is one for specific objectives and conditions.

INTRODUCTION

Emerging technologies require structures with smart capabilities such as self-cooling and self-healing. These smart features can be added to a structure with embedded vascular channels [1-3]. Bio-mimicry is the inspiration for the engineered structures with smart features. However, the objectives and constraints of a biological system may be different than engineered structures. Therefore, instead of copying the design of the mimicked biological structure, the design of the engineered structure should be altered to fit for the objectives and constraints in order to uncover the best possible design with smart features.

Constructal theory shows that changing the design as the objectives, constraints and boundary conditions of a system vary decreases the resistances to the flows (heat, fluid, stress and so on). There is no optimum design but best designs for the given conditions and objectives [4, 5]. Therefore, the use of constructal theory is essential in bio-mimicry in order to design engineered structures which performs the best. The application of the constructal theory is vast, and it is used in diverse fields such as biology, geophysics, social dynamics, physics and engineering [3-14].

White et al. [15] showed that a structure can gain self-healing capability with embedded microcapsules which are filled with healing agents. Later, Therriault et al. [16] illustrated that embedding a three-dimensional vascular channel network in a structure in which healing agent flows can also gain self-healing capability to a structure. Because, in theory, the material can heal itself countless times with embedded vascular channels, self-healing with embedded channels is superior to with microcapsules. In addition, Odom et al. [17] showed that properties of healed structures such as electrical conductivity can be restored with self-healing. Bejan and Lorente [18] illustrated that vascular structures provide volumetric cooling and mechanical strength in addition to gaining self-healing capability to the structure. Kim et al. [19] also showed vascular structures provide volumetric cooling. Wang et al. [20] uncovered how the mechanical strength of a structure is affected by the shape of the embedded vascular channels. Later, Cetkin et al. [3] illustrated how the cooling performance and mechanical strength of a structure is affected by the shape of the vascular channels simultaneously. Cetkin et al. [21] uncovered how the mechanical strength of a structure can be increased by embedding cooling channels in it when the thermal stresses are not negligibly small. In the current literature, a configuration for the cooling channels is selected such as sinusoidal [22-23] or serpentine [24], and the selected design is optimized with given parameters (i.e., volume fraction, boundary conditions and initial conditions). Therefore, there is no comparison on the cooling performance of different configurations in the literature, but the optimized designs for selected configurations and conditions [22-24].

This paper focuses on how the shape of microvascular channels affect the cooling performance for variable boundary conditions and objectives. Even though Soghrati et al. [22] showed how the design of microvascular channel shape affects the cooling performance of an actively-cooled microvascular fin, it lacks of further analysis related with pressure drop penalty. There are two penalties in a system cooled with embedded cooling channels in which fluid flows: due to heat transfer resistances and due to fluid flow resistances. Though the heat transfer surface area increases with sinusoidal microvascular channels, flow resistances increase (i.e. convection heat transfer coefficient decrease) with sinusoidal channels. There is a trade-off between the heat flow and fluid flow resistances. This paper shows which shape (straight or sinusoidal) performs better in which conditions

MODEL

Consider a two-dimensional rectangular domain in which embedded with a sinusoidal microvascular cooling channel, Fig. 1. The length and height of the domain is L= 60 mm and H = 10.4 mm, respectively. The embedded sinusoidal-shaped micro-channel with diameter D = 500 μm, amplitude A = 4 mm, and wavelength λ = 10 mm is shown in Fig. 1 (a). The total volume of the structure and the flow volume are fixed. The domain is subjected to heating from its bottom and other surfaces are in contact with ambient air. The coolant enters from the left side of the sinusoidal channel, cools the domain down and exists from the right side of it.

The coolant flowing in the micro-channels is water with k_f = 0.6 W/(m K), ρ = 1000 kg/m^3, and c_P = 4183 J/(kg K), and the flow rate is Q = 2 ml/min. The rectangular domain is made of epoxy matrix with thermal conductivity k_e = 0.23 W/(m K). The domain is heated from its bottom with a constant power G = 2.1 W. Other boundaries of the domain are in contact with air at ambient temperature T_{amb} = 21 ^{o}C, and the convection heat transfer coefficient is h = 4.9 W/(m K).

The fluid flow is governed by the mass conservation and momentum equations for incompressible steady flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\nabla^2 u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\nabla^2 v \tag{3}$$

Here $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$; furthermore x and y are the spatial coordinates, u and v are the velocity components corresponding to these coordinates, and P, ν and ρ are the pressure, kinematic viscosity and fluid density. The fluid is single phase with constant properties.

