Investigation of the Development of 7th Grade Students' Skills to Define, Construct and Classify Polygons with Cabri Geometry

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Abstract

The aim of the study is to investigate the development of 7th Grade students' skills to define, construct and classify polygons in geometry course with Cabri Geometry II Plus software geometry, an example of dynamic geometry software. The study used qualitative and quantitative research methods in accordance with the research objectives and focus, so it was designed as a mixed method research. The participants of the study were 21 7th Grade students, 11 girls and 10 boys, who were attending a secondary school in Eskişehir city center during 2012-2013 school year. As a source of qualitative data, four students in this class were selected for the interview. The data were collected with "Polygon Identification and Classification Scale", one group pre-test and post-test in order to determine the level of development and significance level of the gender variable, and Cabri Geometry worksheets developed by the researchers. The quantitative data were analyzed with SPSS Statistics 20. Also, t-test and Wilcoxon test were used in data analysis. The data obtained from the interviews were analyzed through descriptive analysis. The qualitative data showed that the mean of correct answers given by the students to the questions in the Polygon Identification and Classification Scale was higher in the post-test than the pre-test. The t-test results for the pre-test and post-test mean scores and the results of the paired samples test showed a significant difference in favor of the post-test. There was no significant difference based on the gender variable. On the other hand, the data obtained from the interviews were coded under five different themes. The activities about the concept of formation showed that incorrect formations caused incorrect generalizations about the shapes. The study found that, as a result of the teaching practice in the study, hierarchical relations among polygons were expressed correctly. Finally, after the practice, the participants succeeded in defining polygons with their own words.

Keywords: *Geometry; polygons; Cabri Geometry; geometric construction*

Introduction

Mathematical thinking involves and develops skills that the individual needs to be successful in his or her daily and professional life such as problem-solving, analyzing, classifying similarities and differences, generalizing, abstract thinking, thinking about possible consequences, and creative thinking (Driscoll, 2007). One of the important research fields in mathematics education that aim to maximize mathematical thinking is geometry. Geometry is one of the fundamental and conceptual cornerstones of mathematics. Geometry is a study area that requires improved skills of visual perception and analysis and developing skills to see relationships between objects, classify and make modern designs.

In addition to helping establish relationships between the geometric structures in geometry universe and the branches of mathematics, geometry also facilitates efficient use of knowledge acquired through the topics of geometry in problem-solving, daily life and other school subjects (Tutak & Birgin, 2008). The effects of dynamic geometry software in teaching-leaning process have been a popular research topic over the recent years. This is because these kinds of software promotes learning through discovery contribute to students' development of problem-solving skills (Ubuz, Üstün & Erbaş, 2009). Also, dynamic geometry software allows for presenting geometric concepts at different levels. According to Güven and Karataş (2009), these concepts are examined under two stages: "finding – discovery" and "proof – verification". Gillis suggests that dynamic geometric software developed to change this structure allows students to construct geometric shapes, perform measurements, change forms and shapes, and discover geometric concepts (Gillis, 2005). Some properties of geometric structures may be independent while we can also often see parameters changing depending on each other and it is only through discovery that students are able to describe this situation and to abstract.

According to Jones, Fujita and Kunimune (2012), although many mathematical objects have geometric shapes that is given meaning in the mind, geometric shapes contain a mathematical essence. In this regard,

- it should be possible to construct geometric representations on paper or computer screen.
- a representation is supposed to be in two states: "finite and various forms" and "ideal objectivity".
- for example, some representations like a square is more peculiar than others. They assist in the process of problem solving.

Geometric thinking need to be supported with different techniques and strategies in the process of teaching geometry. Also, proof skills should be supported and different methods should be recommended. Several studies have shown the positive results of them (Jones, Fujita and Kunimune, 2012).

Problem-solving techniques have gained different aspects in life situations developing and changing with technology. A variety of tools for geometry can be used in teaching and learning activities. Dynamic geometry environments is an example of these tools.

