



## COMBINED TRELLIS CODED QUANTIZATION/CONTINUOUS PHASE MODULATION (TCQ/TCCPM)

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### **Abstract**

In this paper, we applied Continuous Phase Frequency Shift Keying (CPFSK) to Trellis Coded Quantization/Modulation (TCQ/TCM) and thus we called Trellis Coded Quantization/ Continuous Phase Modulation (TCQ/TCCPM) for this new system. In this new scheme we use continuous phase frequency shift keying signal set instead of phase shift keying signal set. As an example, an eight state TCQ/TCCPM system is designed and its error performance is evaluated for different Rician fading parameters K and signal to noise ratios.

### **Keywords**

Trellis Coded Modulation, Trellis Coded Quantization, Continuous Phase Frequency Shift Keying.

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## 1. INTRODUCTION

Trellis Coded Modulation (TCM) [1] is a very effective modulation scheme for band limited channels. Motivated by trellis coded modulation, Trellis Coded Quantization (TCQ) was developed as a computationally efficient scheme for source coding. The main feature of TCQ is the utilization of a structured codebook with an expanded set of quantization levels based on Ungerboeck's notation of set partitioning. The design of trellis coded scalar quantization systems for memoryless and Gaussian/Markov sources is explained in detail in [2] Signal set expansion and partitioning ideas of trellis coded modulation have been applied to quantization problem to develop a source coding technique known as trellis coded quantization. For the AWGN channel, Marcellin and Fisher [2] [3] constructed a joint source/channel coding system by using trellis coding quantization and modulation. They used identical trellis for TCQ and TCM and a consistent labeling between quantization levels and modulation symbols. Since using two identical trellises separately for TCQ and TCM become unnecessarily complex, the combined form of TCQ/TCM is introduced in [4] and thus coding steps and the number of memories reduced.

Continuous phase modulation (CPM) [5] is widely used on radio channels because of its good spectral properties. The phase continuity of CPM signals improves spectral properties, and introduces memory which induces error control capability and therefore power efficiency.

CPM can be decomposed into a continuous phase encoder (CPE) and a memoryless mapper (MM) [5] The CPE is a ring convolutional encoder which produces code-word sequences that are mapped onto waveforms by the MM creating a continuous phase signal. Once the memory of CPM was made explicit, it became possible to design trellis and convolutionally coded CPM systems which allowed the trellis code and the CPE to be combined into a single joint convolutional code. In this paper we derive the BER-SNR curves of TCQ/TCCPM system in severe fading channel using Lloyd-Max and Optimum quantization levels [2].



## 2. TRELLIS CODED QUANTIZATION/MODULATION

General approach to the selection of a combined TCQ/TCM system is to assume that TCQ and TCM bit and symbol rates are equal so that the squared distance between channel sequences is commensurate with squared error in quantization. The mapping from quantization level within a TCQ subset to modulation level within a TCM subset is selected in the obvious way, so that the level/symbol order is consistent. Since the probability of a TCM error is related to the squared Euclidean distance between the allowable paths through the trellis, consistent labeling guarantees that the squared Euclidean distance in modulation symbol space is in line with mean square error in quantization.

The reproduction codebook size (i.e. number of quantization levels) is selected as  $N=2^{R+CEF}$ . There are totally  $N_1 = 2^{R+CEF}$  subset.  $N$  is chosen such that it can be properly divided by  $N_1$ , so each subset has exactly  $N_2 = N/N_1 = 2^{R-r}$  codewords. Here  $R \geq 1$  is the encoding rate in bits/sample,  $r$  and  $CEF$  are positive integers satisfying  $1 \leq r \leq R$  and  $CEF \geq 0$ . The parameter  $CEF$  stands for "codebook expansion factor", since the codebook size is  $2^{CEF}$  times of a normal  $R$  bit/sample scalar quantizer. On the branch of the trellis, both quantization levels  $q_{k,l}$  ( $k=0,1, \dots, N_1-1$ ;  $l=1,2, \dots, N_2$ ) and signal set  $S_j$  ( $j=1,2, \dots, N_1$ ) are placed using Ungerboeck's rules [1]