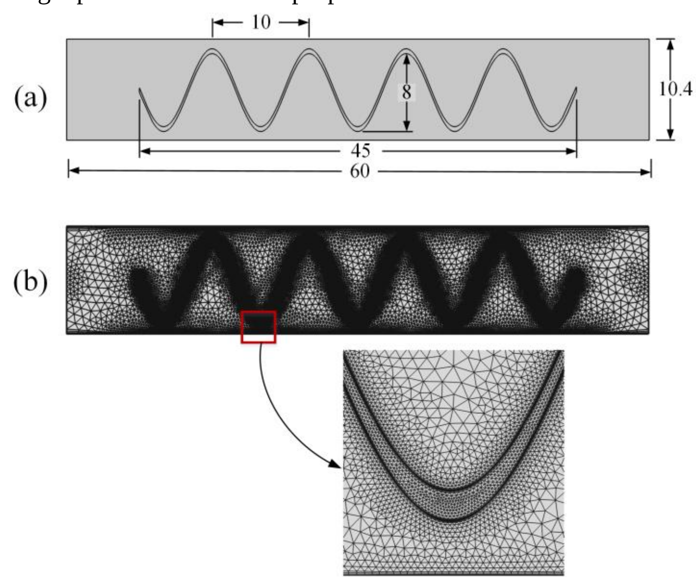

FIGURE 1: GEOMETRY (a) AND COMPUTATION GRID (b) OF A CONDUCTIVE DOMAIN WHICH IS COOLED WITH EMBEDDED SINUSODIAL COOLING CHANNELS.

The temperature distribution inside the flow channels is found by solving the energy equation

$$\rho c_P\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k_f\nabla^2 T \tag{4}$$

where c_P is the specific heat at constant pressure of the fluid, T is the temperature, and k_f is the fluid thermal conductivity. The continuity of heat flux between the solid and fluid interfaces requires

$$k_e\frac{\partial T}{\partial n}=k_f\frac{\partial T}{\partial n} \tag{5}$$

where n is the vector normal to the fluid-solid interface. The temperature distribution inside the solid is obtained by solving $\nabla^2 T=0$.

NUMERICAL MODEL

Consider the two-dimensional domain with sinusoidal cooling channels as shown in Fig. 1 (a). The flow rate and the bottom surface temperature are 0.724 ml/min and 65^{o}C, respectively. Side and top boundaries of the domain is cooled by convection with convection coefficient of 4.9 W/(m K). The ambient temperature is 21 ^{o}C. The maximum temperature difference occurs in the channels affect the thermophysical properties less than 3%. Therefore, thermophysical properties was taken as constant in numerical simulations.

The mass conservation, momentum and energy equations were solved by using a finite element software [25]. The grid was non-uniform both in x and y directions, Fig. 1 (b). Boundary meshes are used at the solid-liquid interfaces and at the boundaries in order to minimize the numerical errors due to variation in the temperature gradients. The mesh size was determined by successive mesh refinement, increasing the number of the mesh elements until the criterion $\left|\left(T^{n}_{max} - T^{n+1}_{max}\right)/T^{n}_{max}\right| < 0.01$ was satisfied. T^{n}_{max} and T^{n+1}_{max} represent the maximum temperature by using the current mesh and the refined mesh, respectively. Table 1 illustrates that mesh independency was achieved with 57051 number of mesh for the given boundary conditions.

TABLE 1: NUMERICAL TESTS SHOWING THAT MAXIMUM TEMPERATURE IS INDEPENDENT OF THE MESH SIZE.

Number of elements	V^{n}_{max}	T^{n}_{max}	T^{n}_{avg}	$\left|\left(T^{n}_{max} - T^{n+1}_{max}\right)/T^{n}_{max}\right|$
10093	0.0461	62.71815	55.01873	
21749	0.05118	32.75513	34.94651	0.477741
35105	0.04862	32.19418	34.68157	0.017126
57051	0.04969	31.91551	34.57476	0.008656

The numerical results obtained by using the current method have been compared with the experimental and numerical results obtained by Soghrati et al. [22] to check the accuracy. Figure 2 (b) shows the match between the results of the current study and the results of Ref. [22] for the location shown with dashed line in Figure 2 (a). The current numerical model is a good approximation of the experimental method.

SINUSOIDAL VS LINE

The thermo-fluid performance of an actively cooled domain is affected by the shape of the coolant channels because of two penalties: resistance to the fluid flow and resistance to the heat flow. Soghrati et al. [22] only considers the effect of heat flow resistances by fixing the flow rate. However, the flow rate in a channel is a function of shape. Therefore, changing the shape (i.e. varying the channel diameter and length) affects the order of the flow rate for a fixed pressure drop which is supplied by a pump. Flow resistance for laminar and incompressible flow in a circular channel can be expressed as follows.