Dynamic Geometry Software

According to National Council of Teachers of Mathematics [NCTM], technology allows teachers to assess the process as well as the product in problem situations and to enrich application processes by examining them in the light of this data. Using dynamic geometry software, students can examine the characteristics of shapes, study the slope and linear relationships, use representations and conduct physical experiments (NCTM, 2000). Dynamic geometry software allows for making significant transformations and makes it possible to experiment with geometric objects. Using concrete models, drawings, and using dynamic geometry software allow students to establish relations with geometrical ideas effectively. By means of well-planned activities, proper tools and support from teachers, students can create their own assumptions and test them. Cabri Geometry, an example of dynamic geometry software, allows students to come up with their own constructions.

Cabri Geometry Software

Cabri Geometry software facilitates defining and comprehending the relationships between shapes helps them acquire meaningful information and relationships. In addition, Cabri facilitates using original methods, making assumptions, and testing assumptions. It allows students to understand the important points about a problem or theorem and to carry out more detailed studies (Pandiscio, 2002). Dynamic geometry software like Cabri Geometry include some stages of geometry such as being deductive, implicational and practical steps in teaching process (Straesser, 2002). A strong connection to be established between drawing and theoretical infrastructure plays an important role in teaching geometry. Köse claims that Cabri Geometry software facilitates understanding the mathematical infrastructure of objects by exposing their properties (Köse, 2008).

This dynamic software allows students to construct, move, rotate and resize shapes. By using the dynamic features of the software in their drawings, students examine the changing or constant conditions in shapes. In this way, they can explore the constant and changing properties of constructions and test the accuracy of assumptions and predictions (Köse, 2008). In addition, it provides teachers with concrete feedback about students and lesson plans.

Aim and Significance

The aim of the study is to investigate the development of 7th Grade students' skills to define, construct and classify polygons in geometry course with Cabri Geometry II Plus software geometry. Thus, the study seeks answers to the following questions about the development of 7th Grade students' skills to define and classify polygons:

1. How do they construct polygons with Cabri Geometry?
2. What strategies do they use when constructing polygons with Cabri Geometry?
3. How do they establish relationships between the constructed with the help of Cabri Geometry and the features of polygons?
4. What kind of verbal expressions do they use when expressing the polygon, triangle and quadrilateral constructed with Cabri Geometry?
5. What kind of verbal expressions do they use when making a hierarchical order of polygons, triangle and quadrilateral constructed with Cabri Geometry? What features do they take into consideration when making a hierarchical order of polygons?

Research has shown that students have problems in understanding and classifying the relationships between mathematical meanings. In recent years, dynamic geometry software has been one of the main tools used in geometry class. This study aimed to observe the process of geometric thinking in students, obtain in-depth knowledge about concept formation and explore students' skills to create and classify formations when faced with constructions.

Research suggests that, in environments using dynamic geometry software, students can improve their understanding of mathematical concepts and the software helps them come up with deductive reasoning. Dynamic geometry software aims to develop visualization, exploring and mathematical ideas. It also facilitates the development of problem solving skills. Activities and problems prepared with Cabri Geometry II Plus provide students with the opportunity to use their skills and creativity and, by means of its dynamic feature, it facilitates viewing the problem or a formation from different angles. This was the reason why this software was preferred for the practice session.

Methodology

The study used qualitative and quantitative research methods in accordance with the research objectives and focus, so it was designed as a mixed method research. Mixed methods researches contains a mixture of qualitative and quantitative research methods or paradigms (Johnson & Christensen, 2004). Mixed research designs involve collecting, analyzing and interpreting qualitative and quantitative data regarding the same basic problems in a study or a series of studies (Leech & Onwuegbuzie, 2007).

Quantitative Method

The study used one group pre-test and post-test in order to determine the level of development and significance level of the gender variable in order to collect data. A group is administered independent variables in the one group pre-test and post-test model.

In this model, data is collected from a single group in the study for the period before the study (pre-test), then the practice stage is performed and, finally at the end of the practice stage, the same group is tested again. If the numerical data obtained indicates a significant difference, this difference is considered to be caused by the practice (Cemaloğlu, 2012).

The participants of the study were 21 7^{th} Grade students, 11 girls and 10 boys, who were attending a secondary school in Eskişehir city center during 2012-2013 school year. The participants for this study were selected as 7^{th} Grade students because the topic of Polygons is covered extensively in the 7^{th} Grade mathematics curriculum.