The overall TCQ/TCCPM scheme determines the most likely coded symbol sequence transmitted and produces the output sequence of quantization levels  $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_L)$  under the assumption that there is one-to-one mapping from quantization level within a TCQ subset to modulation symbol within a TCM subset. Throughout the paper, signal-to-quantization noise ratio (SQNR) is adopted as the performance measure,

$$SQNR = \frac{\sum_{i=1}^L E[x_i^2]}{\sum_{i=1}^L E[(x_i - \hat{x}_i)^2]} \quad (1)$$



The distortion rate function evaluated at the channel capacity provides an upper bound to the overall SQNR performance that is possible. If  $D(R)$  is the distortion rate function for the source, then the optimum performance theoretically attainable (OPTA) is derived by substituting the channel capacity  $C$  at the given signal-to-noise ratio in the source distortion rate function  $D(R)$ . In the case of Gaussian source is used, which is also under investigation here, the SQNR performance is upper bounded by

$$\text{OPTA} = 10 \cdot \log_{10} \left( \frac{1}{2^{-2C}} \right) \quad (2)$$

For the derivation of OPTA curves, it is necessary to define the channel capacity for the considered channel. In AWGN environment, the channel capacity is well known to be

$$C = \frac{1}{2} \log_2 (1 + \gamma) \quad (3)$$

where  $\gamma$  is the signal to noise ratio as  $\gamma = S/BN_0$  with the carrier power  $S$ , channel bandwidth  $B$  and the noise spectral density  $N_0/2$  [6], For the computation of channel capacity over Rayleigh fading channel, which models the microwave channel under investigation, we adopt the model introduced by Yee [7] and developed in [8].

The channel capacity for fading channels must be calculated in average sense. Since fading results in a variation of the above band limited AWGN channel on a slower time scale, the instantaneous capacity given by Eq. (3) has to be averaged over the relevant ensemble. The parameter subject to change is the carrier power or equivalently the signal-to-noise ratio. The average signal-to-noise ratio  $\Gamma$  is expressed as

$$\Gamma = \langle \gamma \rangle = \frac{\langle S \rangle}{BN_0} \quad (4)$$



Denoting  $f_{\Gamma}$  as the probability density function associated with  $\Gamma$ , the average channel capacity for the fading environment is given as,

$$C = \frac{1}{2} \int_0^{\infty} \log_2(1 + \gamma) p_{\Gamma}(\gamma) d\gamma \quad (5)$$

Since we are interested in evaluating performance degradation of TCQ/TCM system due to quantization noise, fading channel parameter estimation error and additive Gaussian noise (AWGN), we need to consider the total effects of these sources over-all system. To achieve this goal, the noise effect must be carried to the output of the Rician channel [6]. Then the modified quantization noise variance  $\sigma_{qm}^2$ , becomes as in [6],

$$\sigma_{qm}^2 = \rho^2 \sigma_q^2 \quad (6)$$

where  $\rho$  is fading parameter, and  $\sigma_q^2$  is quantization variance. Based on Eq. (6), the total variance of of these noise sources can be written as,

$$\sigma_i^2 = \sigma_{qm}^2 + \sigma_{\rho}^2 + \sigma_v^2 \quad (7)$$

where,  $\sigma_{\rho}^2$  is variance of the fading parameter,  $\sigma_v^2$  is variance of AWGN noise. Then the modified signal to noise ratio  $SNR_m$  is then defined as.,

$$SNR_m = \frac{\sigma_x^2}{\sigma_i^2} \quad (8)$$

where,  $\sigma_x^2$  is the variance of the input sequence.



### 3. CONTINUOUS PHASE MODULATION

In [5] Rimoldi derived the tilted-phase representation of CPM, with the information bearing phase given by

$$\Phi(t, \alpha) = 4\pi h \sum_{i=0}^{\infty} \alpha_i q(t - iT) \quad (9)$$

The modulation index  $h$  is equal to  $K/P$ , where  $K$  and  $P$  are relatively prime integers.  $\alpha$  is an input sequence of  $M$ -ary symbols,  $\alpha_i \in \{0, 1, 2, \dots, M-1\}$ .  $T$  is the channel symbol period.