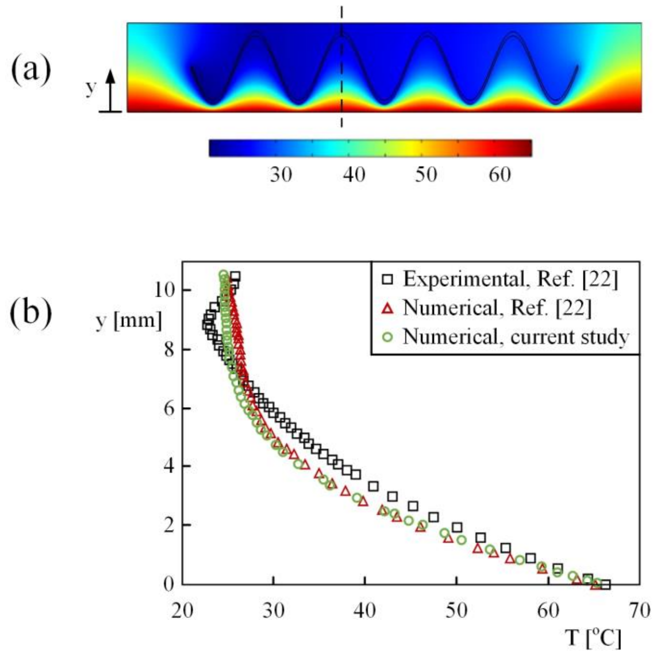

FIGURE 2: TEMPERATURE DISTRIBUTION ON THE DOMAIN (a), TEMPERATURE DISTRIBUTION ALONG THE DASHED LINE FOR THE CURRENT STUDY AND REF. [22] (b).

$$\frac{\Delta P}{\dot{m}} = \frac{128\nu}{\pi}\frac{L}{D^4} \tag{6}$$

Flow resistance formulation shows that mass flow rate is proportional to D^4 and inversely proportional to L. Therefore, the flow rate drops dramatically when the shape of the cooling channels are changed from straight channels to sinusoidal channels if the total volume of the coolant and the pumping power are fixed. The pressure drop in between the inlet and outlet boundaries is fixed at 560 Pa which satisfies the results shown in Fig. 2 (b) in order to uncover the effect of flow resistances.

In addition, the bottom surface of the domain was fixed at 65^{o}C in the validation study, Ref. [22]. This temperature was taken from experimental study when the system reaches to steady state. However, the temperature distribution and the bottom surface temperature is affected by the variation in the designs of the cooling channels because the heat and fluid flow resistances change. Therefore, using the constant temperature boundary condition at the bottom surface does not uncover the effect of the cooling channel shape on the temperature distribution. The bottom surface boundary condition was changed to boundary heat source with total power of 185W which satisfies the temperature distribution shown in Fig. 2.

Figure 3 shows temperature distribution for three designs: sinusoidal (a), straight channel located in the middle of the domain (b), straight channel located at y = 0.7 mm from the bottom (c). The pressure drop in between the inlet and outlet boundaries, the total domain area and the cooling channel area were fixed for these three designs and the bottom surface of the domain is subjected to constant heat flux. Figure 3 shows that the peak temperature is the smallest with the straight channel located at y = 0.7 mm for the given boundary conditions. Furthermore, the peak temperature is the greatest with straight channel located in the middle. Because the flow rate in the sinusoidal channel is a lot smaller than straight channels, the temperature of the coolant fluid increases more along the channel in comparison with the straight channels. Therefore, even though the heat transfer surface area is increased in sinusoidal design (i.e. heat flow resistances are decreased by decreasing the spacing in between the cooling channels and conductive domain), the peak temperature is greater in sinusoidal design than the straight channel designs. In addition, the peak temperatures occur at the solid domain and the liquid temperature always smaller than the boiling temperature (i.e. 100^{o}C for water at atmospheric pressure) in all configurations and solutions.