Qualitative Method

With respect to the qualitative aspect of this mixed methods research, the study was designed as a teaching experiment to examine the development of skills to define, construct and classify polygons with Cabri Geometry software. Teaching experiment is a method that allows researchers and teachers to observe the nature of mathematical thinking and the development of ways of thinking (Czarnocha & Maj, 2008). In addition to observing classroom environment and monitoring teaching and learning process, teaching experiment also facilitates the development of teaching and learning activities (Czarnocha & Prabhu, 2006). The participants chosen with respect to the qualitative aspect of the study, were selected based on achievement in mathematics course, achievement in the scale and interest in computer course. It should also be noted that the researchers attended both mathematics and information and communication courses throughout the school year. The students selected for the interviews were given names similar to their actual names. Table 1 shows the scores received by the interviewed students.

Table 1. Qualitative Study Participants

	Mathematics 1st Exam. Scores	**Pre-test Scores**	**Post-test Scores**	**Interest in computer**
Kerem	88.0	78,8	90.9	good
Nilüfer	76.0	76,0	81,8	good
Burak	70.0	60,6	84,8	very good
Çiğdem	67.0	61,0	63,6	medium

The public school where the study was conducted had a total of 240 students, 13 classrooms, 21 teachers and one computer lab. The computer lab, the computers and other technical equipment used in this lab met all the software and hardware needs.

Data Collection and Analysis

The data were collected with the Polygons Identification and Classification Scale, the worksheets developed by the researchers, the interview protocol, computer logs and the researcher's journal. The quantitative data were analyzed with IBM SPSS 20, Wilcoxon test and t-test.

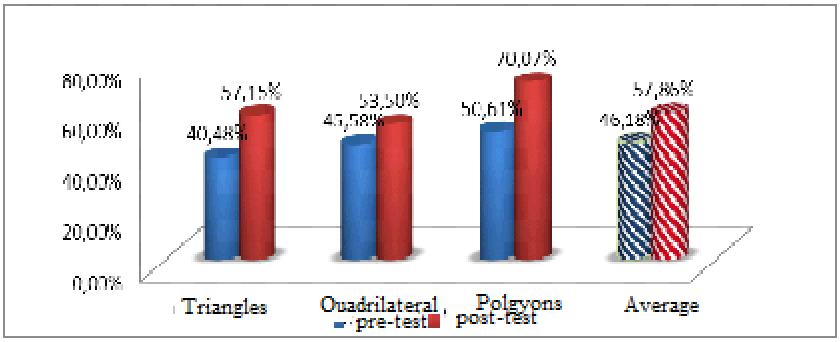
In the analysis of the qualitative data, the qualitative data sets obtained through the interviews were created and analyzed with descriptive analysis. The qualitative data obtained were coded and analyzed by two field experts according to the pre-determined themes. The descriptive analysis included direct quotes from the statements made by the individuals interviewed or observed in order to reflect their views effectively. The reason for this is to describe the data in a systematic way and present the reader with arranged and interpreted data. Then these are explained and interpreted, cause-effect relationships are examined and results are obtained. Interpretation stage involves establishing relationships among the emerging themes, explain their meanings and making future predictions (Yıldırım & Şimşek, 2008). The qualitative data analysis in this study was based on four stages: Stage 1: The data were written in the interview data transcription form, Stage 2: The interview coding key was formed, Stage 3: The interview data were coded, and Stage 4: The codes were compared and the reliability was tested (Köse, 2008).

In the first stage, the interviews conducted with four students were transcribed into the interview form. In the second stage, the theoretical base of the study and the interview questions formed according to it were taken into consideration when forming the coding key. Two researchers together formed the coding keys of the interview forms in which the descriptive data were written and the other parts were left blank. In the third stage, the interview forms of the four students were marked for each of the questions by the researchers independently and, in the final stage, the codes of the two researchers were compared and the reliability of the study was improved. The reliability of the study was calculated by using Agreement/Agreement + Disagreement formula (Miles & Huberman, 1994). In the reliability tests conducted between the researchers, the percentage of reliability was found to be above 70%. This percentage indicates that inter-rater reliability was achieved between the researchers.

Findings

Table 2 shows the pre-test and post-test means based on the topics as a part of the quantitative part of the study. As can be seen in the table, the mean of the correct answers given by the students to the questions in the Polygon Identification and Classification Scale increased by **11.68%** after the teaching practice in comparison with the mean of the scores before the practice. Also, it was found that the mean of each of the groups increased when the questions in the scale were examined in three groups.