The phase response function  $q(t)$  is a continuous and monotonically increasing function subject to the constraints

$$q(t) = \begin{cases} 0 & t \leq 0 \\ 1/2 & t \geq LT \end{cases} \quad (10)$$

where  $L$  is an integer. The phase response is usually defined in terms of the integral of a frequency pulse  $g(t)$  of duration  $LT$ , i.e.,  $q(t) = \int_{-\infty}^t g(\tau) d\tau$  for full response signaling  $L = 1$ , while for partial response systems  $L > 1$ . Finally the transmitted signal  $s(t)$  is given by

$$s(t, \alpha) = \sqrt{2E_s / T} \cdot \text{Cos}(2\pi f_c t + \Phi(t, \alpha) + \Phi_0) \quad (11)$$

Here  $f_c$  is the carrier frequency,  $E_s$  is the energy per channel symbol and  $\Phi_0$  is the initial carrier phase.

#### 4. TCQ/TCCPM SYSTEM MODEL

As shown in Fig.1 the transmitter of the TCQ/TCCPM system consist of convolutional encoder (CE), continuous phase encoder (CPE) and a memoryless mapper (MM). In this system CE encoder realizes both quantization and coding. In order to the CE and CPE to be combined into a single trellis, the number of CPE inputs must match the number of outputs of the CE. For rate  $k/n$  CE's, an equivalent CPE of rate  $n/n(L+1)$  is needed where L is the number of the memory elements of CPE.

In the every modulation interval, analog signal that is coming to CE, which has  $m_{CE}$  memory elements, is once quantized then coded by CE. Output bits of the CE are input to the CPE. The total number of memory elements in the joint code is  $m = m_{CE} + L$ . The  $n(L+1)$  CPE output bits are then mapped to TCQ/TCCPM signals, after that these signals are sent to channel. In the channel, signal is faded and noise is added on it. Thus, with only one TCQ/TCCPM structure, both quantization and coding and continuous phase modulation have been realized. Besides, the complexity of the system has been decreased and the solution has been done in one step. At the receiver side, the received noisy signals first demodulated and then the process of decoding is done to get the estimation of original data bits.

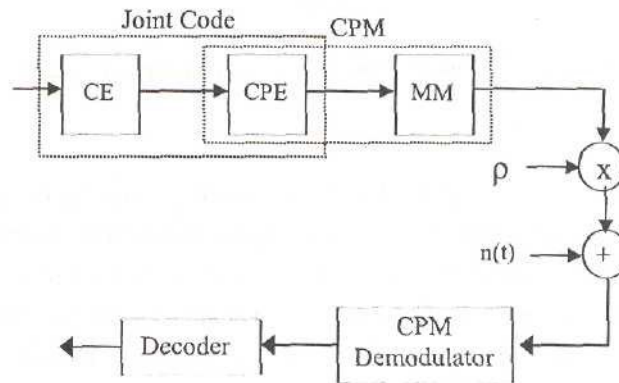


Fig. 1. TCQ/TCCPM system block diagram



**5. SIMULATION RESULTS**

In order to search the error performance of our system we designed an eight state TCQ/TCCPM system with a rate of 1/2 with two memory elements CE and a rate of 2/4 with one memory element CPE which is shown in Fig. 2. The rate of joint code is 1/4. Four output bits of CPE are then mapped to the constellation points, represented by  $n$  vectors of length 16 ( $h=1/2$  16 CPFSK). The fading coefficient  $\rho$  is multiplied and zero mean additive white Gaussian noise with power spectral density  $N_0/2$  is added to each vector component. Then the noisy vectors input to the Viterbi decoder, which its output is the estimated data of the information bits. The decoding step of Viterbi decoder is chosen as 10.