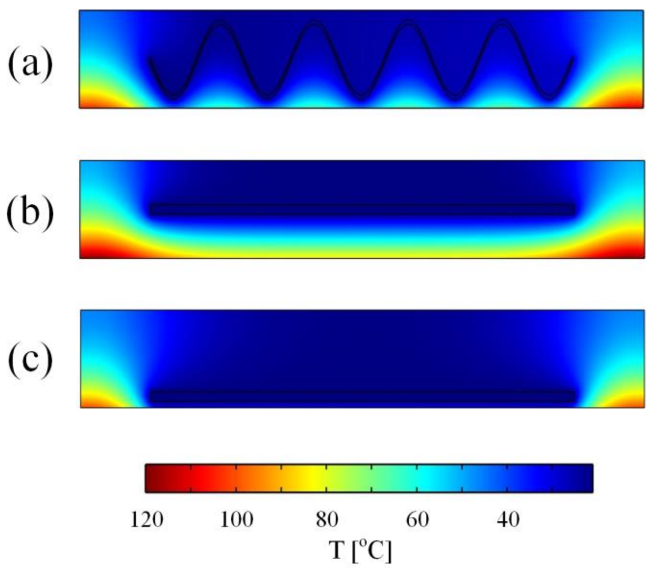

FIGURE 3: STEADY-STATE TEMPERATURE FIELD OF THE COOLED DOMAIN FOR THREE CONFIGURATIONS: SINUSODIAL (a), STRAIGHT CHANNEL LOCATED IN THE MIDDLE (b) AND STRAIGHT CHANNEL LOCATED AT Y = 0.7 mm (c).

In addition, the location of the straight channel affects the order of the peak and surface averaged temperatures, Fig. 4. Figure 4 also shows how the peak and average temperatures are affected by the pressure difference. The sinusoidal channel design is more sensitive to the change in the pressure drop than the straight channels because the flow resistance in sinusoidal design is a lot greater. The peak and average temperatures increase approximately 40 and 20^{o}C for sinusoidal channel as the pressure drop decreases from 560 Pa to 70 Pa. However, the peak and average temperatures do not change more than 1^{o}C when the channels are straight and the pressure drop decreases from 560 Pa to 70 Pa. The heat capacity of the fluid (i.e. flow rate) is great enough to ensure negligibly small increase in the temperature distribution in straight channel designs even the pressure difference is 70 Pa. Figure 4 shows that even varying the pressure difference from 70 Pa to 560 Pa does not change the order of the peak and average temperatures for straight channels, and the effect of varying the location of straight channels is dramatic.

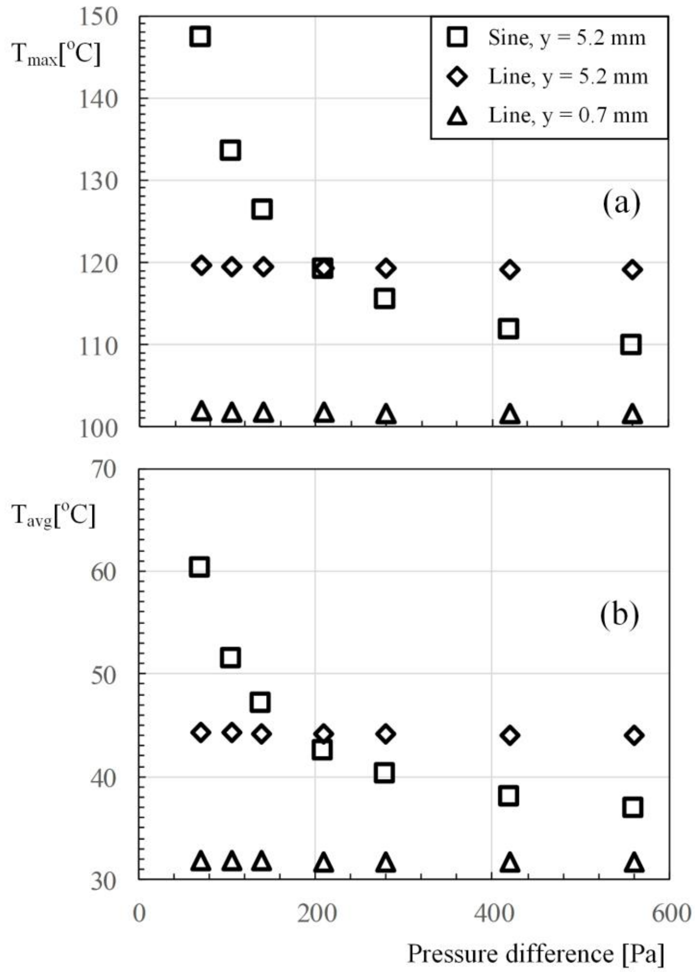

FIGURE 4: THE EFFECT OF PRESSURE DIFFERENCE ON THE PEAK (a) AND AVERAGE (b) TEMPERATURES FOR COOLING CHANNEL CONFIGURATIONS OF FIG. 3.