Table 2. Pre-test and Post-test Means Based on the Topics



As can be seen in Table 3, the t-test results for the pre-test and post-test mean scores and the results of the paired samples test showed a significant difference in favor of the post-test. This indicates that Cabri Geometry applications, an example of dynamic geometry software, has a positive impact on student achievement.

Table 3. t-Test Results for the Pre-test and Post-test Mean Scores Paired Samples Test

Test	**N**	**X**	**SS**	**Sd**	**t**	**p**
Pre- test	21	15,24	5.55	20	- 5.014	.000
Post- test	21	19,19	4,99			

In Table 4, according to the t-test results of the Polygons Identification and Classification Scale Scores Based on Gender, the female students' performance increased slightly more than that of the male students. However, according to the t-test results, this difference was not a significant one. Thus, Cabri geometry applications did not create a significant difference based on the gender variable. The development levels of the female and male students in the study sample were found to be in parallel to each other.

Table 4. t-Test Results of the Polygons Identification and Classification Scale Scores Based on Gender Independent Samples Test

Gender	**N**	**X**	**SS**	**Sd**	**t**	**p**
Girl	11	16,18	5.00	19	0.81	.43
Boy	10	14,20	6.20			
Girl	11	20,36	4.25			
Boy	10	17,90	5,63	19	1,14	.27

With respect to the qualitative aspect of the study, the interviews were structured based on the cases of the formation of polygons, knowledge of their properties, expressing polygons and arranging polygons in a hierarchical order.

The data obtained were categorized into the following themes: *Using Tools for Parallel and Perpendicular Lines, Constructions Based on Other Geometric Structures, Distinctive Constructions and Strategies, Expressing a Polygon with Their Own Words, Identification* and *Hierarchical Order of Polygons*. Figure 1 shows the relationships among the themes.

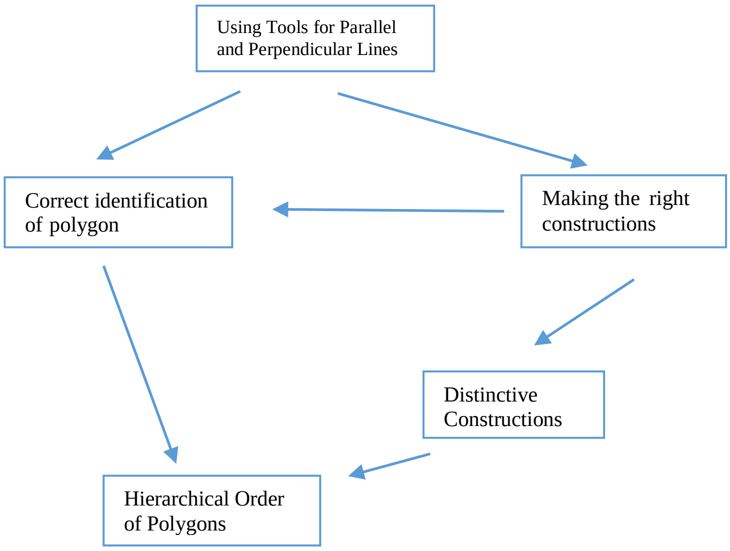


Figure 1. Relationships among the Themes

Using Tools for Parallel and Perpendicular Lines: According to the data obtained from the interviewed students, the students did not pay attention to using tools to construct parallel and perpendicular lines. An example of this was observed with Burak when he was supposed to use parallel lines when constructing a trapezoid (Figure 2).

Researcher : *I want you to draw a trapezoid.*
Burak : *OK.*
Researcher : *Do you remember how I did that in the lesson?*
Burak : *No.*
Researcher : *If you do it in this way, when you draw it, I hold it on a corner and move it. And the shape won't be correct then. We drew two parallel lines for that in the lesson.*

i) ii)

Figure 2. Burak Used Random Points in the Plane

As shown in Figure 2, Burak was not able to remember trapezoid construction. When he was told that he needed to use two parallel lines, he used the parallel line tool and took the lines as perpendicular in the plane. Thus, Burak did not make the mistake made by many students due to their misconception that lines should always be horizontal.