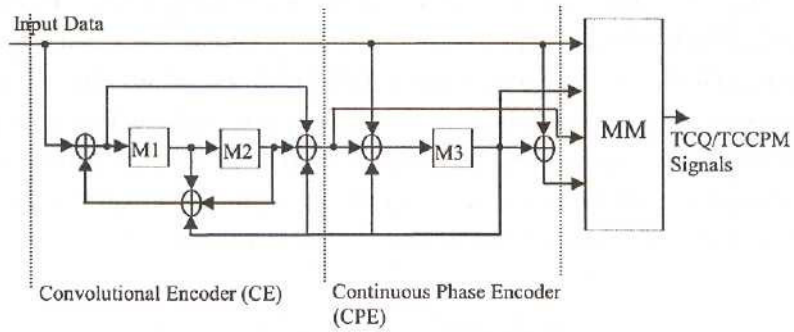


Fig. 2. TCQ/TCCPM Scheme for 16 CPFSK,  $h=1/2$

The system consists of an eight state TCQ/TCCPM scheme. On the branches of the proposed combined structure, there is one-to-one correspondence between the signal set and quantization levels. Two adjacent branches, each of which contains two parallel transitions, emanate from each state. On the transitions of the trellis, quantization levels  $q_{k,l} \in Q_k, k=0,1,2,3; l=1,2$  and modulation symbols  $s_j, j=0,1...15$  are placed as in Fig. 3.



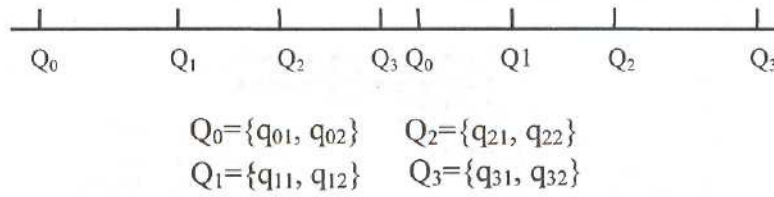


Fig. 3. Quantization levels for 16CPFSK

SQNR-SNR performance for the considered system is investigated through the computer simulation and  $SNR_m$  is defined as a function of quantization noise and channel parameter estimation error given in Eq. (8). For each  $SNR_m$  value,  $10^6$  samples, which are generated from a memoryless Gaussian source of zero mean and unit variance, are processed by the 8 state 16-CPFSK TCQ/TCCPM encoder system (Fig. 2). In Fig. 4, SQNR performance curves are illustrated for Rician fading channel while Lloyd-Max and optimum quantization levels are employed. The utilization of optimum quantization levels improves the performance compared to that of Lloyd-Max levels are used. The effects of the quantization noise for both quantization levels are shown in Fig. 4 for different fading channels.

Fig.5 shows the bit error performance of TCQ/TCCPM system in fading channel while  $K=0$  in another the channel is Rayleigh when optimal and Lloyd-Max quantization levels are used. Fig.6 shows the

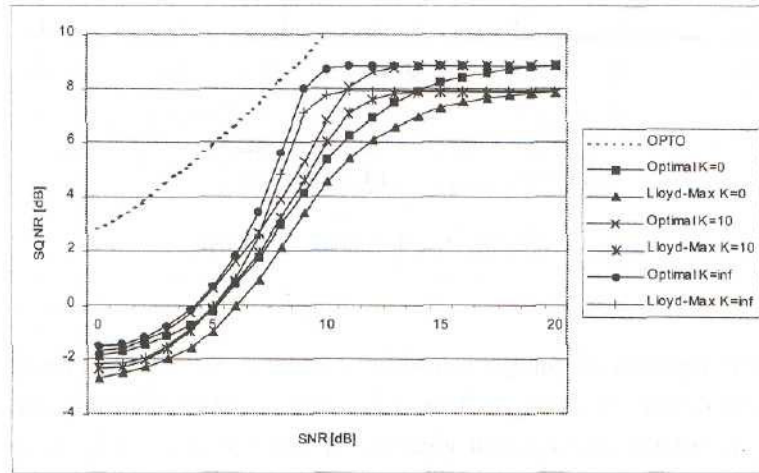


Fig. 4 SQNR-SNR relations

bit error performance of TCQ/TCCPM system while K-10 when optimal and Lloyd-Max quantization levels are used and finally Fig.7 shows the bit error performance of TCQ/TCCPM system in AWGN channel when optimal and Lloyd-Max quantization levels are used. If we compare these results to the 8PSK TCQ/TCM, we can see that our proposed system TCQ/TCCPM has approximately 6dB gain in every fading channels.