Figure 5 shows how the maximum and average temperatures vary relative to the location of the straight cooling channel for various pressure drop and channel thickness values. Maximum and average temperatures decrease as the straight channel moves from the middle of the domain to the bottom of it. At y = 0.7 mm, both maximum and average temperatures are minimum. As the straight channel is moved from y = 0.7 mm to the bottom surface, maximum and average temperatures increase. The reason of this increase is that the cooling channel is shorter (45 mm) than the domain (60 mm), and the uncooled spacing at the left and right side of the domain increases when the cooling channel is located at the bottom. However, if the cooling channel length is the same as the domain length, then the cooling channel should be placed to the bottom surface in order to decrease the peak and average temperatures. This result shows that the cooling channels should be placed near the heated surfaces which is expected because as the distance between the heated surface and the cooling channels increase, the thermal resistances also increase. Therefore, using a sinusoidal cooling channel as proposed in Soghrati et al. [22] is not the best option when the heat is applied on one surface.

Sinusoidal cooling channel configuration is selected in Soghrati et al. [22] because of bio-mimicry. However, the objectives and boundary conditions of the mimicked biological systems do not match with the designed structure. Therefore, the optimal structures for the given problem and the mimicked biological structure should be different. The mimicked structure is the microvascular blood vessels in the human skin. However, in addition to regulating temperature distribution, these vessels carry food and oxygen to the cells. Therefore, they should be in contact with all the cells near them, which makes sinusoidal channel structure a better option rather than straight channels. In addition, heat is not applied from one of the surfaces but generated throughout the volume, and these microvascular channels regulate temperature by carrying hot blood near the cold skin surface. Therefore, the effect of heat generation in the domain should be studied to uncover how the sinusoidal channels perform in comparison with the straight channels.

HEAT GENERATION

Consider that the heat is generated in the domain rather than applying from the bottom surface. The heating power is fixed, and its value is equal to the applied heat from the bottom. Domain area, coolant volume and the location of the straight channel (in the middle) are fixed, and only the shape of the cooling channels and pressure difference in between the inlet and outlet were varied. Figure 6 shows that the order of the maximum and average temperatures decrease as pressure difference increases for the sinusoidal and straight channel configurations. The decrease in the straight channel configuration is negligibly small in comparison with the order of the temperatures. However, the change in the sinusoidal configuration is dramatic. Figure 6 also shows that the peak and average temperatures are smaller with straight channel configuration when the pressure difference is less than 420 Pa. As the pressure difference becomes greater than 420 Pa, the peak and average temperatures become smaller with sinusoidal configuration.

In addition, the effect of flow rate on the maximum and average temperatures with heat generations is also studied. Figure 7 shows how the maximum and average temperatures vary relative to the flow rate for sinusoidal and straight channels. The peak and average temperatures decrease as the flow rate increases, and increasing the flow rate more than 0.4 ml/min does not change the order of the peak and average temperatures significantly in comparison with the order of temperatures. In addition, the peak and average temperatures are the smallest with sinusoidal cooling channels for a given pressure difference value. However, the required pumping power to supply same flow rate is a lot greater in sinusoidal cooling channel configurations, Fig. 6.

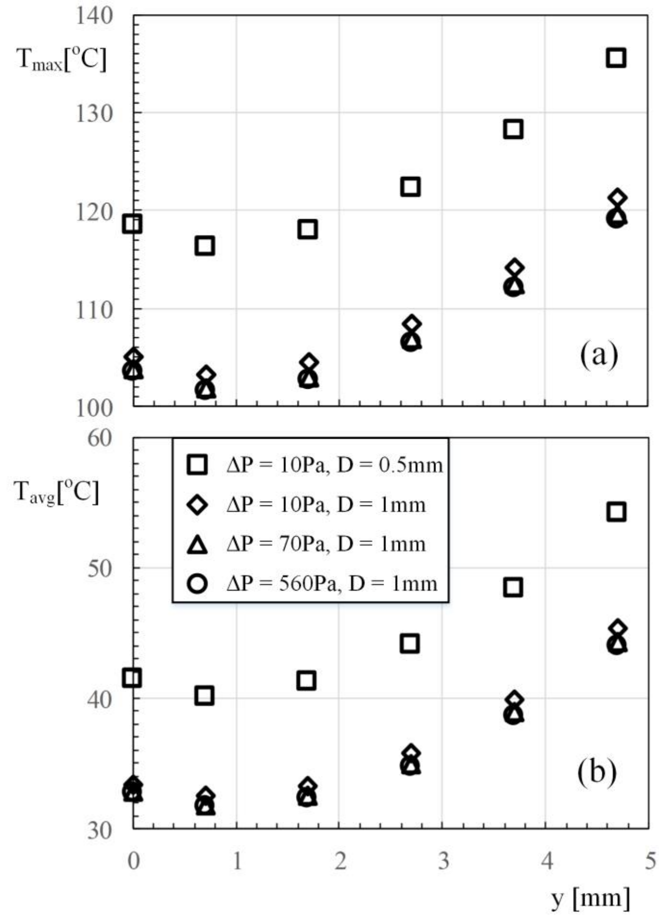

FIGURE 5: THE EFFECT OF CHANNEL LOCATION IN THE Y DIRECTION ON THE PEAK (a) AND AVERAGE (b) TEMPERATURES WITH STRAIGHT COOLING CHANNELS.