Constructions Based on Other Geometric Structures: This theme was about whether the interviewed students constructed the required polygon by using other elements (parallel lines, perpendicular lines, circles, rotation, translation, etc.) or by constructing a shape similar to the quadrilateral shape in their minds.

An example of this was observed in the interview with Kerem about the process he followed in square formation.

Researcher	:	*Now, I want you to construct a square.*
Kerem	:	*I can use the circle. Also, I can do that with 90º rotations*
Researcher	:	*There may be more than one way. Can you show me how you can do it with the circle?*

i) ii) iii) iv)

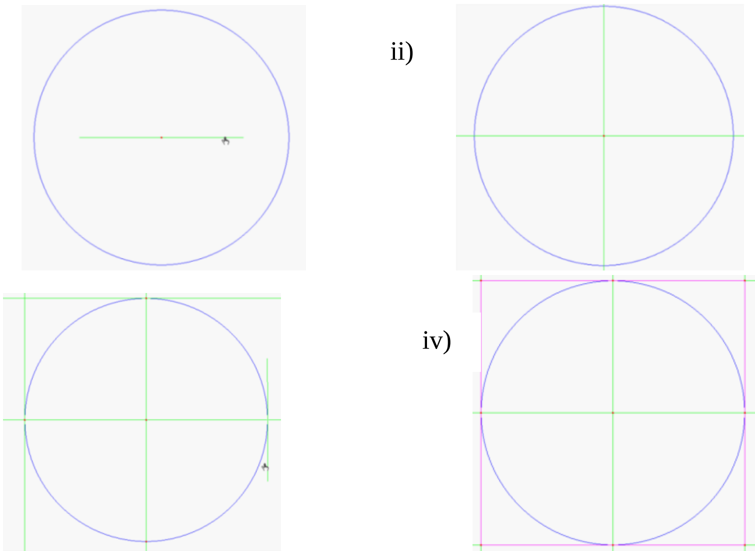


Figure 3. Kerem’s Construction

As can be seen in Figure 3, (i) Kerem took a horizontal line passing through the center of the circle, (ii) he took another line perpendicular to the vertical line and (iii)-(iv) he drew the tangent lines passing through the points where these two lines cut the circle. Thus, the student made use of different geometric structures in square construction and used the circle and perpendicular lines together.

Distinctive Constructions and Strategies: This theme was about distinctive constructions performed by the students when constructing the required polygons. During the practices, the methods presented in the worksheets or their own distinctive products were examined. For example, Nilüfer constructed a rectangle and a square in the same shape and she seemed to have mastered both constructions.

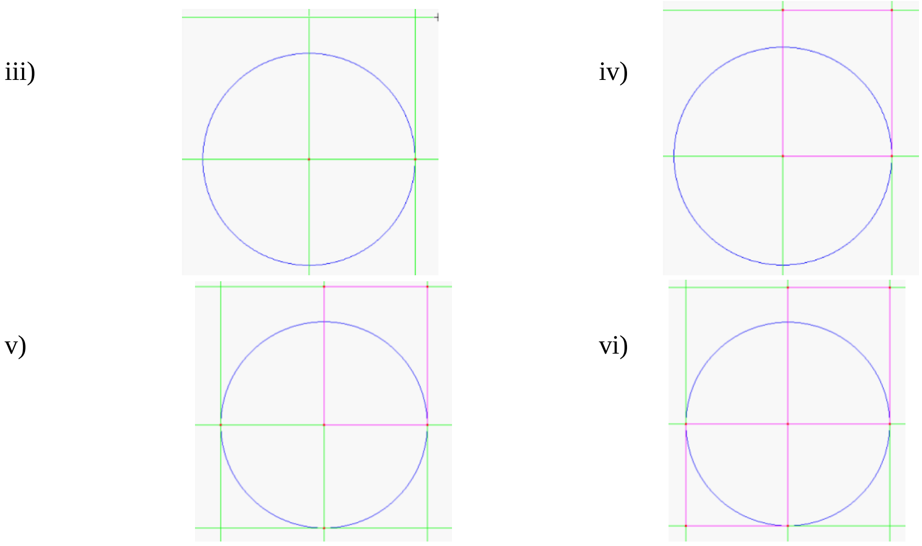


Figure 4. Nilüfer's Construction.