## VI. CONCLUSION

In This paper, combined trellis coded quantization/continuous phase modulation (TCQ/TCCPM) is investigated as an alternative to Trellis coded quantization/modulation (TCQ/TCM). The main contribution of this paper is to demonstrate the performance of the proposed system in several fading environments. As a conclusion, we show that the error performance of TCQ/TCCPM system is very good and has approximately 6dB gain compare to TCQ/TCM.

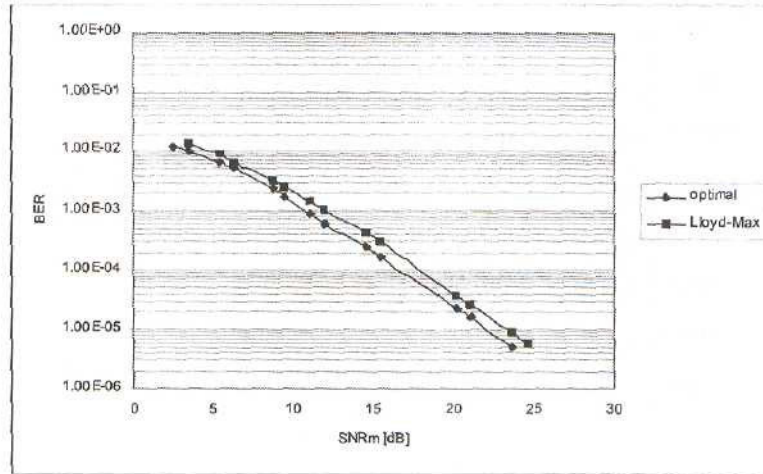


Fig. 5. Error performance of TCQ/TCCPM system in fading channel for K=0dB

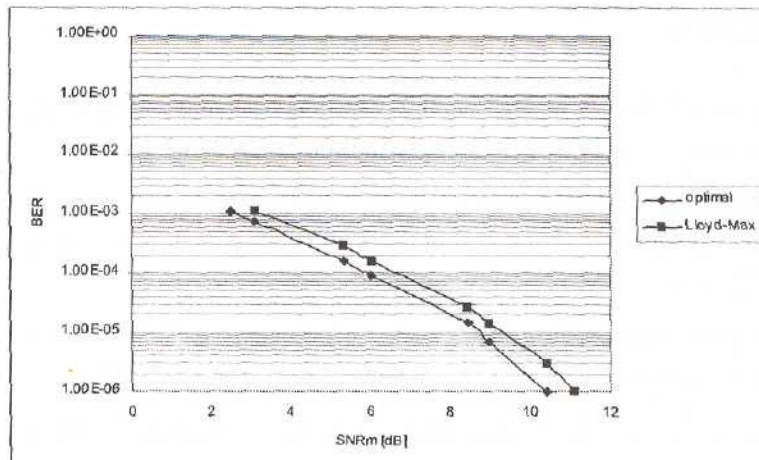


Fig. 6. Error performance of TCQ/TCCPM system in fading channel for K=10dB

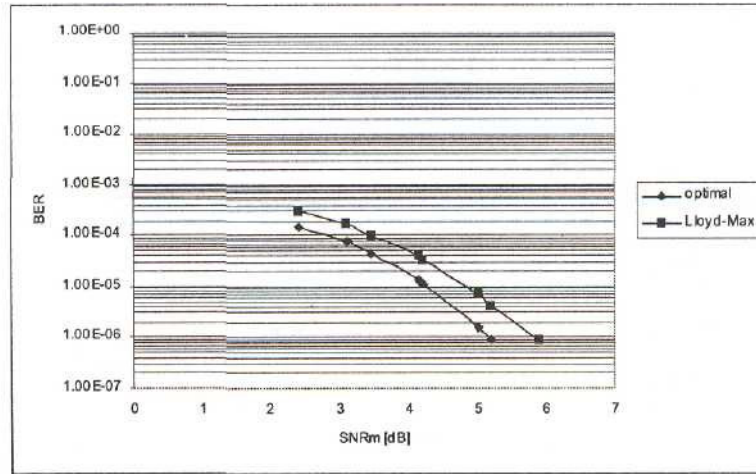


Fig. 7. Error performance of TCQ/TCCPM system in fading channel for  $K = \infty$ dB

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