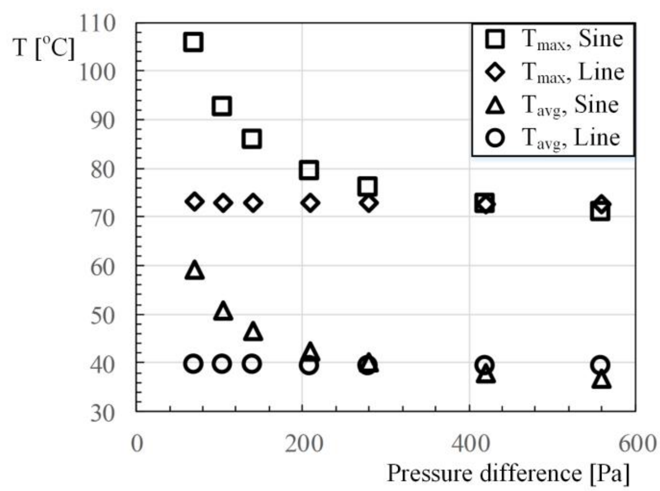

FIGURE 6: THE EFFECT OF PRESSURE DIFFERENCE ON THE PEAK AND AVERAGE TEMPERATURES WHEN THE HEAT IS GENERATED THROUGHOUT THE CONDUCTIVE DOMAIN.

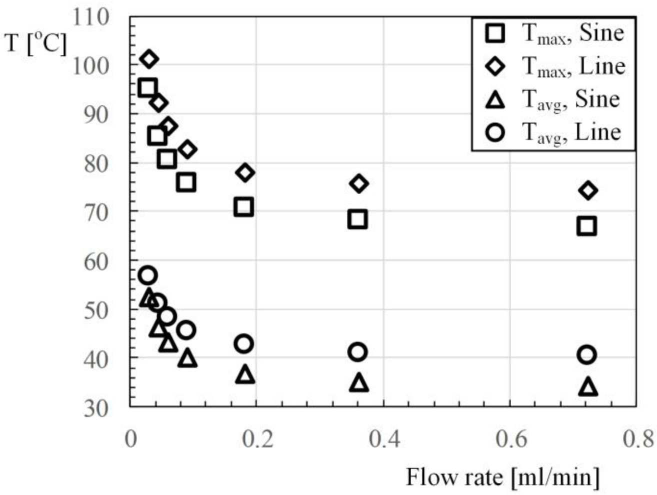

FIGURE 7: THE EFFECT OF FLOW RATE ON THE PEAK AND AVERAGE TEMPERATURES WHEN THE HEAT IS GENERATED THROUGHOUT THE CONDUCTIVE DOMAIN.

Figures 6 and 7 show that the sinusoidal cooling channel configuration performs better when the pressure drop is more than 420 Pa or the flow rate is the same with the straight channel configurations. Even though, the peak and average temperatures are smallest with sinusoidal channel configurations when the pressure drop is greater than 420 Pa, the peak and average temperatures do not vary more than 5°C if the channel becomes straight. In summary, if the objective is to ensure smallest peak and average temperatures, then the structure should be sinusoidal. However, if the objective is to cool the domain under and allowable temperature by using minimum amount of pumping power than the straight cooling channel configuration should be selected.

MULTIPLE COOLING CHANNELS

Next, consider there are multiple straight cooling channels inserted in the conductive domain. The heat is generated on the conductive domain. The fluid and solid volumes are fixed, and they are the same as in the previous sections. The distance in between the cooling channels and pressure drop in between the inlet and outlet boundaries are varied. Increasing the number of cooling channels decreases the diameter of the cooling channels because the fluid volume is fixed. Therefore, the velocity of the coolant which flows through the channels decrease, so is the convection coefficient rate. However, because there are multiple channels, the cooled region is the summation of the thermal boundary layers of these channels. Therefore, there should be an optimal number of cooling channels for each given pressure drop value.