In Figure 4, (i) Nilüfer took a circle in the plane and then (ii) she took a horizontal line passing through the center of the circle and another one perpendicular to that horizontal line. (iii) by drawing the tangent lines passing through the points where these two lines cut the circle, she took the fourth line segment outside the shape. (iv) She constructed the rectangle. After that, (v) she extended the tangent lines and constructed the square. Thus, (vi) she constructed a square and a rectangle in the same shape in a dynamic way. In the interviews conducted with other students, four of the students were able to form 26 complete constructions out of 32 constructions in total and they constructed nine of them with distinctive methods different from what they had practiced in lesson. Also, the students who did not make any construction mistakes generally preferred distinctive methods while the students who had difficulty in constructing a shape tended to recall the practices in lesson. Kerem performed seven complete constructions out of eight constructions and he developed his own strategies in four of them. Nilüfer and Burak did not make any construction mistakes and they used their own distinctive strategies in two constructions. Çiğdem, on the other hand, performed only three complete constructions out of eight constructions and she used a strategy different from the practices in lesson in only one of them.

Expressing a Polygon with Their Own Words: After the required constructions about polygons were completed, the students were asked the question "How do you explain what this polygon is to someone who does not know it?" in order to observe the students' knowledge of the properties of these polygons such as how the students would take the vertex points, how they would not focus on the sides and whether they would use the concept of closed region or not. Figure 5 shows the triangle drawn by Çiğdem during the interview.

Researcher	:	*How can you explain what a triangle is to someone who does not know it?*
Çiğdem	:	*I explain that a triangle has three sides and three vertexs, they are scalene and the angles in a triangle equal 180°. Figure 5. (Çiğdem drew a triangle based on what she told about a triangle.)*

Researcher	:	*How can you explain what a quadrilateral is to someone who does not know it?*
Çiğdem	:	*It is a shape with four sides and opposite sides parallel.*
Researcher	:	*So you say its opposite sides are parallel. How do you know that its opposite sides are parallel?*
Çiğdem	:	*Because they never intersect each other.*
Researcher	:	*Do you think all things that do not intersect each other are parallel?*
Çiğdem	:	*Yes.*
Researcher	:	*OK. Well, can you move that shape?*
Çiğdem	:	*(Nodding, she realized what she had thought at the beginning was wrong.) It is not proper when I move it. They seemed to be parallel at first.*

In Figure 5, Çiğdem drew a quadrilateral using the polygon tool and performed the measurements.

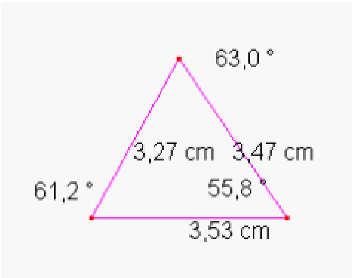


Figure 5. Çiğdem's Quadrilateral

Çiğdem failed to notice that the shape with seemingly parallel lines was actually dynamic and she believed that these lines were parallel and would never change. This student knew about the rule of parallelism correctly and she was confident about that, but she didn't realize her mistake about quadrilaterals. During the dialogue given above, Çiğdem realized that what she had thought at the beginning was wrong and she changed her behavior about this subject (Figure 5).

Identification and Hierarchical Order of Polygons: The interviewed students were asked questions requiring hierarchical order of polygons. Based on the students' constructions, more special and more general statements about polygons were obtained. In addition, the constructions were examined in the following order: "Scalene triangle – Isosceles triangle – Equilateral triangle, Quadrilateral – Trapezoid – Parallelogram – Rectangle – Square". The students' responses were examined in the same order, too. As an example, Nilüfer made a correct hierarchal order of triangles in the interview when she was asked to perform dynamic geometry applications.