Figure 8 shows how the peak temperature in the conductive domain is affected by the locations of two straight cooling channels. Moving the cooling channels further from the centerline decreases the peak temperature until x_d = 2.5 mm. After that, increasing x_d increases the order of the peak temperature. In addition, increasing the pressure difference from 10 Pa to 70 Pa decreases the peak temperature around 8%. However, increasing the pressure difference from 70 Pa up to 210 Pa affect the order of the peak temperature less than 1%. Figure 8 also shows that x_d = 2.5 mm is the location where the peak temperature is minimum independent of the pressure difference value.

Figure 9 shows how the peak temperature is affected by the locations of three cooling channels. As the channels move further from the centerline the peak temperature decreases. The peak temperature is minimum when x_d = 3.5 mm. x_d value which corresponds to the minimum peak temperature has increased from two channels configuration to three channels configuration. The reason of that is x_d defined in three channels configuration is the distance in between two channels. Unlike, it was two times the distance between the cooling channels in two channels configuration. By dividing the x_d value which corresponds to the minimum peak temperature for three channels design to two, the relation between the number of the channels and their location is uncovered. x_d / 2 = 1.75 mm for three channels configuration. This shows that the cooled region by one channel has decreased from 2.5 mm to 1.75 mm, which is expected because the resistance to the fluid flow increases by increasing the number of the cooling channels (because D of Eq. (6) decreases). In addition, the effect of pressure difference has become more visible in three channels configuration. For example, increasing the pressure difference from 10 Pa to 70 Pa decreases the peak temperature around 8% and 17% for two channels and three channels configurations, respectively. The effect of increasing the pressure difference from 70 Pa to 210 has tripled (i.e. from 0.8% to 2.4%) from two channels to three channels.

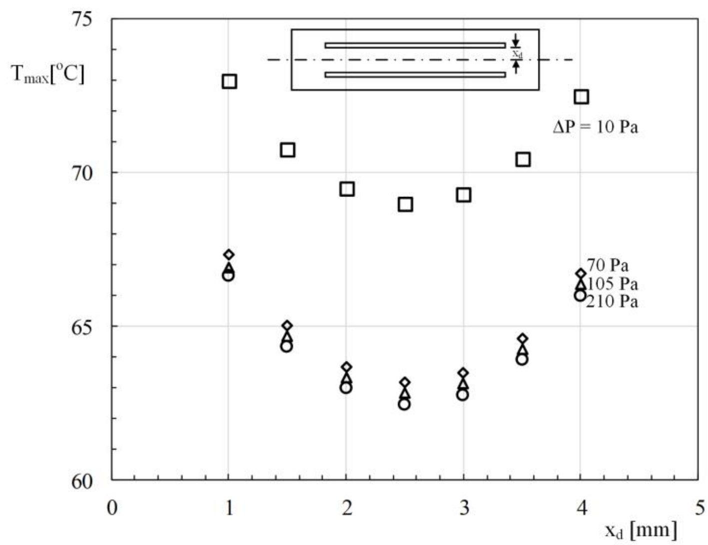

FIGURE 8: THE EFFECT OF THE DISTANCE IN BETWEEN TWO COOLING CHANNELS ON THE PEAK TEMPERATURE.

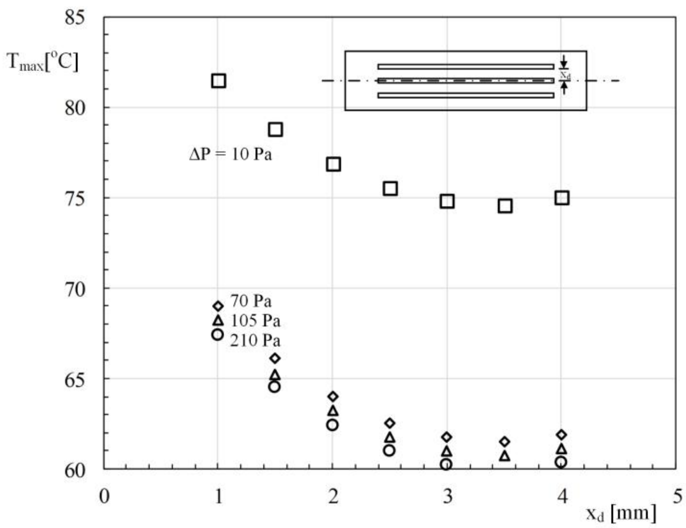

FIGURE 9: THE EFFECT OF THE DISTANCE IN BETWEEN COOLING CHANNELS ON THE PEAK TEMPERATURE WITH THREE COOLING CHANNELS DESIGN.