Researcher	:	*Which of the three triangles on the screen do you think is the most special one?*
Nilüfer	:	*The scalene triangle.*
Researcher	:	*Why?*
Nilüfer	:	*Because all of its angles and side lengths are equal.*
Researcher	:	*OK, which one comes after the scalene triangle?*
Nilüfer	:	*The isosceles triangle.*
Researcher	:	*Which of the triangles would you prefer to teach if you were a teacher?*
Nilüfer	:	*First, the scalene triangle. Then the isosceles triangle. And finally, the equilateral triangle.*

Nilüfer thought that the features of an equilateral triangle would apply to the other triangles and she stated that if she had organized them in a presentation, she would have organized them according to the hierarchical order. After completing the quadrilateral, trapezoid and parallelogram constructions, the same student gave correct answers to the questions about the hierarchical order and she stated that the most general one was the quadrilateral while the most special one was the parallelogram. Kerem thought that the square was the most special one and what distinguished it from the other quadrilaterals was its side lengths (the rhombus was not used in the practice). He also mentioned the rectangle's four right angles. Regarding the difference of the rectangle from the other quadrilaterals, Burak focused on the side lengths of the shape and thought of the concept of parallelism only about the side lengths of the parallelogram. He also added that the square was more special than the rectangle because all of its sides are equal. Finally, Çiğdem stated that the square had more features than the other quadrilaterals and this was similar to the situation of the equilateral triangle about the classification of triangles.

Conclusion and Implications

- All of the students interviewed used the circle for the square construction. Also, the students were observed to tend to use the circle in the isosceles and equilateral triangle constructions. The study found that the fact that the students who performed successful constructions used different elements or followed different methods in the process of constructions were not effective in explaining the features of the shape.
- The students interviewed performed 81.25% (26/32) of the constructions correctly. Out of these correct constructions 34.6% (9/26) were based on distinctive strategies independent from the construction approaches presented in the practices. Thus, it could be suggested that the teaching-learning process based on Cabri Geometry software has a positive impact on the students' original thinking and creative skills.
- The study found that the circle used to construct an isosceles triangle limited the features of the isosceles triangle and misguided the students when the dragging feature of the program was used. The students who completed the construction with this strategy observed the side features of the isosceles triangle they obtained but they generalized the angle features incorrectly. The process of studying with the Dynamic Geometry software was so convincing that the students who participated in the interview sessions and performed their constructions with this method gave correct answers to the questions about the isosceles triangle in the scale but they were affected by this incorrect generalization in the interview sessions. As Laborde (2003) reported, this situation shows the importance of planning dynamic geometry applications at this stage.
- Analysis of the answers given by the students to the questions about how they would define scalene triangle and quadrilateral structures with Cabri Geometry showed that they emphasized vertex points, sides, and closed shape concepts and they used appropriate expressions. The study found that Cabri Geometry applications had a positive impact on the students' definition skills. After the practice with dynamic geometry applications, the students preferred to make mathematical explanations about polygons instead of making definitions. This result is similar to the one reported by Jones (2001).
- The dragging feature of the software played a role in the students' explanations. The study found that, as a result of the practice conducted with Cabri Geometry, the students expressed the hierarchical orders of polygons appropriately. When making the hierarchical orders, the students developed strategies such as "focusing on angle and side features", "ordering from easy to difficult", "taking the development of structures into consideration", "assigning a higher position to structures with more features in the hierarchical order".
- Also, the students compared the square, the most special type of quadrilaterals, and the equilateral triangle, the most special type of triangles. The students explored these two constructions, on top of the hierarchical order of their own groups, and compared their

common features. The results in this study were in parallel to the development of the Q-levels by Fujita (2008).

- In the light of the results, it could be suggested that the teaching-learning process regarding the subject of polygons should involve dynamic geometry software applications.
- This study showed the importance of students' preliminary knowledge in teaching geometric concepts. The students with the required preliminary knowledge were found to be more successful in the practices.
- Thus, practitioners need to take students' preliminary knowledge into consideration when designing and implementing these activities. Cabri Geometry applications should include practices in which they can use their own strategies to promote students' creativity. Students need to be introduced to the concept of geometric construction and they need to be able to define these constructions.
- The study found that the students inferred the features of polygons from the features of the elements in the construction and they developed mathematical perspective in this way. The relevant practices need to be prepared by taking this into account.
- The hierarchical relations among polygons play a key role in teaching geometric concepts. For this reason, students need to be provided with activities designed to encourage students to form the relationships among polygons.
- In conclusion, this study clearly demonstrated the effect of Cabri Geometry software on students' skills to define and classify polygons.
- Future research can potentially address the effect of different geometry software applications on these skills.

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