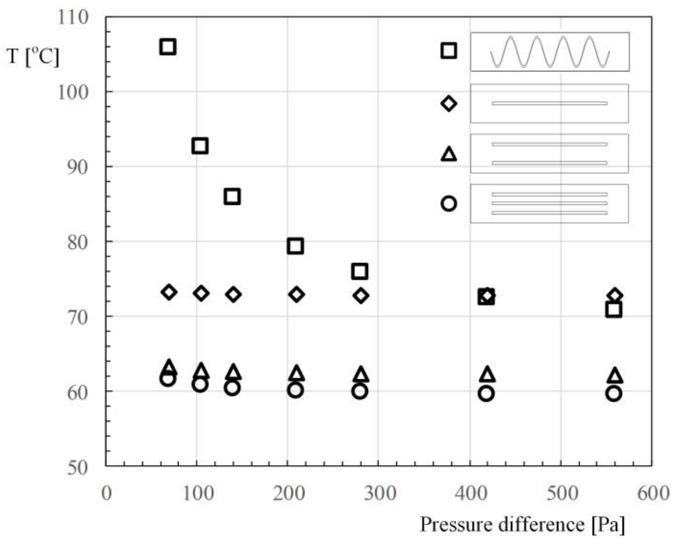

FIGURE 10: THE EFFECT OF PRESSURE DIFFERENCE ON THE PEAK TEMPERATURE WITH HEAT GENERATION FOR THE COOLING CHANNEL CONFIGURATIONS OF SINUSOIDAL, STRAIGHT AND MULTIPLE STRAIGHT.

Figure 10 summarizes how peak temperature is affected by the design. The peak temperature data corresponding to sinusoidal and straight channel was taken from Fig. 6. The two and three channels has x_d = 2.5 and 3.5 values, respectively, which are the locations corresponding to the minimum peak temperature. The solid and fluid volumes and the heat generation rate is the same for all the designs. Figure 10 shows that as the pressure difference increases changing from one channel design to sinusoidal design ensures the smallest peak temperature. However, instead of changing to sinusoidal design, changing to multiple channel configuration promises to smaller peak temperature value. However, it should be noted that the multiple channel configuration curves have the optimized locations, and Fig. 9 shows that if the location is not optimized then the order of the peak temperature is greater than both sinusoidal and one channel configurations.

CONCLUSIONS

This paper shows that a conductive domain can be cooled actively with embedded microvascular channels in it. The shape of the microvascular channels and their locations in the y direction are changed for a fixed cooling channel volume. The peak and average temperatures are the smallest with straight cooling channels located at y = 0.7 mm when the conductive domain is subjected heating from the bottom surface. The fluid flow resistance in sinusoidal channel configuration is a lot greater than in straight channel configurations when the channel volume is fixed because the diameter of the sinusoidal is smaller.

In addition, the heat generation throughout the conductive domain is studied. The peak and average temperatures are the smallest with straight channel when the pressure drop is less than 420 Pa. As the pressure drop becomes greater than 420 Pa, the peak and average temperatures become the smallest with sinusoidal cooling channel. In addition, the peak temperature is the smallest with multiple cooling channels when the pressure difference is fixed. Furthermore, the peak and average temperatures are the smallest with sinusoidal configuration when the flow rate is fixed.

This paper also shows the importance of objectives. Depending on the objective one cooling channel becomes more desirable. If the minimum peak temperature is desired than sinusoidal structure should be selected which requires great pumping power. If the cooling below a limit is desired with minimum pumping power than the straight channels should be selected. In addition, the objectives and conditions of mimicked biological structure is essential to find optimal design of an engineered structure. Because the biological structures are multi-objective such as self-cooling, self-healing, distribution of food and oxygen to cells and so on, mimicking them for structures with sole objective such as only cooling may not result with an optimal design. Therefore, the objectives, constraints and conditions should be known well in order to find the constructal bio-mimicked structure (the best performing structure for the known conditions).

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NOMENCLATURE

A amplitude, mm
c_P specific heat, J kg^{-1} K^{-1}
D thickness of cooling channels, mm
H height of conducting domain, mm

h convection coefficient, W m^{-1} K^{-1}
k thermal conductivity, W m^{-1} K^{-1}
L length of the domain, mm, Fig. 1
$\dot{m}$ mass flow rate, kg s^{-1}
n normal direction
Q flow rate, ml min^{-1}
P Pressure, Pa
G heater power, W
T temperature, K
x, y coordinates, m
x_d distance from the centerline, mm
u,v velocity components, m s^{-1}
V velocity magnitude, m s^{-1}

Greek symbols

ΔP pressure difference, Pa
ρ density, kg m^{-3}
λ wavelength, mm
ν kinematic viscosity, m^2 s^{-1}

Subscripts

amb ambient
avg average
e epoxy
f fluid
max maximum

Superscript

n index of the mesh independency test

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